Sophisticated Monetary Policies

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Barro, Lucas-Stokey Approach to Policy

- Solve Ramsey problem
  - choose $a = \{policies, prices, allocations\}$ to

$$\max \; Utility$$
$$s.t. \;\; a \in Competitive\,Equilibrium$$

Answer: Ramsey outcome a^* function of exogenous shocks

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Answer: Ramsey outcome a^* function of exogenous shocks

- Left open: Implementation
    - Designing policies so Ramsey outcome is unique equilibrium

Our Solution to Implementation Problem

- *Sophisticated policies* can
  - Depend on histories of agents' actions
  - Differ on and off equilibrium path

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- Example
    - If $\pi_t \in \mathit{Acceptable\ Region}$, follow Ramsey policy
    - If not, switch to alternative policy (*reversion*)

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- Main Result
  - Can uniquely implement any desired outcome

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- Note:
  - Differs from "implementation via nonexistence"
  - Here continuation equilibria exist after all deviations

Message of Our Paper

Follow Barro, Lucas-Stokey Approach to Policy

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Follow Barro, Lucas-Stokey Approach to Policy

Implementation Problem Now Solved

Implementation: A Nontrivial Problem in Monetary Models ________

- Sargent-Wallace result
  - Indeterminacy if interest rates depend only on exogenous events
- Indeterminacy risky

Contrasts with Literature

- Our approach: Implementation by discouraging deviations

- Literature: Implementation via nonexistence

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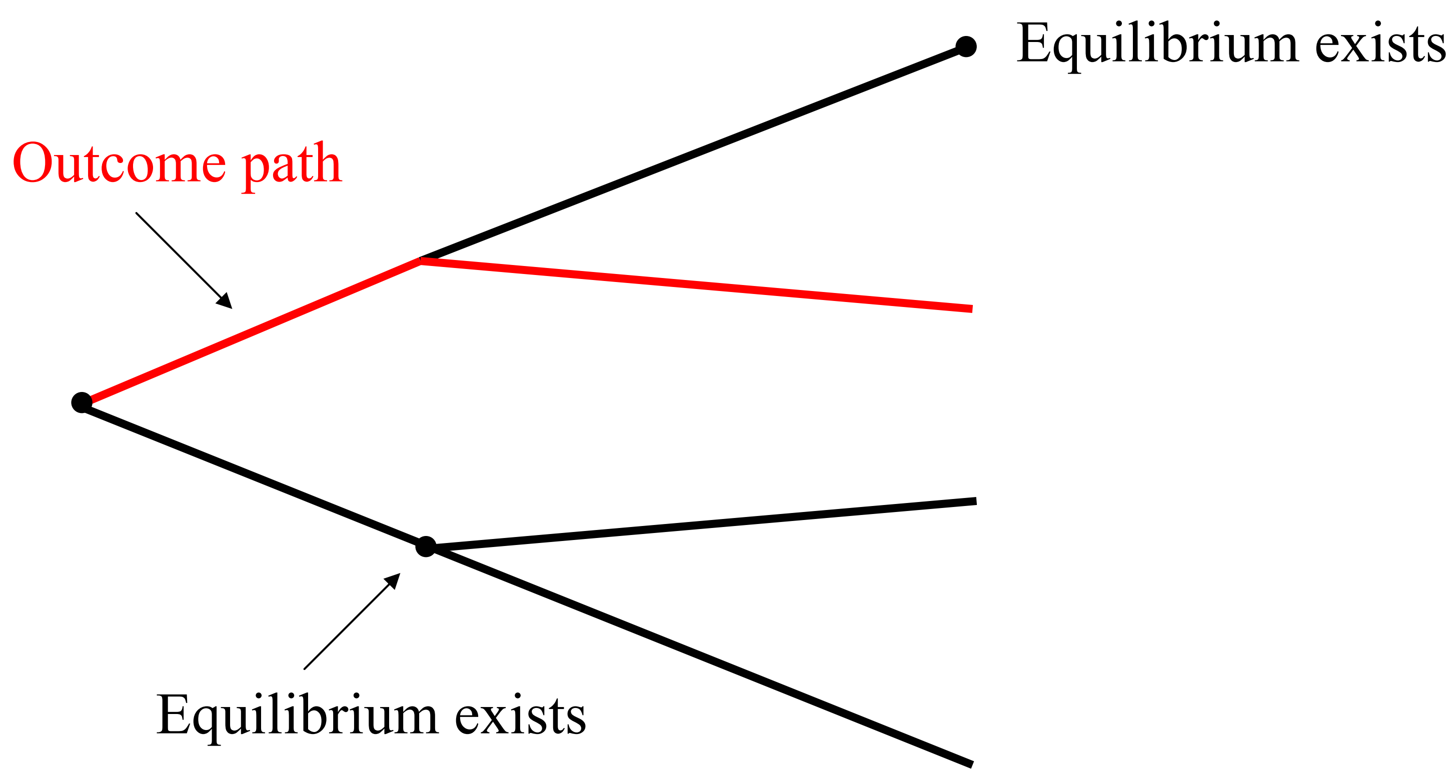
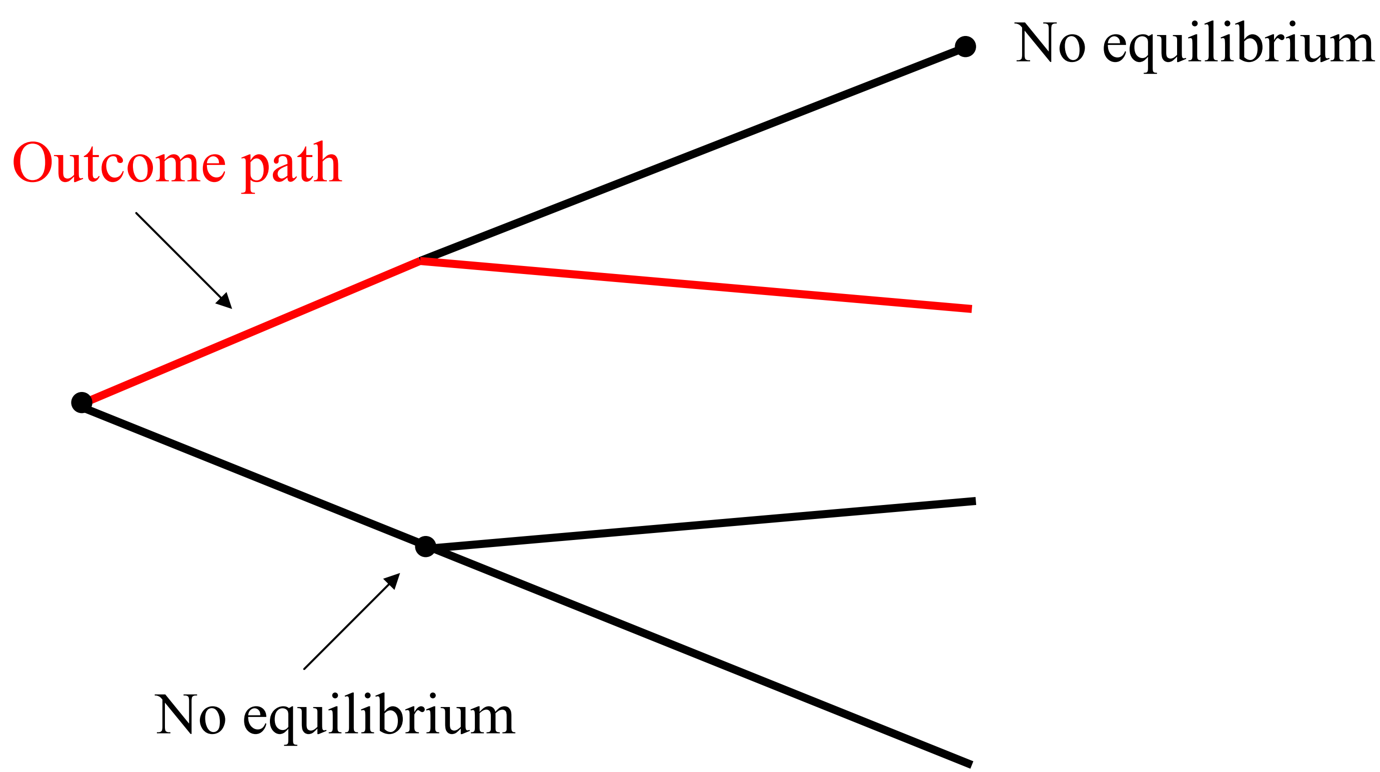
Our Approach: Discourage Deviations



Contrasts with Literature

- Our approach: Implementation by discouraging deviations
- Literature: Implementation via nonexistence

Literature: Nonexistence after Deviations

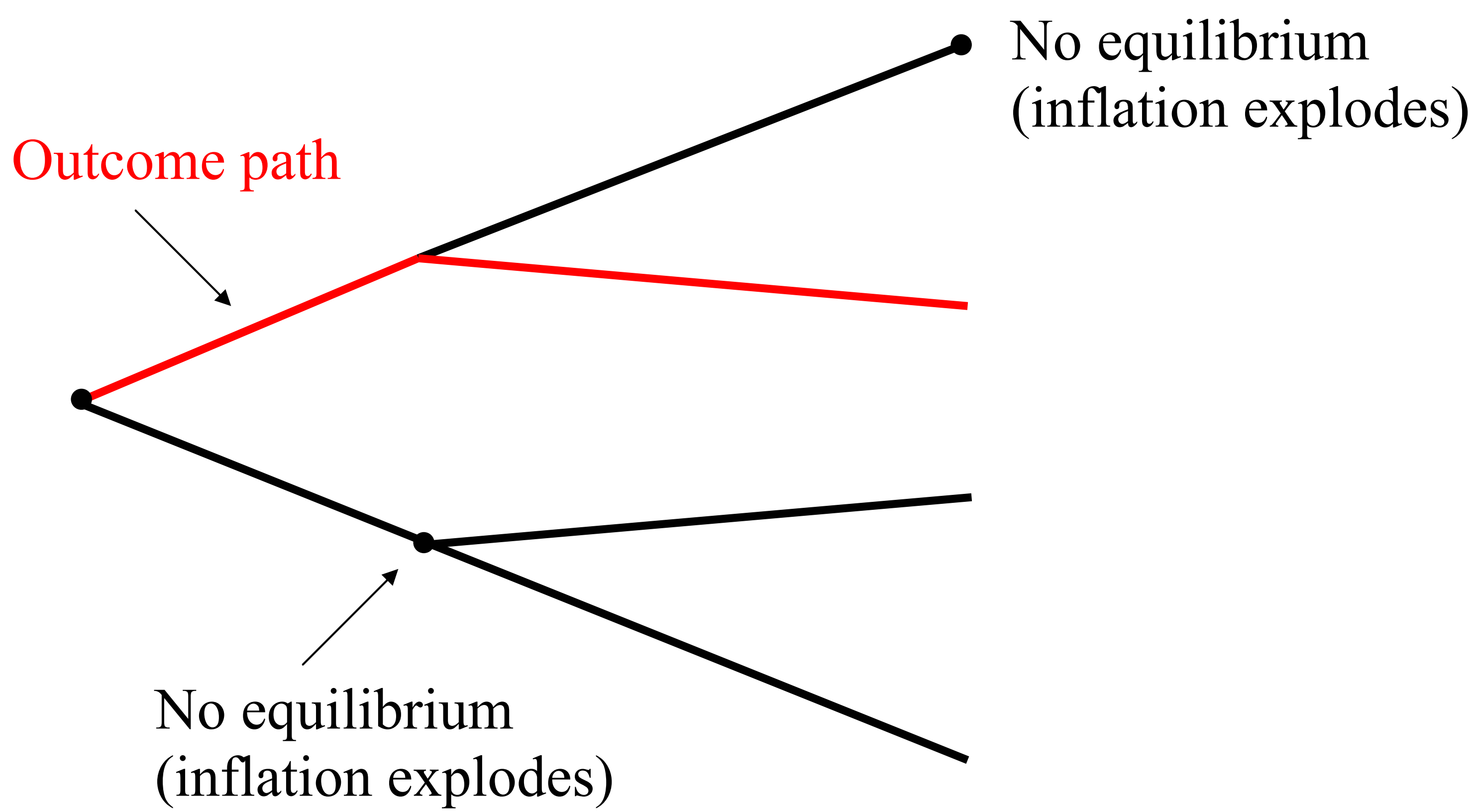


Contrast Concerning Taylor Principle

- Literature: Taylor principle needed for uniqueness
  - Taylor principle

$$i_t = \bar{i} + \phi(\pi_t - \bar{\pi}), \quad \phi > 1$$

raise interest rates more than 1 for 1 with inflation



Main Results

- Simple Sticky Price model
    - Implement with sophisticated policies
    - Indeterminacy with linear feedback rules

- Extend to
    - New Keynesian model
    - Imperfect Information

Simple Sticky Price Model

Outline of Section

- Model Setup and 4 Equilibrium Conditions
- Implement with Sophisticated Policies
- Restricted Policies
  - Equilibrium Characterization
  - Equilibrium Inefficiency

Setup and 4 Equilibrium Conditions

Setup: Technology and Preferences

- Final good technology

$$Y_t = \left[\int Y_t(j)^{\theta} dj \right]^{\frac{1}{\theta}}$$

- Intermediate good technology

$$Y_t(j) = L_t(j)$$

- Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

where $L_t = \int L_t(j)$

- Cash-in-advance

Setup: Technology and Preferences

- Final good technology

$$Y_t = \left[\int Y_t(j)^{\theta} dj \right]^{\frac{1}{\theta}}$$

- Intermediate good technology (some producers sticky *p* some flexible *p*)

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- Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

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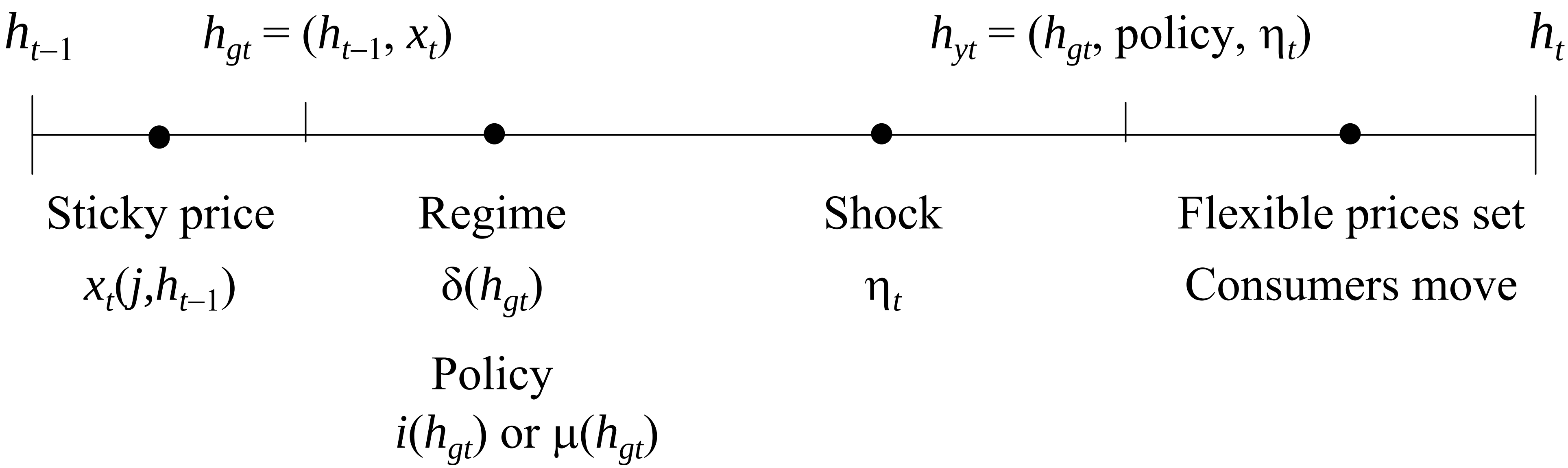
One-Period Stickiness

- Let h_{t-1} be history of past actions and shocks at start of period t



- Strategies of agents and central bank depend on relevant history
- Sticky price producers only interesting strategic players

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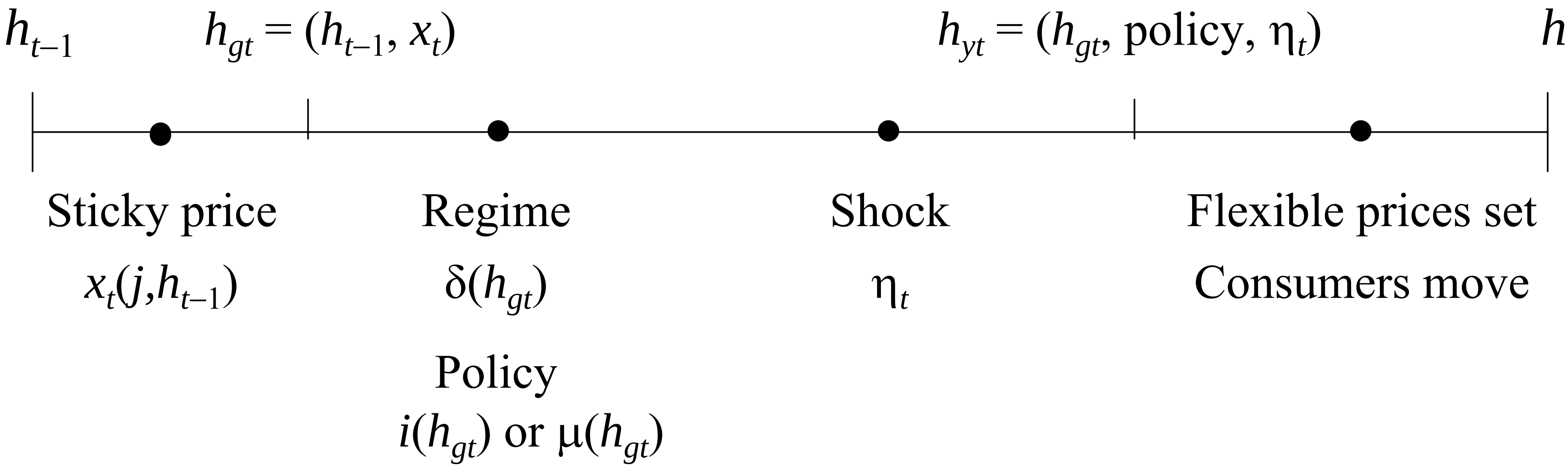

- Strategies of agents and central bank depend on relevant history
- Sticky price producers only interesting strategic players
- Next, 4 equations of this "New Classical" sticky price model

Derive 4 equations of New Classical System

- Sticky price producer's best response

  - price set as markup over "expected" marginal cost

$$P_{st}(j) = \frac{1}{\theta} \frac{E_{t-1}\left[Q_t \ P_t^{\frac{1}{1-\theta}} W_t \ y_t \right]}{E_{t-1}\left[Q_t \ P_t^{\frac{1}{1-\theta}} \ y_t \right]}$$

Derive 4 equations of New Classical System

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when log linearized gives

$$p_{st}(j) = E_{t-1}\left[p_t + \gamma y_t\right]$$

Note: Actions depend on expectations of future.

Derive 4 equations of New Classical System

- Sticky price producer's best response

  - price set as markup over "expected" marginal cost

$$P_{st}(j) = \frac{1}{\theta} \frac{E_{t-1}\left[Q_t \, P_t^{\frac{1}{1-\theta}} W_t \, y_t\right]}{E_{t-1}\left[Q_t \, P_t^{\frac{1}{1-\theta}} \, y_t\right]}$$

letting $x_t(j) \equiv p_{st}(j) - p_{t-1}$ and $\pi_t \equiv p_t - p_{t-1}$

(1) $$x_t(j) = E_{t-1}\left[\gamma \, y_t + \pi_t\right]$$

Derive 4 equations of New Classical System

- New Classical Phillips Curve
  - use flexible price producers' problem and aggregate price index to get

(2) $$\pi_t = \kappa y_t + x_t$$

Derive 4 equations of New Classical System

- Cash-in-advance model delivers
  - Log Linearized Euler equation (η_t is flight-to-quality shock)

(3) $$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \eta_t$$

  - Log Linearized quantity equation (cash-in-advance)

(4) $$\pi_t = \mu_t - (y_t - y_{t-1})$$

New Classical System of 4 Equations

1. Sticky producers' best response

$$x_t(j) = E_{t-1}\left(\gamma\, y_t + \pi_t\right)$$

2. New Classical Phillips curve

$$\pi_t = \kappa\, y_t + x_t$$

3. Euler equation

$$y_t = E_t y_{t+1} - \sigma(i_t - x_{t+1}) + \eta_t$$

4. Quantity theory

$$\pi_t = \mu_t - \left(y_t - y_{t-1}\right)$$

New Classical System with Strategies and Interest Rate Regime______

1. Sticky producers' best response

$$x(j,h_{t-1},x_t) = E\left[\gamma\, y\left(h_{yt}\right) + \pi\left(h_{yt}\right) \mid h_{t-1}, x_t\right]$$

2. New Classical Phillips curve

$$\pi\left(h_{yt}\right) = \kappa\, y\left(h_{yt}\right) + x\left(h_{t-1}\right)$$

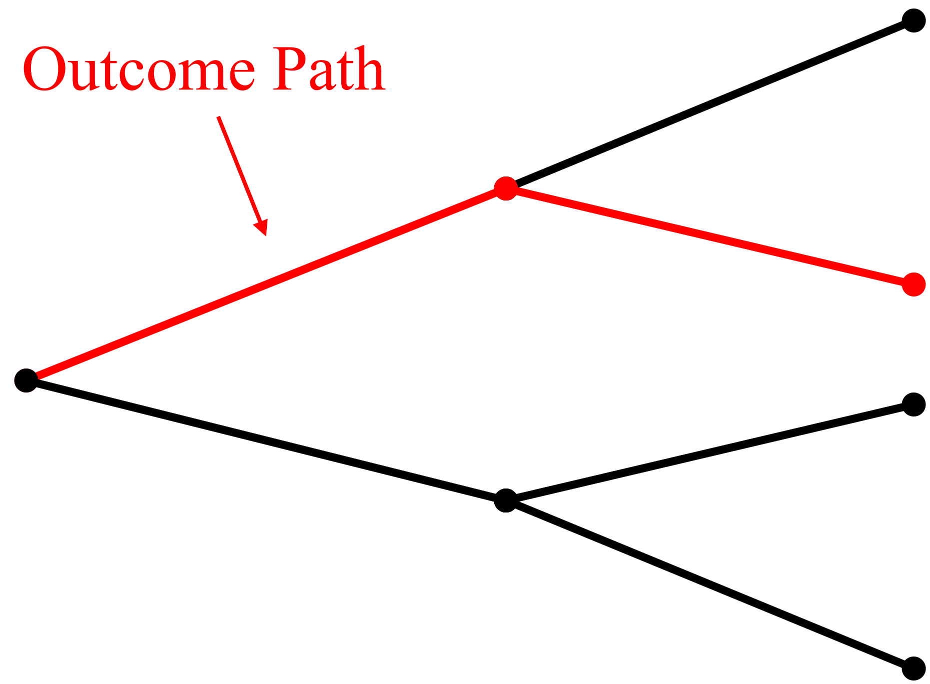
3. Euler equation

$$y\left(h_{yt}\right) = E\left[y\left(h_{yt+1}\right) \mid h_{yt}\right] - \sigma\left(i\left(h_{gt}\right) - x\left(h_t\right)\right) + \eta_t$$

4. Quantity theory

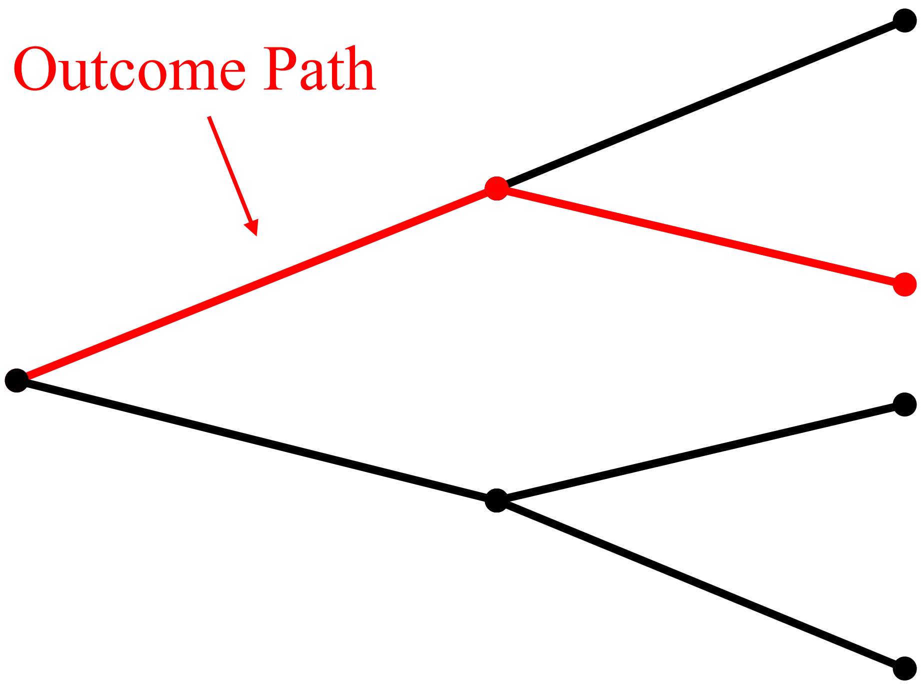
$$\pi\left(h_{yt}\right) = \mu\left(h_{yt}\right) - \left(y\left(h_{yt}\right) - y_{t-1}\right)$$

Outcome Path versus Strategies



- Strategies say what to do at all possible histories
- Outcome path describes what actually happens:

Outcome Path versus Strategies



- Strategies say what to do at all possible histories
- Outcome path describes what actually happens:
- $\left\{x_t\left(\eta^{t-1}\right), i_t\left(\eta^{t-1}\right), y_t\left(\eta^t\right), \pi_t\left(\eta^t\right)\right\} \equiv \left\{a_t\left(\eta^t\right)\right\}$

How Strategies Induce Future Histories

- Fix strategies for all players $\{x_t(\bullet)\}$, $\{\delta_t(\bullet), i_t(\bullet), \mu_t(\bullet)\}$, $\{y_t(\bullet), \pi_t(\bullet)\}$

- Strategies recursively define future histories

  - Given h_{t-1}, history h_t generated from strategies and realization of η_t

$$h_{t-1} \rightarrow x_t = x_t(h_{t-1})$$

$$\Rightarrow h_{gt} = (h_{t-1}, x_t) \rightarrow \delta_t = \delta_t(h_{gt}),\ i_t = i_t(h_{gt})$$

$$\Rightarrow h_{yt} = (h_{gt}, \delta_t, i_t, \eta_t) \rightarrow y_t(h_{yt}),\ \pi_t(h_{yt})$$

and so on

Outcome Path $\left\{a_t\left(\eta^t\right)\right\}$

1. Sticky producers' equilibrium response

$$x_t\left(\eta^{t-1}\right)=E\left[\gamma\, y_t\left(\eta^t\right)+\pi_t\left(\eta^t\right)\mid\eta^{t-1}\right]$$

2. New Classical Phillips curve

$$\pi_t\left(\eta^t\right)=\kappa\, y_t\left(\eta^t\right)+x_t\left(\eta^{t-1}\right)$$

3. Euler equation

$$y_t\left(\eta^t\right)=E\left[y_{t+1}\left(\eta^{t+1}\right)-\sigma\left(i_{t+1}\left(\eta^t\right)-x_{t+1}\left(\eta^t\right)\right)\right]+\eta_t$$

4. Quantity theory

$$\pi_t\left(\eta^t\right)=\mu_t\left(\eta^t\right)-\left(y_t\left(\eta^t\right)-y_{t-1}\left(\eta^{t-1}\right)\right)$$

Implementation with Sophisticated Policies

Implementation with Sophisticated Policies

Policies can differ on and off the equilibrium path

Implementation Theorem

- Suppose some given outcome path

$$a_t^*\left(\eta^t\right)$$

solves 4 equations of New Classical system. There are sophisticated policies with a unique equilibrium which generate given outcome path.

Sketch of Proof

- To implement specific outcome path $\left\{x_t^*\left(\eta^{t-1}\right), i_t^*\left(\eta^{t-1}\right), y_t^*\left(\eta^t\right), \pi_t^*\left(\eta^t\right)\right\}$
  - If $\tilde{x}_t = x_t^*$ stay with original policy i_t^*
  - If $\tilde{x}_t \neq x_t^*$ switch to money for one period and

    choose money to generate original inflation π_t^*

- Proof exploits *controllability with money* i.e.
  - Central bank can induce any best response by individual sticky price producer following aggregate deviation $\tilde{x}_t$

Sketch of Proof

- In money regime π_t and y_t determined by
  - Flexible producers' decisions

$$\pi_t = \kappa y_t + x_t$$

  - Cash-in-advance in first differences

$$\pi_t = \mu_t - (y_t - y_{t-1})$$

- So for any sticky producer choice x_t (with y_{t-1} given)
  - π_t and y_t uniquely determined by μ_t
  - Monotone in μ_t

Sketch of Proof

- Best response of sticky producer

$$x_t(j) = E_{t-1}[\gamma y_t + \pi_t]$$

- So by appropriately varying μ_t can make

$$x_t(j) \neq \tilde{x}_t$$

Sketch of Proof

- Best response of sticky producer

$$x_t(j) = E_{t-1}[\gamma y_t + \pi_t]$$

- So by appropriately varying μ_t can make

$$x_t(j) \neq \tilde{x}_t$$

- Under our reversion scheme: sticky producer *j*'s best response to $\tilde{x}_t$

$$x_t(i) = x_t^* + \frac{1}{\kappa}(x_t^* - \tilde{x}_t)$$

  - Since $x_t(i) \neq \tilde{x}_t$ when $\tilde{x}_t \neq x_t^*$ then deviation deterred. **Q.E.D.**

Recap

- Unique Implementation with Sophisticated Policies

- Next, show why regime switching is necessary

Necessity of Regime Switching

Standard Specification of Policy

- Linear feedback rule

$$i_t = \bar{i}_t + \sum_{s=0}^{\infty} \phi_{xs} x_{t-s} + \sum_{s=1}^{\infty} \phi_{ys} y_{t-s} + \sum_{s=1}^{\infty} \phi_{\pi s} \pi_{t-s}$$

- Necessarily yields indeterminacy
  - Under this rule continuum of competitive equilibria

$$x_{t+1} = i_t + c\eta_t, \ \pi_t = x_t + k(1+\psi c)\eta_t, \ y_t = (1+\psi c)\eta_t$$

indexed by c and x_0.

Standard Specification Includes King Rule

- King rule

$$i_t = i_t^* + \phi(x_t - x_t^*)$$

where i_t^*, x_t^* are desired outcomes.

- Our approach: King rule yields indeterminacy

Recap

- Simple sticky price model
  - Unique implementation with sophisticated policies
  - Necessity of regime switching

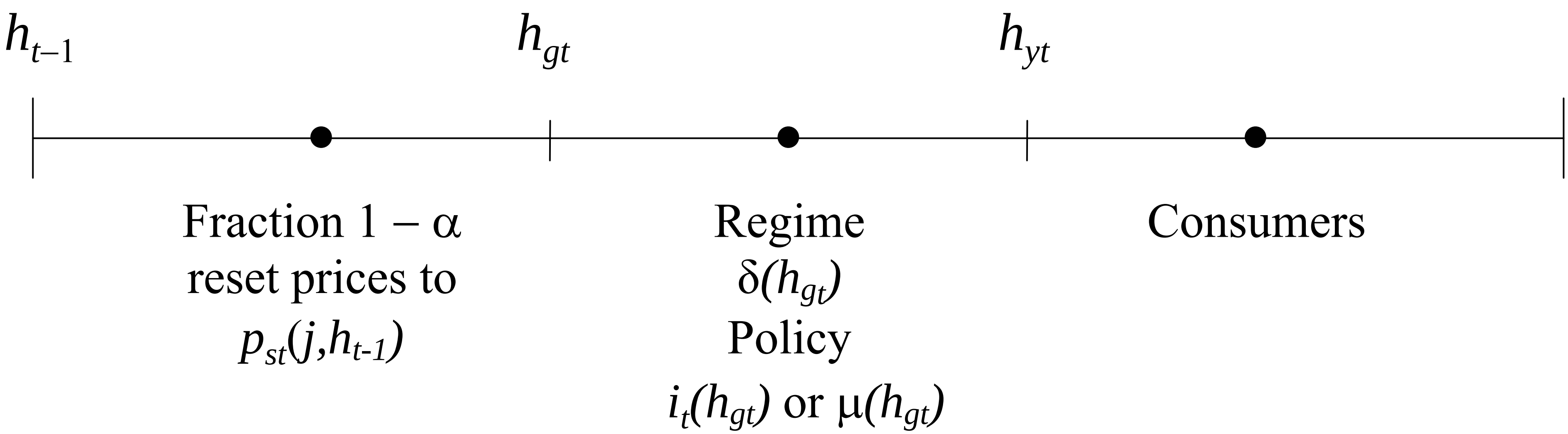
- Next, extend to standard New Keynesian model with
  - Sticky price producers use Calvo-pricing
  - No flexible producers

Standard New Keynesian Model

Model Setup

Standard New Keynesian Model

- Timing (w/o shocks)



- Only 1 equation changes from simple sticky price model
  - Sticky price producers' best response

Sticky Price Producers' Best Response

- Solution using price levels is

$$p_{st}(j) = (1-\alpha\beta)\left[\sum_{r=0}^{\infty}(\alpha\beta)^{r-t}(y_r + p_r)\right]$$

- Implies Transversality condition

$$p_{st}(j) = (1-\alpha\beta)[y_t + p_t] + \alpha\beta p_{st+1}$$

$$\lim_{T\to\infty}(\alpha\beta)^T p_{sT} = 0$$

- Note: $u(c,l) = \log c - b(1-l)$

Sticky Price Producers' Best Response

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- Transversality plays a role later

Sticky Price Producers' Best Response

- Solution using price levels is

$$p_{st}(j) = (1-\alpha\beta)\left[\sum_{r=0}^{\infty}(\alpha\beta)^{r-t}(y_r + p_r)\right]$$

- Controllable with reversion to level of money $\bar{m}$

  - Cash-in-advance implies

$$y_r + p_r = \bar{m}$$

  - So

$$p_{st}(j) = \bar{m}$$

$\Rightarrow$ Unique implementation easy with money

Recap

- Unique implementation with reversion to money

- Next: unique implementation with reversion to interest rates

Standard New Keynesian Model

Unique Implementation with Reversion to Interest Rates

How Do We Implement It?

- Along equilibrium path

$$i_t\left(h_{gt}\right) = i_t^*\left(\eta^{t-1}\right)$$

- Any deviation at time t switch to new regime

$$i_t\left(h_{gt}\right) = \bar{i} \text{ at } t \text{ with } \bar{i} \neq 0$$

and

$$i_s\left(h_{gs}\right) = \hat{\phi} x_s \quad \text{all } s \geq t+1$$

- To work, solution to NK system must be unique under $\hat{\phi}$

Add Economic Boundary Conditions to NK System

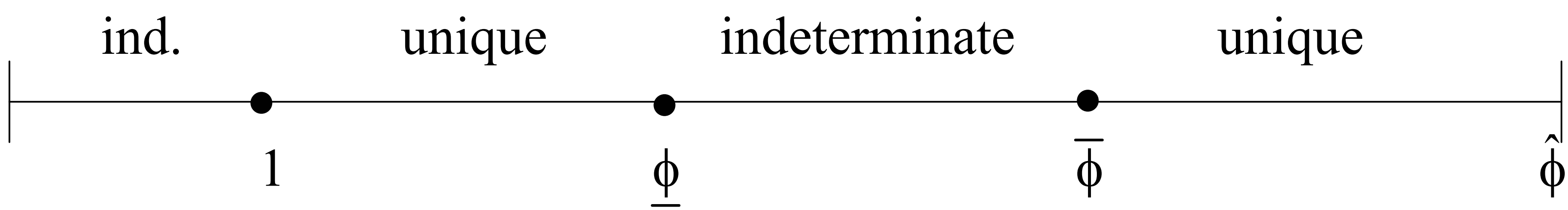
- Transversality condition

- Output bounded above

- Inflation bounded below (interest rates nonnegative)

When are solutions unique under $\hat{\phi}$?

- Economic bounds
  - Regions of indeterminate and unique solutions



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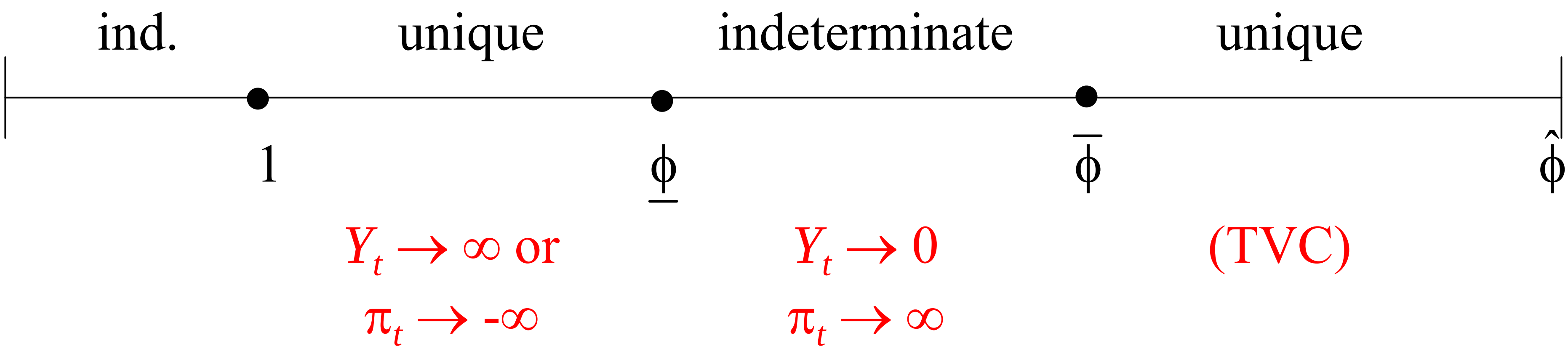

  - Result: Unique implementation when $\bar{i} \neq 0$ and $\hat{\phi}$ in unique region

King Rule Works but Differently from Literature

- King rule

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*), \quad \phi > 1$$

- Our approach
    - Implements bounded outcomes
    - After deviation, returns to desired outcomes

- Literature
    - After deviation, leads to nonexistence (π explodes)

Standard New Keynesian Model
Robust to Imperfect Information

Imperfect Information

- Imperfect monitoring
  - See agents' actions very period with probability q

- Measurement error
  - See agents' actions with measurement error

Imperfect Monitoring

- Central Bank sees
  - price setter's decision x_t with probability q
  - nothing with probability $1 - q$

Imperfect Monitoring

- Central Bank sees
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- Result: If detection probability q sufficiently high in that

$$\frac{1}{1-q} > 1 + \beta q + (1-q)\kappa\sigma$$

then sophisticated policies can uniquely implement any outcomes.

Imperfect Monitoring

- Central Bank sees
  - price setter's decision x_t with probability q
  - nothing with probability $1 - q$
- Result: If detection probability q sufficiently high in that

$$\frac{1}{1-q} > 1 + \beta q + (1-q)\kappa\sigma$$

then sophisticated policies can uniquely implement any outcomes.

- Mechanics
  - q acts like discounting
  - both roots bigger than 1 so uniqueness following a deviation

Measurement Error

- See $\tilde{x}_t = x_t + \varepsilon_t$

- Use modified King Rule

$$i_t = i_t^* + \phi(\tilde{x}_t - x_t^*)$$

- Unique equilibrium has $x_t = x_t^*$, $y_t = y_t^* - \psi\phi\varepsilon_t$

- For small measurement error, can approximately implement bounded outcomes

Conclusion

- Follow Barro, Lucas-Stokey
  - Check controllability of best responses
  - If controllable, move on to next paper
  - If not?...
- Extend to financial crises, fiscal policy, and so on