Sophisticated Monetary Policies

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Barro, Lucas-Stokey Approach to Policy

- Solve Ramsey problem
  - choose \( a = \{\text{policies, prices, allocations}\} \) to
    \[
    \max Utility \\
    s.t. \quad a \in \text{Competitive Equilibrium}
    \]

Answer: Ramsey outcome \( a^* \) function of exogenous shocks
Barro, Lucas-Stokey Approach to Policy

- Solve Ramsey problem
  - choose \( a = \{\text{policies, prices, allocations}\} \) to
    \[
    \text{max } \text{Utility} \\
    \text{s.t. } a \in \text{Competitive Equilibrium}
    \]
  
  Answer: Ramsey outcome \( a^* \) function of exogenous shocks

- Left open: Implementation
  - Designing policies so Ramsey outcome is unique equilibrium
Our Solution to Implementation Problem

- Sophisticated policies can
  - Depend on histories of agents’ actions
  - Differ on and off equilibrium path
Our Solution to Implementation Problem

- *Sophisticated policies* can
  - Depend on histories of agents’ actions
  - Differ on and off equilibrium path

- Example
  - If $\pi_t \in Acceptable\ Region$, follow Ramsey policy
  - If not, switch to alternative policy (*reversion*)
Our Solution to Implementation Problem

- *Sophisticated policies* can
  - Depend on histories of agents’ actions
  - Differ on and off equilibrium path

- Main Result
  - Can uniquely implement any desired outcome
Our Solution to Implementation Problem

• **Sophisticated policies** can
  
  ○ Depend on histories of agents’ actions
  
  ○ Differ on and off equilibrium path

• **Main Result**
  
  ○ Can uniquely implement any desired outcome

• **Note:**
  
  ○ Differs from “implementation via nonexistence”
  
  ○ Here continuation equilibria exist after all deviations
Follow Barro, Lucas-Stokey Approach to Policy
Follow Barro, Lucas-Stokey Approach to Policy Implementation Problem Now Solved
• Sargent-Wallace result
  ○ Indeterminacy if interest rates depend only on exogenous events

• Indeterminacy risky
Contrasts with Literature

- Our approach: Implementation by discouraging deviations
- Literature: Implementation via nonexistence
Contrasts with Literature

- Our approach: Implementation by discouraging deviations
- Literature: Implementation via nonexistence

Our Approach: Discourage Deviations
Contrasts with Literature

- Our approach: Implementation by discouraging deviations
- Literature: Implementation via nonexistence

**Literature: Nonexistence after Deviations**
Contrast Concerning Taylor Principle

- Literature: Taylor principle needed for uniqueness
  - Taylor principle
    \[ i_t = \bar{i} + \phi(\pi_t - \bar{\pi}), \quad \phi > 1 \]
    raise interest rates more than 1 for 1 with inflation

Diagram:
- Outcome path
- No equilibrium (inflation explodes)
- No equilibrium (inflation explodes)
Main Results

- Simple Sticky Price model
  - Implement with sophisticated policies
  - Indeterminacy with linear feedback rules

- Extend to
  - New Keynesian model
  - Imperfect Information
Simple Sticky Price Model
Outline of Section

- Model Setup and 4 Equilibrium Conditions
- Implement with Sophisticated Policies
- Restricted Policies
  - Equilibrium Characterization
  - Equilibrium Inefficiency
Setup and 4 Equilibrium Conditions
Setup: Technology and Preferences

- Final good technology

\[ Y_t = \left[ \int Y_t(j)^\theta \, dj \right]^{1/\theta} \]

- Intermediate good technology

\[ Y_t(j) = L_t(j) \]

- Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \]

where \( L_t = \int L_t(j) \)

- Cash-in-advance
Setup: Technology and Preferences

- Final good technology

\[ Y_t = \left[ \int Y_t(j)^{\theta} \, dj \right]^{\frac{1}{\theta}} \]

- Intermediate good technology (some producers sticky \( p \) some flexible \( p \))

\[ Y_t(j) = L_t(j) \]

- Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \]

where \( L_t = \int L_t(j) \)

- Cash-in-advance
One-Period Stickiness

- Let $h_{t-1}$ be history of past actions and shocks at start of period $t$

\[
\begin{align*}
  h_{t-1} & \quad h_{gt} = (h_{t-1}, x_t) \\
  h_{yt} & = (h_{gt}, \text{policy}, \eta_t) \\
  h_t &
\end{align*}
\]

- Strategies of agents and central bank depend on relevant history
- Sticky price producers only interesting strategic players
One-Period Stickiness

- Let $h_{t-1}$ be history of past actions and shocks at start of period $t$

$h_{t-1}$  $h_{gt} = (h_{t-1}, x_t)$  $h_{yt} = (h_{gt}, \text{policy, } \eta_t)$  $h_t$

- Sticky price
- Regime $\delta(h_{gt})$
- Shock $\eta_t$
- Flexible prices set

$x_t(j, h_{t-1})$  $\mu(h_{gt})$  Policy

- Strategies of agents and central bank depend on relevant history
- Sticky price producers only interesting strategic players
- Next, 4 equations of this “New Classical” sticky price model
Derive 4 equations of New Classical System

- Sticky price producer’s best response
  
  - price set as markup over “expected” marginal cost

\[
P_{st}(j) = \frac{1}{\theta} \frac{E_{t-1}\left[Q_t P_t^{1-\theta} W_t y_t\right]}{E_{t-1}\left[Q_t P_t^{1-\theta} y_t\right]}\]
Derive 4 equations of New Classical System ______________________

- Sticky price producer’s best response

  - price set as markup over “expected” marginal cost

    $$P_{st}(j) = \frac{1}{\theta} \frac{E_{t-1} \left[ \frac{1}{Q_t} P_t^{1-\theta} W_t y_t \right]}{E_{t-1} \left[ \frac{1}{Q_t} P_t^{1-\theta} y_t \right]}$$

    when log linearized gives

    $$p_{st}(j) = E_{t-1} \left[ p_t + \gamma y_t \right]$$

Note: Actions depend on expectations of future.
Derive 4 equations of New Classical System

- Sticky price producer’s best response

  - price set as markup over “expected” marginal cost

\[
P_{st}(j) = \frac{1}{\theta} \left[ \frac{1}{E_{t-1}} \left[ Q_t \frac{P_{t}^{1-\theta}}{P_{t}^{1-0}} W_t y_t \right] \right]
\]

letting \( x_t(j) = p_{st}(j) - p_{t-1} \) and \( \pi_t = p_t - p_{t-1} \)

\[(1) \quad x_t(j) = E_{t-1}[\gamma y_t + \pi_t] \]
Derive 4 equations of New Classical System

- New Classical Phillips Curve
  - use flexible price producers’ problem and aggregate price index to get

\[ \pi_t = \kappa y_t + x_t \]
Derive 4 equations of New Classical System

- Cash-in-advance model delivers

  - Log Linearized Euler equation ($\eta_t$ is flight-to-quality shock)

    $$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \eta_t$$  

  - Log Linearized quantity equation (cash-in-advance)

    $$\pi_t = \mu_t - (y_t - y_{t-1})$$
New Classical System of 4 Equations

1. Sticky producers’ best response

\[ x_t(j) = E_{t-1}(\gamma y_t + \pi_t) \]

2. New Classical Phillips curve

\[ \pi_t = \kappa y_t + x_t \]

3. Euler equation

\[ y_t = E_t y_{t+1} - \sigma(i_t - x_{t+1}) + \eta_t \]

4. Quantity theory

\[ \pi_t = \mu_t - (y_t - y_{t-1}) \]
New Classical System with Strategies and Interest Rate Regime

1. Sticky producers’ best response

\[ x(j, h_{t-1}, x_t) = E\left[ \gamma y(h_{yt}) + \pi(h_{yt}) \mid h_{t-1}, x_t \right] \]

2. New Classical Phillips curve

\[ \pi(h_{yt}) = \kappa y(h_{yt}) + x(h_{t-1}) \]

3. Euler equation

\[ y(h_{yt}) = E\left[ y(h_{yt+1}) \mid h_{yt} \right] - \sigma \left( \mu(h_{gt}) - x(h_t) \right) + \eta_t \]

4. Quantity theory

\[ \pi(h_{yt}) = \mu(h_{yt}) - y(h_{yt}) - y_{t-1} \]
Outcome Path versus Strategies

- Strategies say what to do at all possible histories
- Outcome path describes what actually happens:

![Diagram of Outcome Path and Strategies](image-url)
Outcome Path versus Strategies

- Strategies say what to do at all possible histories
- Outcome path describes what actually happens:

\[ \{x_t(\eta^{t-1}), i_t(\eta^{t-1}), y_t(\eta^t), \pi_t(\eta^t)\} \equiv \{a_t(\eta^t)\} \]

How Strategies Induce Future Histories
• Fix strategies for all players \( \{ x_t (\cdot) \}, \{ \delta_t (\cdot), i_t (\cdot), \mu_t (\cdot) \}, \{ y_t (\cdot), \pi_t (\cdot) \} \)

• Strategies recursively define future histories

  ○ Given \( h_{t-1} \), history \( h_t \) generated from strategies and realization of \( \eta_t \)

\[
h_{t-1} \rightarrow x_t = x_t \left( h_{t-1} \right)
\]

\[
\Rightarrow h_{gt} = (h_{t-1}, x_t) \rightarrow \delta_t = \delta_t \left( h_{gt} \right), i_t = i_t \left( h_{gt} \right)
\]

\[
\Rightarrow h_{yt} = (h_{gt}, \delta_t, i_t, \eta_t) \rightarrow y_t \left( h_{yt} \right), \pi_t \left( h_{yt} \right)
\]

and so on
Outcome Path $\{a_t(\eta^t)\}$

1. Sticky producers’ equilibrium response

$$x_t(\eta^{t-1}) = E\left[ \gamma y_t(\eta^t) + \pi_t(\eta^t) \mid \eta^{t-1} \right]$$

2. New Classical Phillips curve

$$\pi_t(\eta^t) = \kappa y_t(\eta^t) + x_t(\eta^{t-1})$$

3. Euler equation

$$y_t(\eta^t) = E\left[ y_{t+1}(\eta^{t+1}) - \sigma(i_{t+1}(\eta^t) - x_{t+1}(\eta^t)) \right] + \eta_t$$

4. Quantity theory

$$\pi_t(\eta^t) = \mu_t(\eta^t) - (y_t(\eta^t) - y_{t-1}(\eta^{t-1}))$$
Implementation with Sophisticated Policies
Implementation with Sophisticated Policies

Policies can differ on and off the equilibrium path
Implementation Theorem

- Suppose some given outcome path

\[ a_t^* (\eta^t) \]

solves 4 equations of New Classical system. There are sophisticated policies with a unique equilibrium which generate given outcome path.
Sketch of Proof

- To implement specific outcome path \( \{x_t^*(\eta^{t-1}), i_t^*(\eta^{t-1}), y_t^*(\eta^t), \pi_t^*(\eta^t)\} \)
  
  o If \( \tilde{x}_t = x_t^* \) stay with original policy \( i_t^* \)
  
  o If \( \tilde{x}_t \neq x_t^* \) switch to money for one period and choose money to generate original inflation \( \pi_t^* \)

- Proof exploits controllability with money i.e.
  
  o Central bank can induce any best response by individual sticky price producer following aggregate deviation \( \tilde{x}_t \)
Sketch of Proof

- In money regime $\pi_t$ and $y_t$ determined by
  - Flexible producers’ decisions
    $$\pi_t = \kappa y_t + x_t$$
  - Cash-in-advance in first differences
    $$\pi_t = \mu_t - (y_t - y_{t-1})$$
- So for any sticky producer choice $x_t$ (with $y_{t-1}$ given)
  - $\pi_t$ and $y_t$ uniquely determined by $\mu_t$
  - Monotone in $\mu_t$
• Best response of sticky producer

\[ x_t(j) = E_{t-1}[\gamma y_t + \pi_t] \]

• So by appropriately varying \( \mu_t \) can make

\[ x_t(j) \neq \tilde{x}_t \]
- Best response of sticky producer

\[ x_t(j) = E_{t-1}[\gamma y_t + \pi_t] \]

- So by appropriately varying \( \mu_t \) can make

\[ x_t(j) \neq \tilde{x}_t \]

- Under our reversion scheme: sticky producer \( j \)'s best response to \( \tilde{x}_t \)

\[ x_t(i) = x_t^* + \frac{1}{\kappa}(x_t^* - \tilde{x}_t) \]

○ Since \( x_t(i) \neq \tilde{x}_t \) when \( \tilde{x}_t \neq x_t^* \) then deviation deterred. Q.E.D.
Recap

- Unique Implementation with Sophisticated Policies

- Next, show why regime switching is necessary
Necessity of Regime Switching
• Linear feedback rule

\[ i_t = \bar{i}_t + \sum_{s=0}^{\infty} \phi_{xs} x_{t-s} + \sum_{s=1}^{\infty} \phi_{ys} y_{t-s} + \sum_{s=1}^{\infty} \phi_{\pi s} \pi_{t-s} \]

• Necessarily yields indeterminacy

○ Under this rule continuum of competitive equilibria

\[ x_{t+1} = i_t + c\eta_t, \quad \pi_t = x_t + k(1+\psi c)\eta_t, \quad y_t = (1+\psi c)\eta_t \]

indexed by \(c\) and \(x_0\).
· King rule

\[ i_t = i_t^* + \phi(x_t - x_t^*) \]

where \( i_t^*, x_t^* \) are desired outcomes.

· Our approach: King rule yields indeterminacy
Recap

- Simple sticky price model
  - Unique implementation with sophisticated policies
  - Necessity of regime switching

- Next, extend to standard New Keynesian model with
  - Sticky price producers use Calvo-pricing
  - No flexible producers
Standard New Keynesian Model

Model Setup
Standard New Keynesian Model

- Timing (w/o shocks)

\[ h_{t-1} \quad h_{gt} \quad h_{yt} \]

Fraction 1 – \( \alpha \) reset prices to \( p_{st}(j,h_{t-1}) \)

Regime \( \delta(h_{gt}) \)

Policy \( i_t(h_{gt}) \) or \( \mu(h_{gt}) \)

- Only 1 equation changes from simple sticky price model

  ○ Sticky price producers’ best response
Sticky Price Producers’ Best Response

• Solution using price levels is

\[ p_{st}(j) = (1 - \alpha\beta) \left[ \sum_{r=0}^{\infty} (\alpha\beta)^{r-t} (y_r + p_r) \right] \]

• Implies Transversality condition

\[ p_{st}(j) = (1 - \alpha\beta)[y_t + p_t] + \alpha\beta p_{st+1} \]

\[ \lim_{T \to \infty} (\alpha\beta)^T p_{st} = 0 \]

• Note: \( u(c,l) = \log c - b(1-l) \)
Sticky Price Producers’ Best Response

- Solution using price levels is

\[ p_{st}(j) = (1 - \alpha \beta) \left[ \sum_{r=0}^{\infty} (\alpha \beta)^{r-t} (y_r + p_r) \right] \]

- Implies Transversality condition

\[ p_{st}(j) = (1 - \alpha \beta)[y_t + p_t] + \alpha \beta p_{st+1} \]

\[ \lim_{T \to \infty} (\alpha \beta)^T p_{st} = 0 \]

- Transversality plays a role later
Sticky Price Producers’ Best Response

- Solution using price levels is

\[ p_{st}(j) = (1 - \alpha \beta) \left[ \sum_{r=0}^{\infty} (\alpha \beta)^{r-t} (y_r + p_r) \right] \]

- Controllable with reversion to level of money \( \bar{m} \)
  
  - Cash-in-advance implies
    \[ y_r + p_r = \bar{m} \]
  
  - So
    \[ p_{st}(j) = \bar{m} \]

  \( \Rightarrow \) Unique implementation easy with money
Recap

- Unique implementation with reversion to money

- Next: unique implementation with reversion to interest rates
Standard New Keynesian Model

Unique Implementation with Reversion to Interest Rates
How Do We Implement It?

- Along equilibrium path
  \[ i_t(h_{gt}) = i_t^*(\eta^{t-1}) \]

- Any deviation at time \( t \) switch to new regime
  \[ i_t(h_{gt}) = \bar{i} \text{ at } t \text{ with } \bar{i} \neq 0 \]

and

\[ i_s(h_{gs}) = \hat{\phi} x_s \text{ all } s \geq t + 1 \]

- To work, solution to NK system must be unique under \( \hat{\phi} \)
Add Economic Boundary Conditions to NK System

- Transversality condition

- Output bounded above

- Inflation bounded below (interest rates nonnegative)
When are solutions unique under $\hat{\phi}$?

- Economic bounds
  - Regions of indeterminate and unique solutions

![Diagram showing regions of indeterminate and unique solutions]
When are solutions unique under $\hat{\phi}$?

- Economic bounds

  - Regions of indeterminate and unique solutions

  
  \[
  \begin{align*}
  &\text{ind.} \quad \text{unique} \quad \text{indeterminate} \quad \text{unique} \\
  &1 \quad \phi \quad \bar{\phi} \quad \hat{\phi}
  \end{align*}
  \]

  \[
  \begin{align*}
  &Y_t \to \infty \text{ or } Y_t \to 0 \quad (TVC) \\
  &\pi_t \to -\infty \quad \pi_t \to \infty
  \end{align*}
  \]

  - Result: Unique implementation when $\bar{i} \neq 0$ and $\hat{\phi}$ in unique region
King Rule Works but Differently from Literature

- King rule

\[ i_t = i_t^* + \phi(\pi_t - \pi_t^*), \quad \phi > 1 \]

- Our approach
  - Implements bounded outcomes
  - After deviation, returns to desired outcomes

- Literature
  - After deviation, leads to nonexistence (\( \pi \) explodes)
Standard New Keynesian Model
Robust to Imperfect Information
Imperfect Information

- Imperfect monitoring
  - See agents’ actions very period with probability $q$

- Measurement error
  - See agents’ actions with measurement error
Imperfect Monitoring

- Central Bank sees
  - price setter’s decision $x_t$ with probability $q$
  - nothing with probability $1 - q$
Imperfect Monitoring

- Central Bank sees
  - price setter’s decision $x_t$ with probability $q$
  - nothing with probability $1 - q$

- Result: If detection probability $q$ sufficiently high in that
  \[
  \frac{1}{1 - q} > 1 + \beta q + (1 - q) \kappa \sigma
  \]
  then sophisticated policies can uniquely implement any outcomes.
Imperfect Monitoring

- Central Bank sees
  - price setter’s decision $x_t$ with probability $q$
  - nothing with probability $1 - q$

- Result: If detection probability $q$ sufficiently high in that

$$\frac{1}{1-q} > 1 + \beta q + (1 - q)\kappa\sigma$$

then sophisticated policies can uniquely implement any outcomes.

- Mechanics
  - $q$ acts like discounting
  - both roots bigger than 1 so uniqueness following a deviation
Measurement Error

- See $\tilde{x}_t = x_t + \varepsilon_t$

- Use modified King Rule

$$i_t = i_t^* + \phi (\tilde{x}_t - x_t^*)$$

- Unique equilibrium has $x_t = x_t^*, \ y_t = y_t^* - \psi \phi \varepsilon_t$

- For small measurement error, can approximately implement bounded outcomes
Conclusion

• Follow Barro, Lucas-Stokey
  ○ Check controllability of best responses
  ○ If controllable, move on to next paper
  ○ If not?...

• Extend to financial crises, fiscal policy, and so on