

## Dual Selves

Based on work with David K. Levine

McIntosh [1969]:

*“The idea of self-control is paradoxical unless it is assumed that the psyche contains more than one energy system, and that these energy systems have some degree of independence from each other.”*

Dual selves: Thaler-Shefrin [1981], Gul-Pesendorfer [2001],[2004], [2007], Bernheim-Rangel [2004], Benhabib-Bisin [2005], Noor [2007], [2011], Dekel, Lipman and Rustichini [2009], Ozdenoren, Salant and Silverman [2009], Dekel-Lipman [2011], Noor-Takeoka [2010a,b].

Multiple selves: Ambrus-Rozen [2011], Green-Hojman [2009], Baldiga-Green [2010], Jackson-Yariv [2010].

**Fudenberg and Levine [2006]:** Decisions made by a “Long run self” who maximizes:

$$\sum_{\ell=0}^{\infty} \delta^{\ell} (u_{\ell} - \gamma(\Delta_{\ell}))$$

$\Delta_{\ell}$ : the difference between the maximum feasible utility in the current period and the utility actually received

$\gamma$ : “control cost function.”

Idea: LR self has to control the impulses of a ‘short-run self’ who cares only about the present.

Solution to this optimization corresponds to the equilibrium of a game between a long-run self and a sequence of short-run selves.

Paper shows dual-self models explain many things, including

- Preference for commitment in decision problems ( Homer, Schelling [1958], Laibson [1997])
- “Excessive” procrastination (O’Donoghue-Rabin [1999]).
- “present bias” in timing of cash payments, i.e. prefer \$10 now to \$11 next week, but prefer \$10 in 10 weeks to \$11 in 11 weeks. (Herrnstein [1961], Elster [1979], Lowenstein-Prelec [1992], Laibson [1997])

These previously explained using quasi-hyperbolic utility (Phelps-Pollak [1968]), where observed decisions correspond to the equilibria of a game with one player per period.

Multiple equilibria typical in infinite horizon models; predicted behavior very sensitive to the exact length of a finite horizon.

Dual-self models don't have those problems, and generate additional predictions in other domains.

Our approach to these models differs from the prevailing methodologies in behavioral economics and in decision theory:

We are looking for theories that are portable, tractable, and make sensible predictions in a range of contexts. This differs from both the “collection of tools” approach advocated by Camerer-Lowenstein [2003] and the “axioms or it doesn't matter” view of some decision theorists.

## Polemics, 1:

- Better to have a small number of models that explain a large number of facts than the reverse.
- Important enough to trade-off versus other criteria like accuracy or intuition: A model that approximates  $n$  facts may be better than  $n$  special-case models each of which better fits one fact and/or better fits prior intuition.
- Models should not only be consistent with the data, but help us extrapolate from one setting to another.
- This requires some stability of the parameters across settings.
- Parameter stability crucial for policy analysis. (Lucas)

## Polemics, 2:

- Representation theorems can help us understand implications of a functional form, but very complex axioms may not be that helpful.
- Axioms that seem well motivated in a normative setting may not describe “behavioral” choices.
- Not clear how to evaluate axioms for “behavioral” agents.
- Our suggested agenda: first find functional forms that are portable, tractable, and make sensible predictions in a range of contexts. Once the dust settles a bit, then look into representations.

## Convex Control Costs:

More than twice as hard/costly to resist twice the temptation.

### *Implications:*

- Certain (not all!) violations of WARP. Specifically explains the “compromise effect” : pick fruit from {fruit, small desert} but small desert from {fruit, small desert, large desert. }
- Allais paradox (*related to “stochastic temptation”*) convex control cost leads to “Allais type” violation of independence. (but not the reverse violation, which is rare/non-existent.)

*Aside:* Prospect theory fits Allais, small stakes risk aversion, and the fact that some people buy both lottery tickets and insurance (which our theory can't do w/o introducing S-shaped utility functions, which lead to other problems.) However prospect theory can't fit all the above with the same parameters (Neilson and Stowe [2002]), and doesn't explain demand for commitment or dynamic choice reversal. Indeed, the dynamic version of prospect theory is still being worked out...

## Convex control costs also explain

- co-variance of effort/hours worked and consumption
- effect of cognitive load on self-control: (1 example coming)
- interaction of risk and delay (2 examples coming)

## Cognitive load+ self control

### **Shiv and Fedorikhin [1999]:**

Subjects asked to memorize either a two- or a seven-digit number, and then walk to a table with a choice of two deserts, namely chocolate cake and fruit salad.

Subjects would then pick a ticket for a desert and report the number and their choice in a second room.

seven-digit number: 63% choose cake

two-digit number: 41% choose cake.

Ward and Mann [2000] report a similar effect of cognitive load. Dual-self explanation: control cost depends on the sum of cognitive load and foregone utility. *Many other explanations...*

## **Fudenberg and Levine [2011]:**

- Qualitative risk-domain implications of convex cost.
- Calibrate the model to experiments and macro data: Used macro data to calibrate subjective discount rate, fraction of income on durables, past experimental studies for range of relative risk aversion, then estimated self-control parameters (quadratic cost) on data from experiments on small-stakes risk aversion. Informally tested how well these parameters fit experiments on the Allais paradox in the US and in Chile.
- Qualitative implications of the **combination of risk and delay** in the presence of convex control costs.

**Keren and Roelofsma [1995]:**

100 Florins            now (82%) vs 110 in 4 weeks;

100            in 26 weeks (37%) vs 110 in 30 weeks:

Many more choose 100 when “now” than when all choices pushed off half a year- not consistent exponential discounting.

But when all money payments replaced with .5 chances of same payment, the fractions choosing 100 were .39 and .33: almost no present bias.

Confirmed by Weber and Chapman [2005] with real stakes.

Explained by convex control cost: costs less than  $\frac{1}{2}$  as much to resist a  $\frac{1}{2}$  probability of reward.

**Baucells and Heukamp** [2010]:common-ratio Allais paradox with immediate payoffs, then with all payoffs to be in 3 months.

The reversal rate is **36%** without delay, **22%** with delay.

Again, FL [2011] show this is qualitatively consistent with convex control costs: Allais paradox comes from convex control costs, less temptation about future payoffs.

But model predicts no paradoxical choices when all payoffs are outside the time horizon of the short-run self. So to fit this data the “period length” for some agents must be more than 3 months- too long to be consistent with other evidence.

**Thaler [1981], Myerson and Green [1995], etc.:** How much (hypothetical) money now to make indifferent to hypothetical \$1,000 after delay  $t$ ?

For each  $t$  estimate marginal interest rate

- Geometric discounting: marginal interest rates time invariant.
- Standard dual-self model: Initial marginal interest rate high, but subsequent marginal interest rates equal and low.
- Data shows smooth decline, both overall and within subject. (Andersen et al [2008] find smooth but smaller decline using lotteries with real stakes.)

## Problems:

- One-period “temptation horizon” too stark, makes unrealistic predictions about the timing of decision and payoff.
- “Period” is artificial and not observed. Choices should depend on the real time between decision and information nodes, independent of how this time is cut into periods.

## **Fudenberg and Levine [2012] “Timing and Self Control”:**

*Summary:* Replace “short run selves” with non myopic “shorter-run” selves.

Explains gradual effect of changing timing of temptations, and why it is cheaper to commit to distant temptations than immediate ones.

Works well when cost of self control is linear in forgone value.

With convex costs, the marginal control cost should depend on self-control used in recent past.

So add “cognitive resources.”

Temptation by future axiomatized in Noor [2007], [2011] in setting with linear costs.

We extend to non-linear costs, relate preferences to the real time interval between each period instead of periods *per se*, add structure so obtain sharper predictions.

## Non-myopic short run selves

- Discrete time periods  $n = 1, 2, \dots$ ; period length  $\tau$ .
- space  $Y$  of states, state-dependent space of actions  $A(y_n)$ .  
(+ *technical conditions e.g. finite dimension, measurability.*)
- Markov chain: distribution on  $y_{n+1}$  determined by  $(y_n, a_n)$ .
- Shorter-Run Self(ves): *not necessarily myopic*
- Per -period utility  $u$  independent of period length.
- SR **self**: discount factor  $\delta\mu = \exp(-(\rho + \eta)\tau)$ .
- Alternatively SR **selves** discount  $\delta = \exp(-\rho\tau)$ , survival probability  $\mu = \exp(-\eta\tau)$ .
- SR's objective function from any period  $n$  on is  
$$(1 - \delta\mu)E \sum_{\ell=0}^{\infty} (\delta\mu)^{\ell} u(y_{n+\ell}, a_{n+\ell}).$$

*Long Run Self:* Discounts future utility by  $\delta$ .

Incurs a “control cost” to change behavior of SR self.

**Key issue:** defining this cost of self control

- a) how does cost depend on SR’s expectations about future outcomes?
- b) how does cost depend on past actions, notably self control used in the recent past?

*Temptation value*  $\bar{U}(y_n)$ :

maximum feasible average present value for SR at state  $y_n$ .

(assume finite)

*Foregone value:*

$$\Delta(y_n, a_n) = \bar{U}(y_n) - \left( (1 - \delta\mu)u(y_n, a_n) + \delta\mu E_{y_n, a_n} \bar{U}(y_{n+1}) \right)$$

*Note:* Action  $a_n$  can change the distribution of  $y_{n+1}$  and so lower maximum possible future utility- this has a current cost.

*LR* may choose future actions that lower future utility; associated loss in value cost will be recognized when utility is sacrificed.

*Interpretation:* future temptation value is SR's prediction of future utility, and SR doesn't think about the way current actions might change future self-control.

*(in the linear case other interpretations fit too...)*

**Next:** specify how the control cost depends on foregone value.

Start with linear cost, then convex.

Then introduce spillover of control cost between periods...

**Linear Cost:** LR's objective at any period  $n$  is

$$\sum_{\ell=0}^{\infty} \delta^{\ell} \left( (1 - \delta)u_{n+\ell} - \Gamma\Delta_{n+\ell} \right).$$

State matters only through effect on foregone value.

Normalization: multiply  $u$  but not  $\Gamma$  by  $(1 - \delta)$ .

- w/o self control a constant flow of  $u$  is worth  $u$  for any  $\tau$ .
- Action that foregoes 1 util in *every* period costs  $\Gamma$  for any  $\tau$ .
- Foregoing 1 util in 1 period costs  $\Gamma(1 - \delta\mu)$ :  $O(\tau)$ .

Infinite sequence of 1-period 1-util sacrifices costs

$\Gamma(1 - \delta\mu) / (1 - \delta) > \Gamma$ : Cheaper to resist future temptations now than as they arise.

Apply to “simple versus persistent temptations”

*Simple temptation*: Choose accept/reject

Reject: no change in utility flow

Accept: get  $u_g > 0$  for  $N$  periods,  $-u_b < 0$  thereafter.

$$\text{LR value: } P = (1 - \delta^N)u_g - \delta^N u_b$$

$$\text{SR value: } S = (1 - (\delta\mu)^N)u_g - (\delta\mu)^N u_b$$

Assume  $S > 0 > P$  .

Hold real time  $T = N / \tau$  fixed as  $\tau$  varies, so  $S, P$  independent of  $\tau$ : separate “period length” from real time.

*Solution*: Reject if  $|P| > \Gamma S$  (regardless of  $\tau$ ).

**Persistent temptation:** if resist, postpone the loss of  $P$ , then face same choice next period.

Foregone value if resist is  $S - \delta\mu S = (1 - \delta\mu)S$ .

So resist if  $|P|(1 - \delta) > \Gamma(1 - \delta\mu)S$ .

Or as  $\tau \rightarrow 0$  :  $\rho|P| > \Gamma(\rho + \eta)S$ .

So resist simple temptation and accept persistent one if

$$(\rho + \eta)\Gamma S > \rho|P| > \rho\Gamma S$$

$\rho|P|$ : value of postponing P for an interval  $dt$ ,

$(\rho + \eta)\Gamma S$ : flow cost of resisting the persistent temptation

$\rho\Gamma S$ : flow equivalent of one-time payment of  $\Gamma S$

The persistent temptation more costly to resist than the simple one because declining a simple temptation commits to avoid future temptations.

Same logic implies agent can decline a “bundle” of two simple temptations, one occurring tomorrow and one next week, even though he would take each of them if presented in isolation, as in data of Ainslie [2001], Kirby and Guatsello [2001].

The linear-costs model also explains gradual decline of incremental interest rates (*modulo cash vs. consumption*):

Specify  $u(c) = c$ , find  $c_n$  that makes agent indifferent between  $c_n$  at  $n$  and 1 util now.

*Temptation:* consume 1 now; no control cost, LR value  $1 - \delta$ .

*Foregone value*  $\Delta_n$  of waiting is

(SR value of 1 now) - (SR value of  $c_n$  at  $n$ ) =

$$(1 - \delta\mu) - (1 - \delta\mu)(\delta\mu)^{n-1}c_n$$

LR value of  $c_n$  is  $(1 - \delta)\delta^{n-1}c_n - \Gamma\Delta_n$ .

Compute  $c_n$  that gives indifference. The resulting marginal interest rate is  $\rho$  plus a term that declines exponentially at rate  $\eta$ .

“Kick me contracts”: contracts that impose fine on agent if he takes a tempting action (smoking, not going to the gym, etc.”

Laibson (slides) “When academic researchers design commitment contracts, take-up is almost never greater than 50% (despite demand effects).”

Less demand for commitment than quasi-hyperbolic model predicts.

Eg StickK.com (Ayres, Golberg and Karlan); Gine, Karlan and Zinman [2009]: 11% of smokers offered chance to post “quite smoking” bonds choose to do so.



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Peysakhovich [2011]: dual-self agents have less demand for kick-me contracts- in fact no demand at all if the SR self is completely myopic. *Reason:* myopic SR not influenced by future punishments; only way the fine could change behavior is if LR chooses to exert self control in the future- which can't make LR better off. Even with non-myopic SR, kick-me contracts only optimal if the penalties are large enough that the SR self feels no temptation: at the optimal kick-me contract no self-control is exercised.

Peysakhovich also shows that a dual self agent asked to pick a time  $t$  to incur a cost, with benefit at  $t+T$ , chooses  $t>0$ .

He argues this is why he and I like to make commitments to give seminars at dates that are reasonably far off in the future.

*Cost of delay:* push back net payoff of the investment (this is a cost if investment would be optimal w.o. self control costs.)

*Benefit of delay:* self-control costs go down. Interior optimum with linear or convex costs...

## Convex Control Costs

Generalize the cost  $\Gamma \Delta$  to a convex function  $g(\Delta)$

Now objective in any period  $n$  is

$$\sum_{\ell=0}^{\infty} \delta^{\ell} ((1 - \delta)u(y_{n+\ell}, a_{n+\ell}) - g(\Delta(y_{n+\ell}, a_{n+\ell})))$$

*stochastic simple temptation:*

Accept gives probability  $q$  of the simple temptation,  $1-q$  of 0.  
Reject gives 0.

Resisting foregoes value  $qS$ , so reject if  $q|P| > g(qS)$ .

This can hold even though  $|P| < g(S)$  so accept when  $q=1$ .

Violates independence axiom in same direction as Allais paradox- more self control when odds lower.

## *Two Simple Temptations*

In periods  $n_1 = 1$  and  $n_2 \geq 1$  agent can accept or reject a simple temptation.

- Sequential decisions:  $n_2 > 1$ .

The two decision problems are identical;

take both temptations if  $|P| < g(S)$  and resist both if  $|P| > g(S)$ .

This is true in particular if  $n_2 = 2$

- Simultaneous decisions:  $n_2 = 1$ .

Temptation is to take both, get  $2P$

Take 1: get  $P - g(S)$ .

Take 0: get  $-g(2S)$ .

When  $g$  is convex, it is possible that

$$P < g(0) - g(S) = -g(S) \quad \text{and}$$

$$P > g(S) - g(2S)$$

Then when choice is simultaneous, resist one temptation and give in to the other.

But resist both when choices sequential, REGARDLESS OF THE REAL TIME BETWEEN PERIOD 1 AND PERIOD 2!

**Conclusion:** The non-linearity of costs should “spill over” from one period to the next if periods are short.

## Cognitive Resources and Self Control

Costs are convex because self control uses scarce cognitive resources (Baumeister et al [1998], Muraven et al [1998]).

Soon after resisting one temptation, stock of these resources is lower, so marginal cost of resisting is higher.

Add cognitive resources to the model. (extension of Ozdenoren, Salant, and Silverman [2009]-how to eat cake of fixed size with a stock of resources that is used but never replenished.)

This requires many new building blocks, lots of modeling details such as how the stock is depleted by self-control but then replenishes. Skip the details, settle for vague summary...

## Relation to previous model:

Cognitive resources let the model track variations in marginal cost of self control.

Three possible sources of this variation:

- depletion
- benefit of resources
- replenishment

When all 3 of these are linear the stock is irrelevant, and decisions are the same as in the model w/o cognitive resources.

## Simultaneous and Near-Simultaneous Decisions

Agent faces two simple temptations, with linear depletion and replenishment.

**Key:** Very little replenishment occurs in a small time interval. So time path of resources is about the same if agent resists temptation of  $2S$  in a single period, or resists  $S$  twice in rapid succession.

Thus strict preference for 0, 1, or 2 with simultaneous decisions implies same strict preference when decisions are close together.

## Waiting to Give In

May be optimal to resist a while and then give in once the marginal benefit of resources is sufficiently high.

Consistent with perfect foresight, no need to assume agent is naïve about self-control: arguments that this shows naïveté implicitly assume the problem is time-seperable.

Paper characterizes optimal stopping time, and gives conditions for interior solution. Here the optimum satisfies a standard first-order condition: give in when the marginal benefit of delay (postponing the loss) just equals the marginal cost (PV of marginal reduction in cognitive resources.)

## Waiting to Commit

Persistent temptation with added option of permanently taking the temptation off the table for cost  $F < |P|$ .

So 3 choices each period: take, decline for now, or decline forever.

For some parameter values, optimal to decline until stock falls enough (so that the marginal benefit of cognitive resources is sufficiently high), then pay the cost to commit.

Consistent with data of Houser et al [2011]:

Subjects were paid to complete certain tasks while they had access to a web browser, but they could pay a cost to remove web access.

*Problem:* The experimental instructions did not specify when and whether opportunities for commitment might occur in the future..

## Actions that Directly Self-Control “Technology”

No cognitive resources (equivalently, full replenishment).

Allow control cost to depend directly on the state.  
(ruled out until now):

Linear cost in period 1 ,  $\bar{\Gamma} > 0$ .

Period 1: Agent chooses whether to pay a cost  $F$ .

Cost paid: no control cost at all in future periods,  $\Gamma = 0$ .

Not paid: marginal cost of self control remains equal to  $\bar{\Gamma}$ .

Period 2: Agent accepts/rejects simple temptation with  
 $P < -\bar{\Gamma}S$ .

*Solution:*

Period 2: If didn't pay in period 1, then take, if paid, don't take.

Period 1:

$$\Delta(\text{pay}) = \delta\mu S - (\delta\mu S - F(1 - \delta\mu)) = F(1 - \delta\mu).$$

LR will pay if  $F$  is sufficiently small- as SR not associates no future loss with future increase in self control.

So our model doesn't capture St. Augustine's request:

“Give me chastity and continence, but not yet.”

Endogenous self-control poses a challenge for our approach.

## Conclusions:

Non-myopic short-run self provides natural way to parameterize the effect of timing and delay, and explains why commitments to avoid far-off temptations are more attractive than commitments to avoid imminent ones.

When non-linearities are important, need to track “spillovers” from one period’s self-control to future self-control costs; can do this with a stock of cognitive resources.

Adding resources lets us explain why people may “wait to commit” and “resist and then give in.”

*Ideas for future work:*

- calibrate non-myopic SR model to lab and field data
- stochastic but exogenous self-control cost plus rational habit formation
- characterization by readily understood axioms
- Endogenous self control, sophisticated SR...?