INFORMATION IN REPEATED GAMES

Larry Samuelson

Cowles Foundation Lunch Talk, March 26, 2008

I. Point of Departure: The Folk Theorem

	C	D
C	$2, 2$	$-1, 3$
D	$3, -1$	$0, 0$

The folk theorem (Fudenberg and Maskin, 1986 *Econometrica*, with many precursors and extensions, including Hörner and Olszewski, 2006 *Econometrica*): For any feasible, strictly individual rational payoff profile u, there is a discount factor $\delta(u) < 1$ such that for any higher discount factor, there is a subgame-perfect equilibrium with payoff u.

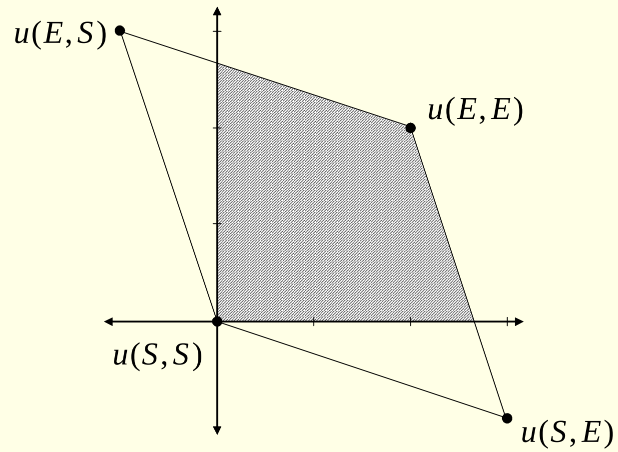

Figure 1: Feasible payoffs (the polygon and its interior) and folk-theorem outcomes (the shaded area, minus its lower boundary) for the infinitely-repeated prisoners' dilemma.

Questions:

0. Multiple equilibria?

$1, 1$	$0, 0$
$0, 0$	$1, 1$

$-1, -1$	$3, 0$
$0, 3$	$0, 0$

Questions:

0. Multiple equilibria?

1. Equilibrium selection?

Questions:

0. Multiple equilibria?

1. Equilibrium selection?

2. Behavior?

Questions:

0. Multiple equilibria?

1. Equilibrium selection?

2. Behavior?

3. Transfers?

Questions:

0. Multiple equilibria?

1. Equilibrium Selection?

2. Behavior?

3. Transfers?

4. Are initial periods too important?

II. Equilibrium Selection, Take 1: “Good” Equilibria and Reputations

It is common to focus on “good” equilibria, and reputation models provide one way of making this criterion precise.

	h	ℓ
H	$2, 3$	$0, 2$
L	$3, 0$	$1, 1$

A reputation model introduces the possibility that player 1 might be a commitment type - one who always plays H.

Reputation result (Fudenberg and Levine, 1989 *Econometrica*, 1992 *Review of Economic Studies*): For every $\epsilon > 0$, there exists discount factor $\delta < 1$ such that if player 1 is at least as least as patient as δ, then in every Nash equilibrium, 1's payoff is at least $2 - \epsilon$.

But:

1. This gives us little insight into the behavioral implications of reputations. For example, there is no reputation building.

2. Reputations are temporary (Cripps, Mailath and Samuelson, 2004 *Econometrica*, 2007 *JET*):

If (i) monitoring is noisy and (ii) player 2 has a strict best response to the commitment type of player 1, then almost surely 2's posterior probability of the type of player 1 converges to the truth.

Why?

Do we care about convergence?

Two next directions, in progress:

1. Reputation models based on separation (rather than pooling) and churning types (Mailath and Samuelson, 2001 *Review of Economic Studies*).

2. Common learning - will people who learn something commonly learn it?

	A	B	W
A	$1, 1$	$-c, -c$	$-c, 0$
B	$-c, -c$	$-c, -c$	$-c, 0$
W	$0, -c$	$0, -c$	$0, 0$

Parameter θ_A

	A	B	W
A	$-c, -c$	$-c, -c$	$-c, 0$
B	$-c, -c$	$1, 1$	$-c, 0$
W	$0, -c$	$0, -c$	$0, 0$

Parameter θ_B

Common learning (Cripps, Ely, Mailath and Samuelson, 2008 *Econometrica*):

Suppose two agents observe a sequence of signals sufficiently informative that they will each almost surely learn the value of an unknown underlying parameter. Then:

1. If agents are observing a finite set of signals, then they almost surely commonly learn the parameter.

2. If the agents observe an infinite set of signals, then there are situations in which commonly learning is impossible.

3. If the agents observe a finite set of signals whose generating process follows a Markov process, then commonly learning may be impossible.

4. Conjecture - a "grain of stationarity" suffices.

III. Equilibrium Selection, Take 2: "Simple" Behavior and Markov Equilibria

"Simple" equilibria rival "good" equilibria as a selection criterion.

Early models of simplicity (Rubinstein, 1986 *JET*, Abreu and Rubinstein, 1988 *Econometrica*, Binmore and Samuelson, 1992 *JET*, Binmore, Piccione and Samuelson 1998 *JET*) motivated simplicity as an implication of costly information processing, typically captured by automata representations of strategies.

A proposed alternative: simplicity as an implication of costly attention.

A illustrating diversion:

Consider the following symmetric version of the hawk-dove game:

	H	D
H	$\frac{V}{2} - C, \frac{V}{2} - C$	$V, 0$
D	$0, V$	$\frac{V}{2}, \frac{V}{2}$

An *evolutionarily stable strategy* in a symmetric game is a strategy s that is

(i) a best response to itself

(ii) a better response to any alternative best response s' than is s' itself.

In the (symmetric) hawk-dove game, there is a (unique) mixed ESS.

But,

1. In asymmetric games, only pure strategies can be evolutionarily stable (Selten, 1980 *JTB*).

2. All equilibria are “essentially” strict (Harsanyi, 1973 *IJGT*).

Binmore and Samuelson (2001 *GEB*, 2001 *JTB*): Consider a game with perturbed payoffs and noisy role identification.

Both of the pure equilibria and the mixed equilibrium have positive global invasion barriers.

The former are relatively high when payoff perturbations are relatively large and role identification relatively noisy.

The repeated-game version (in progress):

Consider a dynamic game. In each period agents potentially observe the history of behavior and details of the current stage game.

Suppose that attention is congested (Samuelson, 2001 *JET*). Then “states” will be endogenously (but not uniquely) determined, and will tend to be characterized by relatively relevant information.

IV. Transfers

What if players in repeated games can make transfers?

To look at this, we examine a "favor game."

"Favor banks" are repeated relationships characterized by reciprocal non-monetized favors and possibly nonstationary continuation values (Ellickson, 1991, *Order without Law*).

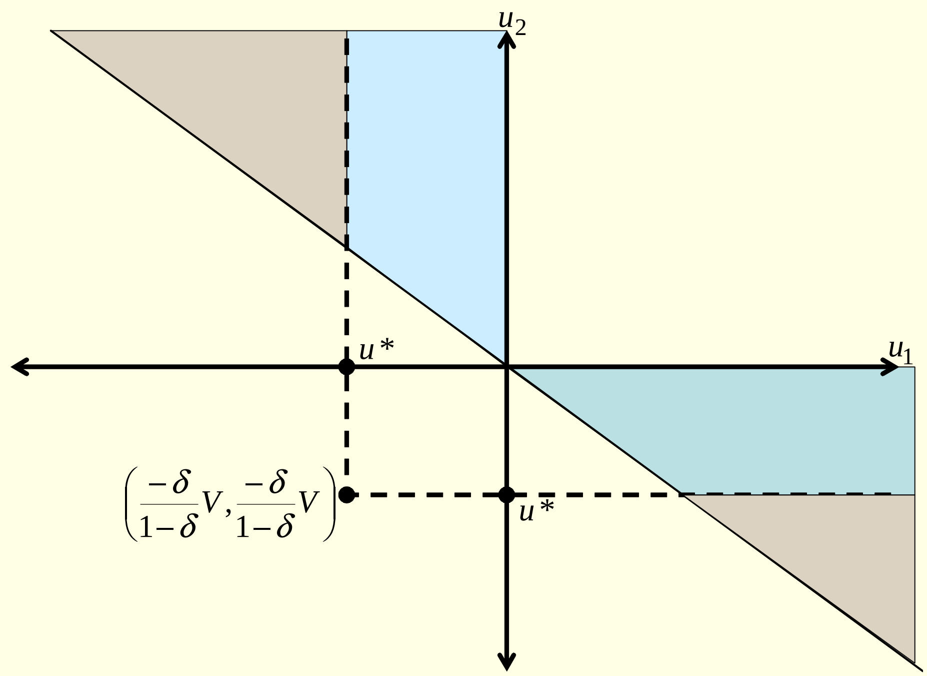

Figure 2: Best stationary equilibrium.

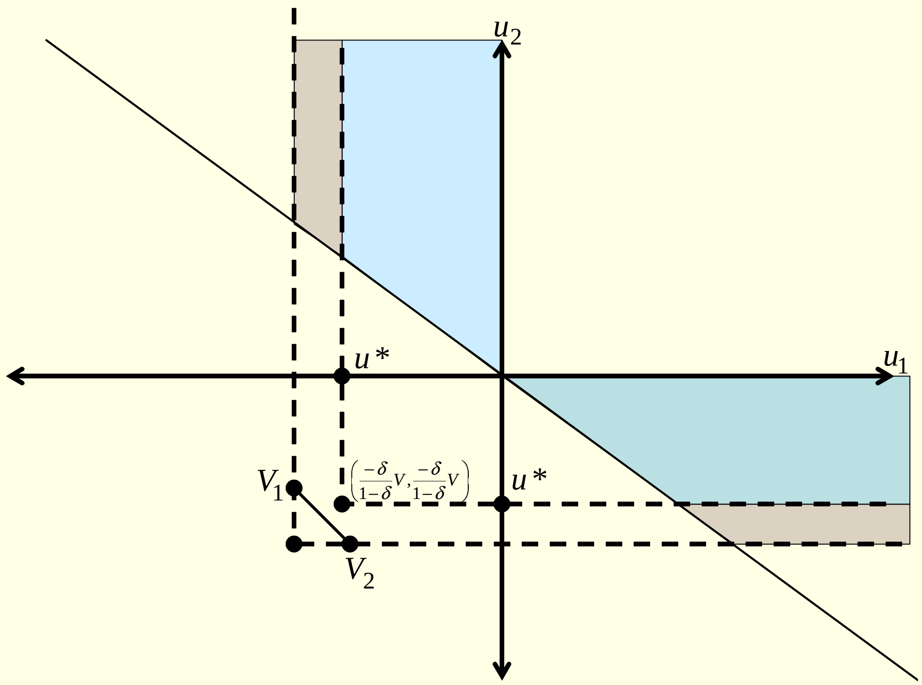

Figure 3: Nonstationary equlibrium.

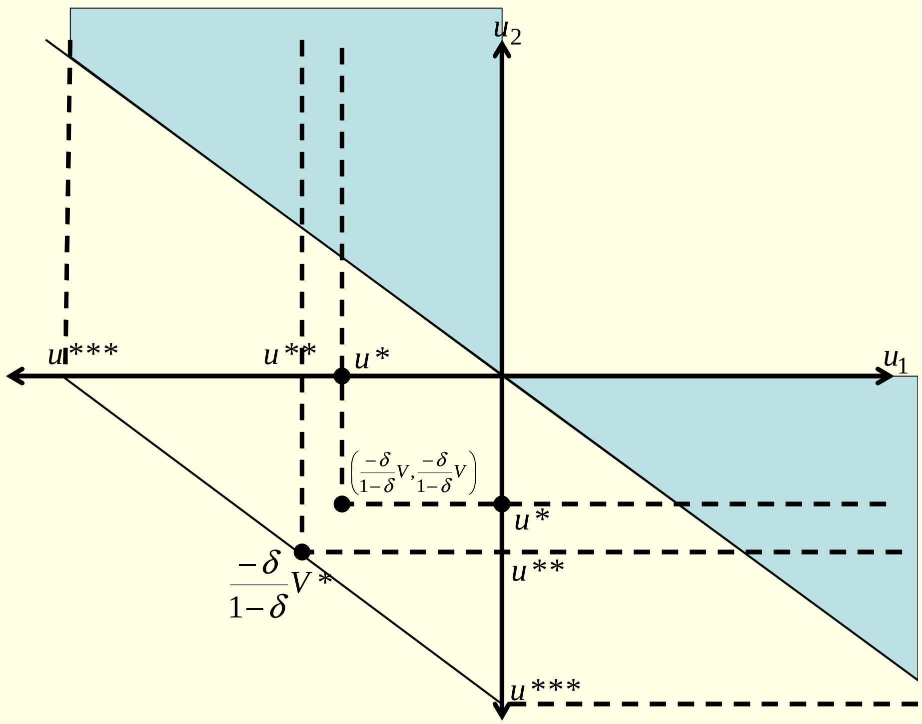

Figure 4: Stationary equilibrium with transfers.

Transfers eliminate the nonstationarity and (possibly) the necessity for repetition. There are similar results in other contexts (e.g., Levin, 2003 *AER*).

Why don't transfers render repetition moot?

One possibility is limitations on ability to make transfers (Levin,, 2003 *AER*, Abdulkadiroğlu and Bagwell).

In progress - a monopolistic-competition based theory of reciprocal favors.

V. Are Initial Periods too Important?

Reconsider the prisoners' dilemma with the independent private-monitoring technology given by

$$\begin{aligned} \text{Pr}\{y_i = c | a_j = C\} &= \text{Pr}\{y_i = d | a_j = D\} = 1 - \epsilon \\ \text{Pr}\{y_i = c | a_j = D\} &= \text{Pr}\{y_i = d | a_j = D\} = \epsilon, \end{aligned}$$

where $y_i \in \{c, d\}$ is the signal received by player i and $a_i \in \{C, D\}$ the action played by j.

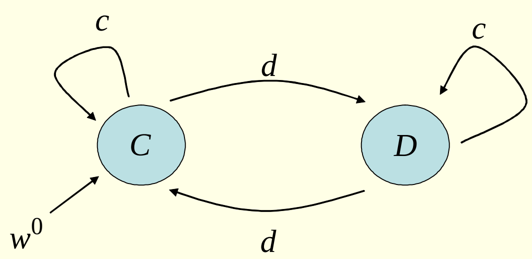

Figure 5: Perfect monitoring equilibrium.

Suppose both agents play the strategy shown in Figure 5. We can then view the players as being in one of four joint states in each period, with the states given by

$$\begin{aligned} s_1: &\quad C_1C_2 \\ s_2: &\quad C_1D_2 \\ s_3: &\quad D_1C_2 \\ s_4: &\quad D_1D_2. \end{aligned}$$

Having entered a period in a given state and observed their signals, the players move to a new state according to the transition matrix:

	C_1C_2	C_1D_2	D_1C_2	D_1D_2
C_1C_2	cc	cd	dc	dd
C_1D_2	cd	cc	dd	dc
D_1C_2	dc	dd	cc	cd
D_1D_2	dd	dc	cd	cc

The probabilities attached to these transitions are then:

$$\begin{array}{ccccc} & C_1C_2 & C_1D_2 & D_1C_2 & D_1D_2 \\ C_1C_2 & (1-\epsilon)^2 & (1-\epsilon)\epsilon & \epsilon(1-\epsilon) & \epsilon^2 \\ C_1D_2 & \epsilon^2 & \epsilon(1-\epsilon) & (1-\epsilon)\epsilon & (1-\epsilon)^2 \\ D_1C_2 & \epsilon^2 & \epsilon(1-\epsilon) & (1-\epsilon)\epsilon & (1-\epsilon)^2 \\ D_1D_2 & (1-\epsilon)^2 & (1-\epsilon)\epsilon & \epsilon(1-\epsilon) & \epsilon^2 \end{array} \tag{1}$$

This transition matrix induces a stationary distribution over states. Let n_i be the probability attached to state s_i to by this stationary distribution. The stationary distribution thus induces correlation into the agent's states with it being more likely that they are in the same state than in mismatched states.

Proposition 1 *Fix $\delta \in (1/2, 1)$. Then for sufficiently small ϵ, it is an equilibrium for players 1 and 2 to each play the strategy shown in Figure 5, with a correlated choice of initial states govern by (n_1, n_2, n_3, n_4).*

We can interpret this as a correlated equilibrium of a new game, or a sequential equilibrium of a continuation game.

For the latter interpretation to be helpful, we need the game to stretch to $-\infty$ as well as ∞.

This example has used the special structure of this game quite heavily.

VI. Conclusion