88.1.3 Structural Estimation under Partial Identification, proposed by P.C.B. Phillips. Consider the following single equation of a simultaneous system:

\[ y_1 = Y_2 \beta_2 + Y_3 \beta_3 + X_1 \gamma + u \]  \hspace{1cm} (1)

with reduced form

\[
\begin{bmatrix}
1 & n_2 & n_3 \\
K_1 & K_2 \\
1 & n_2 & n_3 \\
\Pi_{11} & \Pi_{12} & \Pi_{13} \\
\Pi_{21} & \Pi_{22} & \Pi_{23} \\
1 & n_2 & n_3 \\
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
\end{bmatrix}
+ \begin{bmatrix}
v_1 \\
v_2 \\
v_3 \\
\end{bmatrix}
\]  \hspace{1cm} (2)

or

\[ Y = X\Pi + V. \]  \hspace{1cm} (3)

Suppose \( \Pi_{22} = 0 \), rank \( \Pi_{22} \) = \( n_2 \), \( \beta_2 = 0 \), and \( K_2 \geq n_2 + n_3 \). Assume that standardizing transformations have been carried out, so that the rows of
\[ y_1, y_2, y_3 \] are i.i.d. \( N(0, \Sigma) \), where \( n = 1 + n_2 + n_3 \). Let \( (\hat{\beta}_2, \hat{\beta}_3) \) denote the two-stage least squares (2SLS) estimator of \( (\beta_2, \beta_3) \).

(i) Find the distribution of \( \hat{\beta}_3 \). What happens to this distribution as \( T \uparrow \infty \)?

(ii) Assuming that \( X'X/T \) tends to a positive definite matrix as \( T \uparrow \infty \), how would you analyze the asymptotic distribution of \( \hat{\beta}_3 \)? How do your results accord with conventional asymptotic theory?