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GOLD, LIQUIDITY AND SECURED LOANS

IN A MULTISTAGE ECONOMY

PART II: MANY DURABLES LAND AND GOLD

by

M. Shubik and S. Yao

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1. INTRODUCTION

In a previous paper (Shubik and Yao, 1988) we examined a multistage exchange economy with  $m$  perishable goods and one infinitely durable gold used as money. We considered an economy without credit and one with one hundred percent secured loans. In this paper we consider an economy with  $m_1$  goods which have finite lives and  $m_2$  goods which are of infinite durability. Historically the two durables which have been prominent in economic activity have been gold and land, although one might wish to include platinum and some other items.

The proofs of existence of equilibrium for the finite and infinite horizon and existence of a stationary state for the infinite horizon model (under appropriate circumstances) are somewhat more complicated than in the strategic market games with only perishables and gold. However with the introduction of many durables several basic new phenomena of economic interest appear which illustrate the relationship between money and other stores of value. In particular in this paper we consider several phenomena which depend upon the presence of durables. They are: (1) the distinction and

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difference in efficiency between asset and rental markets; (2) the difference in the possibility for the existence of a stationary state with rental or asset markets; (3) the meaning of and distinction between the conditions for 100% gold or money backed loans and 100% general asset backed loans; (4) the use of assets as a store of value and (5) the possibilities for and distinctions among fiat money, 100% land backed money and 100% gold backed money.

## 2. TRADE WITHOUT CREDIT

### 2.1. Spot, futures, asset and rental markets

The general equilibrium model of exchange does not call for the economist to be specific in counting the number of markets in existence. All markets are implicitly present. If a gaming point of view is adopted the number of markets must be specified and a description of how price is formed in each market must be supplied.

Suppose that gold is of infinite durability and for simplicity in exposition<sup>1</sup> all  $m$  other goods last for  $T$  periods and as they may be subject to wear and tear, a good  $t$  years old may be regarded as a different good from one that is  $t^*$  years old. Furthermore let us assume that contracts cannot be written for more than  $T$  periods into the future. During any period there will be  $mT$  goods and gold. If gold is the money there are  $mT$  spot markets. There are  $mT(T-1)$  futures markets, where for the

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<sup>1</sup>In the proofs we use generally different lengths of life. Furthermore in this paper in order to separate out difficulties we limit our study to infinitely lived traders. The results for overlapping generations are considerably different. Among other new phenomena introduced by considering a strategic game with OLG, we must specify inheritance rules to dispose of goods left over by a deceased trader.

sake of specificity, an individual pays gold now for the future delivery of a commodity  $j$  at time  $t > 1$ . There are  $T$  money and credit markets where gold can be borrowed now to be repaid anywhere from the start of period 2 to  $T+1$ .

Once trade is contemplated among durables a new set of markets must be considered. Do we rent or do we buy the asset itself? Both are feasible. For goods which only last one period rental and the purchase of the asset are economically the same.<sup>2</sup> Counting rental and asset markets the number of markets in any period is:

- $m$  new good spot markets for assets,
- $m$  new good rental markets,
- $m(T-1)$  second hand asset markets,
- $m(T-1)$  second hand rental markets,
- $m(T-1)^2$  futures markets for assets,
- $mT(T-1)$  futures markets for rentals .

Figure 1 illustrates this count for an economy with a single finite lived durable which lasts for 3 periods. NS and NR are new asset and rental markets (1 each); S and R are second hand spot rental and asset markets (2 each); A are the asset futures (4) and F the rental futures (6). This count has only one period rental markets. Many more could be added. There is also 1 short term money market and  $T-1$  long term loan markets.

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<sup>2</sup>But legally the distinction between owning and renting is considerable.

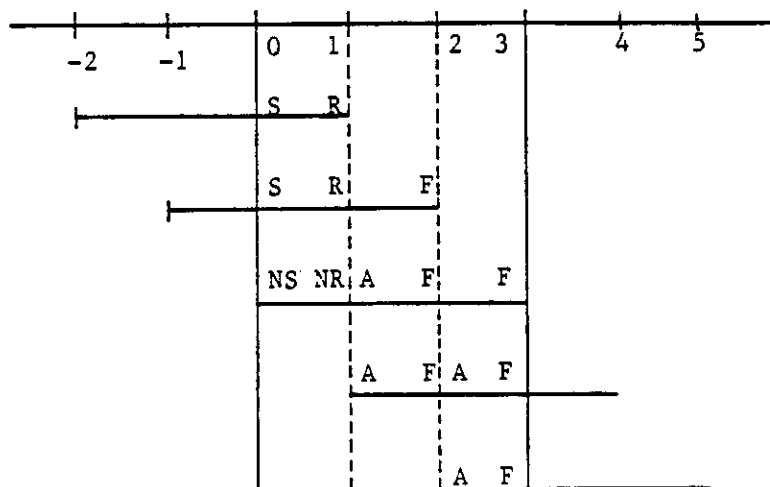


FIGURE 1  
Rental, Asset and Futures Markets

The market for borrowing gold is clearly only a rental market. The use of the gold is for a limited time. In legal fact a further distinction must be made. When a Bezzant or Sovereign is borrowed, the borrower assumes ownership while the bank holds his note. He is only required to return a like coin (plus interest). When an automobile is rented ownership does not change and the identical item must be returned.

There is the possibility of even more markets involving contracts for example arranged at time  $t$  for payment at time  $t^*$  and delivery at  $t^{**}$ . In actuality a futures contract may involve the payment of earnest money at time  $t$  then the payment of the rest of the agreed upon price at time  $t^{**}$ .

Currently one of the preoccupations of the financial community is the invention of new instruments and markets. At the level of abstraction of general equilibrium theory or strategic market games without explicit introduction of transactions costs and exogenous uncertainty many of the distinctions among close substitute financial instruments disappear and at

some levels of analysis are not relevant. But here even at the level of abstraction given below the distinction between asset markets and rental is of importance.

## 2.2. The rental of assets for gold

The first model considered involves rental markets only. Assets are rented for gold. This implies that the ownership of all durables (except gold) remains unchanged. However, each period many assets are offered for rental. Under reasonably general conditions given below we establish the existence of an NE (noncooperative equilibrium) and for instances with a stationary input of new durables the existence of a stationary state equilibrium (SNE) can be shown.

Let us first describe the model in more detail. Assume that there are  $m_1$  goods of finitely long life (the  $j^{\text{th}}$  lasts for  $T_j$ ) and  $m_2$  goods of infinitely long life.

The utility function<sup>3</sup> for  $i$  is

$$\pi^i = \sum_{t=0}^{\infty} \beta^t \phi^i(x^i, X^i)$$

where

$$x^i = (x_1^i, \dots, x_{m_1}^i), \quad x_j^i = (x_{j1}^i, \dots, x_{jT_j}^i)$$

$$X^i = (X_1^i, \dots, X_{m_2}^i), \quad X_{m_2}^i - \text{amount of gold.}$$

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<sup>3</sup>In this paper we confine our remarks to individuals with infinite lives. The results for overlapping generations we suspect are considerably different especially when an asset "lives" longer than its owner. But in keeping with our approach of dividing difficulties we defer consideration of the OLG model.

The endowment at  $t$  is

$$a^i(t) = (a_1^i(t), \dots, a_{m_1}^i(t); A_1^i(t), \dots, A_{m_2}^i(t))$$

$$a_j^i(t) = (a_{j,1}^i(t), \dots, a_{j,T_j}^i(t))$$

strategies are:

$$s^i(t) = (q_1^i(t), \dots, q_{m_1}^i(t); p_1^i(t), \dots, p_{m_2-1}^i(t));$$

$$b_1^i(t), \dots, b_{m_1}^i(t); c_1^i(t), \dots, c_{m_2-1}^i(t)$$

prices are:

$$p_{j,r}(t) = \frac{\int b_{j,r}^i(t)}{\int q_{j,r}^i(t)}, \quad \bar{p}_j(t) = \frac{\int c_j^i(t)}{\int r_j^i(t)}$$

reallocations are given by:

$$x_{j,r}^i(t) = a_{j,r}^i(t) - q_{j,r}^i(t) + b_{j,r}^i(t)/p_{j,r}(t)$$

$$x_j^i(t) = A_j^i(t) - r_j^i(t) + c_j^i(t)/\bar{p}_j(t)$$

$$\begin{aligned} x_{m_2}^i(t) = & A_{m_2}^i(t) - \sum_{j=1}^{m_1} \sum_{r=1}^{T_j} b_{j,r}^i(t) - \sum_{j=1}^{m_2-1} c_j^i(t) \\ & + \sum_{j=1}^{m_1} \sum_{r=1}^{T_j} q_{j,r}^i(t) p_{j,r}(t) + \sum_{j=1}^{m_2-1} p_j^i(t) \bar{p}_j(t) \end{aligned}$$

updated holdings are:

$$a_{j,r_j}^i(t+1) = x_{j,r-1}^i(t) , \quad r \geq 2 , \quad 2 \leq r_j \leq T_j$$

$$a_{j,1}^i(t+1) \text{ is new}$$

$$A_j^i(t+1) = X_j^i(t) .$$

Theorem 1. Assume the utility functions are all gold separable:

$\emptyset^i = f^i + g^i$  with  $g^i$  being CIS $_{\omega}$ .<sup>4</sup> Then for any given  $G > 0$  , there is a proper distribution of  $G$  , say  $A = (A^1, \dots, A^n)$  such that the game  $\Gamma_{\omega}(a,A)$  of selling services has an SNE.

The proof of Theorem 1 is the same as the proof of the existence of SNE in our first paper, here what we have to do is to regard the same type of goods of different ages as different commodities lasting for only one period.

### 2.3. The sale of assets for gold

In contrast with the rental markets we may assume that only assets are for sale. Heuristically we should suspect that this could be less efficient than having rental markets. Rental markets provide more flexibility in financing than asset markets and may require less money to operate.

Definition 1. In a game  $\Gamma_{\omega}(a,A)$  of selling assets, a Nash equilibrium is said to be essentially stationary (ESNE) if after finitely many periods the strategy selection and allocation are stationary.

In general we cannot expect that  $P_{\omega}(a,A)$  has an ESNE except when the  $\emptyset^i$  have some special properties.

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<sup>4</sup>CIS = continuous, increasing and strictly concave.

Theorem 2. Assume that the utility functions all have the following properties:

- (i) gold separable:<sup>5</sup>  $\phi^i = f^i + g^i$ , with  $g^i$  being CIS $^\infty$ .
- (ii) for any good  $j$ ,  $\phi^i$  is symmetric with respect to  $x_{j,1}^i, \dots, x_{j,T_j}^i$  (i.e. the new and the old are of same value to  $i$  in one period).

Then when  $G > 0$  is properly distributed:  $A = (A^1, \dots, A^n)$ , the game  $\Gamma_\infty(a,A)$  of selling assets has an essentially stationary Nash equilibrium (ESNE).

For the proof of Theorem 2, we need the following:

Lemma 1. Assume that  $\Gamma_\infty(a,A)$  has an ESNE. Assume that the stationary allocation is  $(x_1^i, \dots, x_{m_1}^i; X_1^i, \dots, X_{m_2}^i)$  with  $x_j^i = (x_{j,1}^i, \dots, x_{j,T_j}^i)$ , and the associated prices are  $p_{j,r_j}$  ( $j = 1, \dots, m_1, r_j = 1, \dots, T_j$ ) and  $\bar{p}_j$  ( $j = 1, \dots, m_2-1$ ). Then

$$(i) \quad \frac{\partial f^i}{\partial x_{j,r_j}^i} = \frac{p_{j,r_j}}{1 - \beta^{(T_j - r_j + 1)}} \frac{\partial g^i}{\partial X_{m_2}^i} \quad (j = 1, \dots, m_1, r_j = 1, \dots, T_j)$$

$$(ii) \quad \frac{\partial f^i}{\partial X_j^i} = \frac{\partial g^i}{\partial X_{m_2}^i} \bar{p}_j \quad (j = 1, \dots, m_2-1)$$

The proof is straightforward, we omit it.

With the help of Lemma 1, the proof of Theorem 2 is similar to the proof of the existence of SNE in our first paper.

We may now observe that given two economies which differ only in the

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<sup>5</sup>This condition is sufficient but we suspect not necessary.

feature that one has trade only in rental markets and the other in asset markets the former may be more efficient than the latter if the possession of the commodity money yields any utility. We establish this observation by means of an example.

In essence we select a utility function strictly concave in each variable. The market with only services is such that the equilibrium is a stationary state in services and gold transacted. The second market allows only asset transfers. The traders who were previously buying services must now buy assets. There will be an ESNE (essentially stationary noncooperative equilibrium) after some periods at which the capital good replacement policy is repeated; however initially there will be a flow of gold to the traders with the capital goods and given the concavity of the utility functions this will be less efficient than the fully stationary state. The sellers of the assets benefit from a "front end load." Although the flow of gold may be only for a few periods they are the first periods and may remain significant for the whole infinite horizon.

Example 1. good  $x$  : durable for 3 periods;

good  $y$  : consumable;

gold  $z$  : infinitely long life.

Two types of traders  $(i_1, i_2)$ ,  $i_1 \in [0, 1/2]$ ,  $i_2 \in [1/2, 1]$ .

$$\varnothing^{i_1} = \varnothing^{i_2} = \sum_{t=0}^{\infty} \beta^t [(x_1 + x_2 + x_3)^{1/2} y^{1/3} + \log(1+z)] .$$

Endowments:  $(2, 2, 2, 0; A^1)$  for  $i_1$ ;  $(0, 0, 0, 6; A^2)$  for  $i_2$ .

$$\beta = \frac{\sqrt{5} - 1}{2} = 0.618.$$

Case 1. Market game of selling services.

$$\frac{1}{2} \left( x_1^{i_1} + x_2^{i_1} + x_3^{i_1} \right)^{-1/2} y^{i_1 \frac{1}{3}} = \frac{1}{1-\beta} \cdot \frac{P_j}{1+A} \quad (j = 1, 2, 3)$$

$$\frac{1}{3} \left( x_1^{i_1} + x_2^{i_1} + x_3^{i_1} \right)^{1/2} y^{i_1 - \frac{2}{3}} = \frac{1}{1-\beta} \cdot \frac{P_4}{1+A} \quad (j = 1, 2, 3)$$

$$= \frac{P_1}{P_4} = \frac{3}{2} \cdot \frac{y^{i_1}}{x_1^{i_1} + x_2^{i_1} + x_3^{i_1}} .$$

Similarly,

$$\frac{P_1}{P_4} = \frac{3}{2} \cdot \frac{(6 - y^{i_1})}{(6 - x_1^{i_1} - x_2^{i_1} - x_3^{i_1})} .$$

Note that  $p_1(6 - x_1^{i_1} - x_2^{i_1} - x_3^{i_1}) = p_4 y^{i_1}$

Therefore  $x_1^{i_1} - x_2^{i_1} - x_3^{i_1} = 1.2$  ,  $y^{i_1} = 3.6$  ;

$$x_1^{i_2} - x_2^{i_2} - x_3^{i_2} = 0.8$$
 ,  $y^{i_2} = 2.4$  .

$$A^1 = \frac{2 \cdot \sqrt[6]{3.6} p_1}{1-\beta} - 1$$
 ,  $A^2 = \frac{2 \cdot \sqrt[6]{2.4} p_1}{1-\beta} - 1$  .

On the other hand, the transaction of gold is  $2.4p_1$  ;

$$2.4p_1 \leq \frac{2 \cdot \sqrt[6]{2.4} p_1}{1-\beta} - 1$$

$$\Rightarrow \boxed{p_1 \geq 0.29}$$

$$A^1 \geq 0.782 ; A^2 \geq 0.694$$

$$G_S \geq 1.486$$

Case 2. Market game of selling assets

$$\frac{1}{2} \left( \frac{i_1}{x_1} + \frac{i_1}{x_2} + \frac{i_1}{x_3} \right)^{-1/2} y \frac{i_1}{3} = \frac{1}{1-\beta^3} \cdot \frac{P_1}{1+A^1}$$

$$\frac{1}{2} \left( \frac{i_1}{x_1} + \frac{i_1}{x_2} + \frac{i_1}{x_3} \right)^{-1/2} y \frac{i_1}{3} = \frac{1}{1-\beta^2} \cdot \frac{P_2}{1+A^1}$$

$$\frac{1}{2} \left( \frac{i_1}{x_1} + \frac{i_1}{x_2} + \frac{i_1}{x_3} \right)^{-1/2} y \frac{i_1}{3} = \frac{1}{1-\beta} \cdot \frac{P_3}{1+A^1}$$

$$\frac{1}{3} \left( \frac{i_1}{x_1} + \frac{i_1}{x_2} + \frac{i_1}{x_3} \right)^{1/2} y \frac{i_1}{3} = \frac{1}{1-\beta} \cdot \frac{P_4}{1+A^1}$$

$$\Rightarrow P_2 = \frac{1+\beta}{1+\beta+\beta^2} P_1 = \frac{P_1(1+\beta)}{2}, \quad P_3 = \frac{P_1}{2}, \quad P_4 = \frac{P_1}{3}$$

From second period,  $(2 - x_1) p_1 = y \frac{i_1}{3} p_4$

On the other hand,  $\frac{P_1}{P_4} = \frac{3}{2} \frac{y \frac{i_1}{3}}{\frac{i_1}{x_1} + \frac{i_1}{x_2} + \frac{i_1}{x_3}} (1 + \beta + \beta^2)$

$$\Rightarrow x_1 \frac{i_1}{3} + x_2 \frac{i_1}{3} + x_3 \frac{i_1}{3} = 1, \quad y \frac{i_1}{3} = 3$$

$$A^1 = \frac{2 \cdot \sqrt[6]{3} P_1}{1-\beta^3} - 1, \quad A^2 = A^1$$

Transaction of gold in second period =  $p_1$

$$A^2 = \frac{2 \cdot 6\sqrt{3} p_1}{1 - \beta^3} - 1 \geq p_1 \Rightarrow p_1 \geq 0.50$$

Transaction of gold in first period =  $p_1 + p_2 + p_3$ , we must have

$$\frac{2 \cdot 6\sqrt{3} p_1}{1 - \beta^3} - 1 - \frac{4+\beta}{2} p_1 \geq 0$$

from which we obtain

$$\boxed{p_1 \geq 1.44}$$

$$\bar{A}^1 \geq p_1 = 1.44$$

$$\begin{aligned} \bar{A}^2 &= A^2 + p_1 + p_2 + p_3 - 3p_4 = \frac{2 \cdot 6\sqrt{3} p_1}{1 - \beta^3} - 1 + \frac{2+\beta}{2} p_1 \\ &= 5.2 \end{aligned}$$

$$G_A \geq 1.44 + 5.2 = 6.64$$

$$G_S > G_A .$$

#### An aside on the float

In the example above there is no transactions time lag; i.e. there is no float. Individuals are paid immediately and the person who holds the gold at the end of the period is given full credit for any utility which may

accrue to holding that amount for the full period.<sup>6</sup> In our previous paper (Shubik and Yao, 1988) we had a one period time lag in payments where we assumed that the gold was in transit for the whole period and this represented a loss to the whole community. Although float or transactions time loss is real and often important this example does not depend on this type of loss.

#### The nonexistence of ESNEs with asset markets

Although the first example made use of the existence of an ESNE for the exchange economy with asset markets the next example shows that no ESNE may exist.

Example 2. Two types of traders:  $i_1$ ,  $i_2$ , with different utility functions:

$$\begin{aligned} \emptyset^{i_1} &= \left( x_1^{i_1} + x_2^{i_1} \right)^{1/2} y^{i_1 \frac{1}{3}} + \log z^{i_1} \\ \emptyset^{i_2} &= \left( 2x_1^{i_2} + x_2^{i_2} \right)^{1/2} y^{i_2 \frac{1}{3}} + \log z^{i_2} \end{aligned}$$

$x_1^{i_1}$  - amount of good 1 which is new

$x_2^{i_1}$  - amount of good 1 which is 1 year old

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<sup>6</sup>One of the nagging but rarely made explicit problems in monetary theory is where is the float, what is its size and who lives off it. In every indirect transaction there are at least four parties who partake in the use of the money paid by A to B in one period. They are A, B, the intermediary and "limbo" or the transit system. As we make our time periods shorter and shorter there are many ways in which we can pass to the limit to continuous time. It is a matter of logic to rule out some suggestions; but it is an empirical problem to characterize the appropriate limiting process.

$y^i_1$  - amount of consumer good.

Initial endowments:  $(2, 2, 0, A)(0, 0, 4, B)$  .

Claim. No nontrivial ESNE exists.

Proof. Assume there exists a nontrivial ESNE. Assume after  $k$  periods, the stationary allocation are

$$(a, b, c, A^1)(2-a, 2-b, 4-c, A^2)$$

the associated prices are  $p_1, p_2, p_3$  .

By Lemma 1, we must have

$$(1) \quad \begin{cases} \frac{1}{2}(a+b)^{-1/2}c^{1/3} = \frac{1}{1-\beta^2} \frac{p_1}{A^1} \\ \frac{1}{2}(a+b)^{-1/2}c^{1/3} = \frac{1}{1-\beta} \frac{p_2}{A^1} \end{cases}$$

and

$$(2) \quad \begin{cases} (6-2a-b)^{-1/2}c^{1/3} = \frac{1}{1-\beta^2} \frac{p_1}{A^2} \\ \frac{1}{2}(6-a-b)^{-1/2}c^{1/3} = \frac{1}{1-\beta} \frac{p_2}{A^2} \end{cases}$$

From (1),  $p_1 = (1+\beta)p_2$  ; from (2)  $p_1 = 2(1+\beta)p_2$  a contradiction.

#### Asset or rental markets?

The above analysis suggests that at this level of abstraction rental markets are better than asset markets. Yet when we glance at any economy both exist in significant proportions. In essence rental markets provide

more flexible credit and a means to smooth cash flows. In an actual economy these factors may be dominated by other features which have not been modeled here. In particular the costs of transfer of ownership or the writing of rental agreements have not been considered. Perhaps more important are problems concerning maintenance of property. Owners tend to be more concerned about their maintenance procedures than renters.

#### 2.4. Trading, markets and enough money

In the service market, just as in our first paper, when the marginal utility of gold tends to  $\infty$  as the amount tends to zero, then any amount of gold is enough to guarantee the existence of a SNE provided the gold is properly distributed. If gold is not properly distributed, an SNE can be achieved when a secured loan between traders is available (see Section 3). When the marginal utility of gold is finite, we have:

Theorem 2. Assume that the utility functions  $\phi^i$  are gold separable:

$$\phi^i = f^i(x^i) + g^i(X^i)$$

and  $g^i$  have the property CIS. Let  $E(f^i, a^i)$  be a pure exchange economy and  $(\dot{a}, \dot{p}) > 0$  be a CE of  $E(f^i, a^i)$ . Let<sup>7</sup>

$$\dot{p}_g = \min_{i,j,r_j} \left\{ \frac{\dot{p}_{j,r_j} \lim_{X^i \rightarrow 0} g^{i'}(X^i)}{(1-\beta)\partial_{j,r_j} f^i(\dot{a}^i)} \right\}.$$

Assume that  $\dot{X}^i$  satisfies

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<sup>7</sup>The notation  $g^{i'}$  is the derivate of  $g^i$  with respect to  $X^i$ .

$$\frac{y^{i'}(\dot{X}^\alpha)}{(1-\delta)\dot{p}_g} = \frac{\partial_{x_j^i} f^i(\dot{a}^i)}{\dot{p}_{j,\tau_j}} .$$

Then the game has an SNE provided

$$G \geq \sum_i \dot{X}^i + \dot{p}_g^{-1} \sum_i \dot{p}_{j,\tau} \max(0, \dot{a}_{j,\tau_j}^i - a_{j,\tau_j}^i) .$$

In an asset market, things are quite different. As we already see, in general, an SNE need not exist. Even if the utility functions are all symmetric with respect to any same type of goods of different ages and a ESNE does exist, the prices and allocations are different from the corresponding service market, and the ESNE needs more gold for transaction (see Example 1).

When there is no float loss or time lag in payments the meaning of enough money when rental markets are considered is that the CE's are attainable as NE's.

### 3. TRADE WITH SECURED LOANS

#### 3.1. One hundred percent gold backed loans

In a service market, if at the beginning gold is not properly distributed, we can introduce a fully secured loan between traders so as to achieve an SNE allocation. At the beginning of each period, a poorer trader  $i$  can borrow an amount  $v^i$  of gold from the richer traders and he should return  $\beta^{-1}v^i$  next period. Just as in Part 1, we have the following theorem.

Theorem 4. Assume that only gold is forever durable. Assume that the  $\phi^i$  are CIS. Then the service market always has a stationary Nash equilibrium with secured lending when  $\beta$  is closed to 1 sufficiently.

Proof. Let  $u^i$  be the solution of the following system of differential equations:

$$(1) \quad \begin{cases} \partial_{x_j^i} u^i(x^i, X^i) - \partial_{x_j^i} \phi^i(x^i, X^i) \\ \partial_{X_j^i} u^i(x^i, X^i) - \frac{1}{1-\beta} \partial_{X_j^i} \phi^i(x^i, X^i) \\ u^i(a^i, A^i) - \phi^i(a^i, A^i) \end{cases}$$

Regard goods of different ages as different goods, consider the pure exchange economy  $E(a, A, u)$  with  $T_1 + T_2 + \dots + T_{m_1+m_2}$  different kinds of goods. Let  $(\dot{a}, \dot{A})$  be a CE associated with prices

$$P_{1,1}, \dots, P_{1,T_1}, \dots, P_{m_1,1}, \dots, P_{m_1,T_{m_1}}, P_1, \dots, P_{m_2}.$$

It is not difficult to check that  $(\dot{a}^i, \dot{A}^i)$  ( $i = 1, \dots, n$ ) form a stationary Nash equilibrium with secured lending (SLSNE). Trader  $i$  borrows an amount  $v^i = \dot{A}^i - A^i$  of gold at each period and returns  $\beta^{-1}v^i$  next period.

Note that if there are more than one good having infinitely long life, in general we cannot guarantee that every CE of  $E(a, A, u)$  can be achieved as an SLSNE since the problem of enough money occurs.

If the  $\phi^i$  are gold separable, we can state the following result:

Theorem 5. Assume that there is a partition of the  $n$  types of traders:

$I_1 = \{1, \dots, k\}$ ,  $I_2 = \{k+1, \dots, n\}$  with two sets of nonnegative numbers:  $u^1, \dots, u^k$  and  $v^{k+1}, \dots, v^n$  such that  $\beta \sum_{i=1}^k u^i = \sum_{i=k+1}^n v^i$ .

Assume that there exist  $x^i$ ,  $X^i$  ( $i = 1, \dots, n$ ) and  $p_{1,1}, \dots, p_{m_1 T m_1}$ ,

$p_1, \dots, p_{m_2}$  such that

$$\begin{aligned}
 \text{(i)} \quad & \frac{\partial_{x_j^i} f^i(x^i, X^i)}{p_j r_j} = \frac{g^{i'}(X_{m_2}^i)}{(1-\beta)p_{m_2}} \\
 \text{(ii)} \quad & \frac{\partial_{X_j^i} f^i(x^i, X^i)}{p_j} = \frac{g^{i'}(X_{m_2}^i)}{p_{m_2}} \\
 \text{(iii)} \quad & \sum_{j=1}^{m_1} \sum_{r=1}^T p_j r_j x_j^i + \sum_{j=1}^{m_2} p_j X_j^i \\
 \text{(iii)} \quad & = \begin{cases} \sum_{j=1}^{m_1} \sum_{r=1}^T p_j r_j a_{j,r}^i + \sum_{j=1}^{m_2-1} p_j A_j^i + p_{m_2} (A_{m_2}^i + \beta u^i) & (i \in I_1) \\ \sum_{j=1}^{m_1} \sum_{r=1}^T p_j r_j a_{j,r}^i + \sum_{j=1}^{m_2-1} p_j A_j^i + p_{m_2} (A_{m_2}^i + v^i) & (i \in I_2) \end{cases} \\
 \text{(iv)} \quad & X_{m_2}^i - A_{m_2}^i = \begin{cases} u^i & (i \in I_1) \\ -\beta^{-1} v^i & (i \in I_2) \end{cases}
 \end{aligned}$$

and

$$(v) \quad \sum_{j=1}^{m_1} \sum_{\tau_j=1}^T p_j r_j \max\{0, x_{j\tau_j}^i - a_{j\tau_j}^i\} + \sum_{j=1}^m p_j \max\{0, X_j^i - A_j^i\}$$

$$\leq \begin{cases} p_{m_2} (A_{m_2}^i - \beta u^i) & (i \in I_1) \\ p_{m_2} (A_{m_2}^i - v^i) & (i \in I_2) \end{cases}$$

Then  $(x^i, X^i)$  is an SLSNE allocation. Condition (v) gives an implicit constraint concerning "enough money." Since there is another good with infinitely long life, even when  $\beta \rightarrow 1$ , (v) is not always guaranteed automatically.

### 3.2. Gold and land as stores of value

Gold and land are the two classical consumer-producer durables with essentially unbounded life and hence no depreciation. The economist's concept of utility or value, as psychologically crude as it may be, is ideally defined without references to prices, trade or production. Thus in particular industrial intermediate products have no direct consumer value, but their worth is derived as inputs into processes which eventually lead to the production of final consumer products.

Humans who are participants in a modern organized economy are, in essence, socialized individuals whose concept of value or worth is shaped within the economic system. The concept of the utility of gold or land to Robinson Crusoe is captured by utility theory alone as there are no markets for him to consider where he might sell items of no utility to him but of great direct or indirect value of others.

As direct primary sources of consumer economic value a few acres of land and a few pounds (or ounces?) of gold would probably satiate Crusoe's

direct desires. The ownership of vast estates where the owner may never even see the land he owns must be considered in terms of power, social position and implicitly stored wealth. If one considers the fact that the utilization of any asset requires time and some form of maintenance and responsibility, then at least for Robinson Crusoe saturation should set in. If free disposal or nonutilization of the resource is feasible then we may imagine a marginal utility of zero as a lower bound (otherwise a negative marginal utility has to be considered).

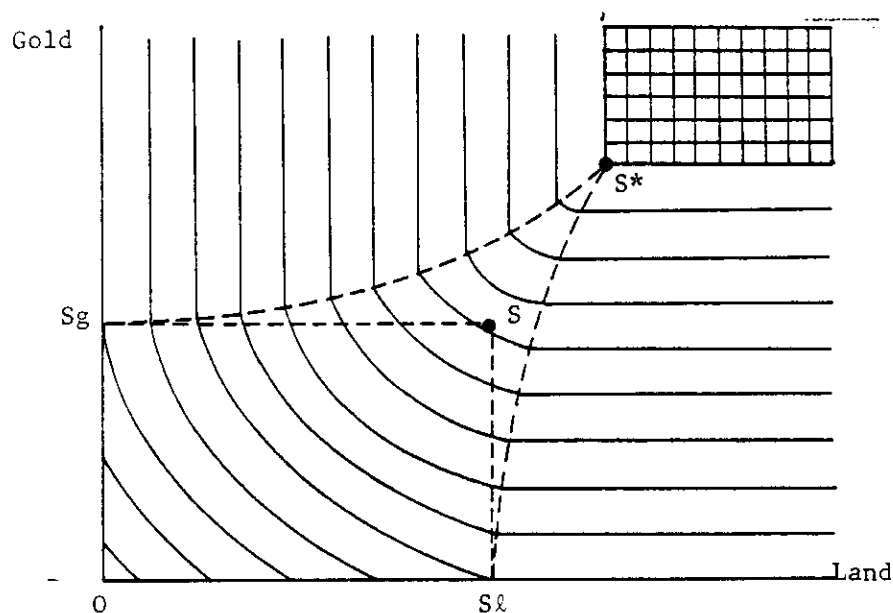


FIGURE 2  
Indifference Curves on Gold-Land Plane

A priori there is little that we can say about the specific shape of the utility functions or preference maps of individuals for small amounts of gold and land. Allowing for free disposal of both land and gold we may expect a saturation surface between gold and land (holding all other assets fixed). Figure 2 shows the preference map. The rectangle bounded by  $OS_g S_l$

is the zone where there is neither saturation of preferences for gold or land.  $S_g$  is the lowest saturation point for gold,  $S_l$  for land and  $S^*$  for both. Above and to the right of  $S^*$  the indifference surface is thick as is indicated by the cross hatching.

If there were competitive markets for land and gold and the supplies of both were large enough to saturate the needs of all, the price would drop to zero. But if the total supply were smaller than saturation for all, then there would be a positive price for each. Thus it is reasonable to expect some individuals to hold supplies of gold and land beyond their saturation point where their purpose for holding the gold and land is as a store of value where income may be derived from renting or selling the asset.

There are approximately five billion individuals on the earth (as of around 1987). The land area of the earth is around fifty-two million square miles and the amount of mined gold is around a hundred thousand metric tons. Counting all land (regardless of locations and productivity) the per capita area of land amounts to around 6.66 acres. A crude estimate limiting ourselves to economically productive land is that around 10-15% is of some economic worth.<sup>8</sup> Thus the per capita estimated amount of productive land is around between .65 to 1 acre.

The number of Troy ounces in a metric ton is 32,148. Thus the per capita amount of gold is .64 of a Troy ounce. The bounds of 3 or 4 ounces of gold and 5 or 10 acres of land per capita appear reasonable as estimates of the levels around which saturation sets in for an isolated individual. With

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<sup>8</sup>Of a land area of 115 million square kilometers around 13% are cultivated. There is an estimated 64 million square kilometers of forest, 15% of which has been converted to cultivation; 28 million of grass land 20% of which is cultivated and 16 million of desert 2% of which is under cultivation.

these estimates (or higher) there is neither sufficient land nor gold to saturate the preferences of all economic agents. Thus the price of both will be positive, but as both are indefinitely durable we may expect to observe situations where individuals own gold and land far beyond their saturation points, but as they are stores of value they can usefully be loaned or rented.

The proofs in Sections 2.2 and 2.3 require modification to establish that they still hold even though there may be saturation for some individuals of both land and gold, but that saturation is not universal.

Let  $u^i$  be a solution of the system of differential equations:

$$\begin{cases} \frac{\partial}{\partial x_j^i} u^i(x^i, X^i) - \frac{\partial}{\partial x_j^i} \varphi^i(x^i, X^i) \\ \frac{\partial}{\partial x_j^i} u^i(x^i, X^i) - \frac{1}{1-\beta} \frac{\partial}{\partial x_j^i} \varphi^i(X^i) \quad (j = 1, 2) \\ u^i(a^i, A^i) = \varphi^i(a^i, A^i) \end{cases}$$

where  $A_1^i$ ,  $A_2^i$  are initial amounts of land and gold, respectively.

Regarding goods of the same kind but of different ages as different goods, consider the exchange economy  $E(a, A, u)$  with  $T_1 + \dots + T_{m_1} + 2$  different kinds of goods. Let  $(\dot{a}, \dot{A}_1, \dot{A}_2)$  be a CE of  $E(a, A, u)$  associated with prices  $p_j r_j$ ,  $p_1$ ,  $p_2$ . Under our assumption, we should have  $p_1 > 0$  and  $p_2 > 0$ . Moreover, if the following conditions holds

$$\begin{aligned}
 (1) \quad & \sum_{j=1}^{m_1} \sum_{r_j=1}^T p_j r_j \max(0, \dot{a}_{j r_j}^i - a_{j r_j}^i) + p_1 \max(0, \dot{A}_1^i - A_1^i) \\
 & \leq p_2 [A_2^i - \beta |\dot{A}_2^i - A_2^i|]
 \end{aligned}$$

then the CE allocation can be achieved as an SLSNE allocation.

Note that (1) is an implicit condition of "enough money," it is not automatically guaranteed even if  $\beta$  is close to 1, it also depends on the distribution of land at the very beginning.

### 3.3. Land or gold backed currency

As bills issued upon money security are money, so bills issued upon land are, in effect coined land.

Benjamin Franklin

Historically both gold and land have been considered as items with which a government could back a paper currency issue. Gold is more or less easy to mold into uniform ingots which are fungible. Land, in contrast, is highly variable and difficult to assess or classify. This may have been one of the reasons contributing to the failure of John Law's assignat scheme.

We may conceive of a "gold bank" (or "land bank") as follows. Gold is to be the reserve backing for a paper currency. All gold is deposited in the central bank. Bank notes are issued to all in exchange for the gold they deposit. The price of a unit of gold is fixed at one unit of paper, thus for example, the bank note might bear the inscription "The bank owns one ounce of gold against this note."

Gold is demonetized. Each period the bank rents all gold for one period. The bank also issues shares along with its notes to the depositors. Thus an individual who has contributed  $x\%$  of the gold obtains  $x\%$  of both

the banknote issue and the shares. Formally each individual begins with initial resources of  $(a_{11}^i, \dots, a_{m1}^i, a_{m+1,1}^i, 0, 0)$  where  $a_{m+1,1}^i$  is gold and the two zero entries are gold backed paper money and shares. All are required immediately to turn in their gold in exchange for banknotes and shares. Thus the bank has as initial capital:  $A = \sum_{i=1}^n a_{m+1,t}^i$  units of gold. Each period it puts up all  $A$  units for auction on a loan basis (i.e. the gold is leased not sold). Each period  $t$  a strategy by a trader  $i$  is to bid  $(b_{1t}^i, \dots, b_{m+2,t}^i; q_{1t}^i, \dots, q_{m+2,t}^i)$  where  $0 \leq b_{1t}^i$ ,  $\sum_{j=1}^{m+2} b_{jt}^i \leq 1$ ,  $0 \leq q_{jt}^i \leq a_{jt}^i$ .

The separation of the bank stock from currency gives a zero interest bearing currency with the bank earnings from leasing gold going to the owners of shares and an implicit loan market created by the sale and purchase of shares. However it is important to note that this implicit loan market does not require any default conditions as it is an equity market in which the ownership of stock changes hands for currency

$$a_{m+1,t}^i = 0, \quad x_{m+1,t}^i = b_{m+1,t}^i M_t^i / p_{m+1,t}$$

$$a_{m+2,t}^i = a_{m+2,t-1}^i + b_{m+2,t-1}^i M_{t-1}^i / p_{m+2,t-1} - q_{m+2,t-1}^i$$

$$M_t^i = a_{m+3,t}^i = a_{m+3,t-1}^i (1 - \sum_{j=1}^{m+2} b_{j,t-1}^i) + \sum_{j=1}^m p_{j,t-1} q_{j,t-1}^i \\ + p_{m+2,t-1} q_{m+1,t-1}^i + \frac{a_{m+2,t-1}^i}{a_{m+2}} p_{m+1,t-1} a_{m+1}^i$$

where the last term is the bank dividend paid to those who hold stock at the start of time  $t-1$ . This implies that stocks are traded ex-dividend.

The existence of a Nash equilibrium path for the gold back currency

model is obvious (the strategy set for an individual is convex and compact, in the case with a continuum of traders, no concavity problem occurs). The existence of an SNE can also be proved when at the beginning gold is properly distributed. In fact, the SNE allocation is the CE allocation.

#### 4. FURTHER PROBLEMS

##### 4.1. Comments on fiat money

We defer the analysis of fiat money in a multistage economy to a future paper, but a few comments which contrast with the commodity money models are in order.

Schemes such as the issue of currency backed in part by gold, silver or land have cropped up over the last few hundred years. but as soon as 100% reserves, or fully secured loans are abandoned then a central problem is manifested in at least three forms. Who or what device polices the government, the banks or private individuals in the case of default. Furthermore if there is default two further questions arise, is the penalty inflicted economic or extra-economic and are the rules designed to compensate the creditors.

We limit the discussion and in this section comment on a fiat money where the backing is only in the rules concerning punishment for default. The switch from a commodity money to unbacked paper can take place in several different but related ways. We concentrate only on the government issue of fiat, although some comments on other alternatives are made below.

We can imagine that each individual issues his own banknotes denominated in an arbitrary unit of account fixed by the government. There are several problems encountered in trying to well define this as a playable

game. We can consider the possibility that the community clubs together and authorizes the issue of banknotes in a mutual bank where the shares are distributed among the members of the community in some manner or the other. For example, if there are  $n$  individuals each might receive  $1/n$  of the shares. The shares may or may not be negotiable. The bank then exchanges banknotes for individual debt. The government specifies the unit of account, places a limit on the issue of banknotes and specifies the penalties for default. A third alternative which is the one analyzed here is that the government issues the notes itself.

In previous work Shubik and Wilson (1977) and Dubey and Shubik (1979) considered a one period exchange model with the introduction of outside fiat. They showed that with sufficiently high bankruptcy penalties no one would default and that the (endogenous) rate of interest would be zero. In a note Dubey and Shubik (1987) showed that the bankruptcy penalty could be made economically endogenous. However an attempt to extend these results to a multistage economy introduces several problems which are not present in the one period model. In particular because in the one period model the rate of interest emerges as zero there is no need to make a distinction between an outside or inside bank as there are no problems with "who gets to spend the bank earnings on what." Furthermore in a one period game the bank need only function as a bank of issue. When there are two or more periods it might be desirable to have it as a bank of issue and deposit, but as soon as it starts to act as a bank of deposit unless the rules are appropriately constraining it may be possible that the bank could default in which case bank default rules are needed. In the one period models it was relatively easy to finesse or ignore questions concerning the control and motivation of

the bank. The multistage models require more specification.

#### 4.2. Comments on generally secured loans

In Shubik and Yao (1988) and in Section 3.1 the possibility for 100% gold secured loans was discussed. This possibility depended in its general form upon gold being the single commodity money. Because it is a commodity of intrinsic value individuals are motivated to hold some of it for consumption; therefore any promise to pay in gold in one period's time an amount less than or equal to the amount that the individual intends to hold for consumption purposes can be guaranteed. The creditor in lending lends the amount he expected to get back diminished by the interest rate. But as the interest rate is formed prior to the other prices this is independent of the rest of the price system as it is limited strictly to one dimension, gold for gold. If an array of assets is utilized as security there is no guarantee that their prices (which are measured relative to gold) will remain within the ranges necessary to guarantee that under all circumstances an individual will have 100% coverage of his loans beyond the gold for gold coverage. For example, all other prices could drop arbitrarily close to zero. In order to deal with loans covered by many assets we need to introduce conditions on the expected range of price movements. This is deferred until a subsequent paper.

#### 4.3. Comments on loans in a stochastic environment

If the supply of gold is nonrandom and it is used for money then it is still possible to have 100% backed loan even if the rest of the environment is stochastic. However conditions for both generally secured and unsecured loans become considerably more complex and problematical. For 100% secured

loans the bounds on potential price changes in assets must account for the variability of the environment. When the loans are less than 100% secured then default or bankruptcy arrangements which were adequate for a non-stochastic environment are not necessarily adequate for an environment in which strategic default and default through ill fortune are confounded. It is feasible, but costly and improbable to have a set of default rules which are state dependent. In actual practice two or three contingencies may be taken into account by the courts (such as force majeure, or the court's estimate as to whether fraud was intended in a default). If however the default conditions are not sufficiently state dependent an outcome analogous to the CE will generally not be attainable as an equilibrium and the nonco-operative equilibrium conditions may involve active bankruptcy (see Dubey, Geanakoplos and Shubik, 1988).

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