92.1.5. *Limit Theory in Cointegrated Vector Autoregressions*, proposed by Peter C.B. Phillips and Hiro Y. Toda. Consider the trivariate system

\[ y_t = A y_{t-1} + u_t, \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \]

where \( u_t \) is i.i.d. \((0, \Sigma)\) and \( \Sigma = (\sigma_{ij}) \). Suppose \( A = (a_{ij}) \) is estimated by an unrestricted vector autoregression with one lag leading to \( \hat{A} = (\hat{a}_{ij}) \).

(a) Find the joint asymptotic distribution of \((\hat{a}_{12}, \hat{a}_{13})\).

(b) Find the limit distribution of the Wald test of the noncausality hypothesis.

\[ H_0: a_{12} = a_{13} = 0. \]

(c) Discuss your results.