91.3.3. *Testing for Stationarity in the Components Representation of a Time Series*, proposed by D. Kwiatkowski, P.C.B. Phillips, and P. Schmidt. Let the time series \((y_t)_{t}^{n}\) have the following components representation

\[ y_t = \gamma_0 + \gamma_1 t + r_t + u_t; \quad u_t = \text{i.i.d. } N(0, \sigma_u^2) \]

where

\[ r_t = r_{t-1} + v_t; \quad v_t = \text{i.i.d. } N(0, \sigma_v^2), \quad r_0 = 0 \]

and \(u_t\) is independent of \(v_s\) for all \(s\) and \(t\).
(a) Set up the likelihood function for this model.
(b) Derive an LM statistic for testing the hypothesis
\[ H_0: \sigma^2 = 0. \]
(c) Find the limit distribution under \( H_0 \) of the LM statistic derived in (b).
(d) How do your results depend on the normality assumption?