Book Reviews


For 30 years, Rex Bergstrom has been the world’s leading exponent of continuous-time econometric modelling. This book puts together 10 of the author’s authoritative papers in this field (published originally over the period 1962–89), adds two new papers and an introductory essay that connects the various strands of his research, and provides useful links to the work of other researchers. The papers are arranged in three parts, covering economic models of cyclical growth (Part I), econometric methodology of estimation, forecasting and control (Part II) and empirical applications (Part III). Within each part the papers appear in natural chronological order of first appearance, which aids readability and reveals the logical development of ideas in this field.

Part I has two papers (Chapters 1 and 2) on cyclical growth. These provide a theoretical foundation for prototype empirical models. As economic models they represent an important advance over earlier trade cycle models which set out to explain cyclical behaviour in real variables through the multiplier–accelerator mechanism alone. Bergstrom’s cyclical growth models provide an active role for prices in this cyclical mechanism by including a Keynesian-type adjustment mechanism that operates via real liquidity, interest rates and investment to real output. At the same time, these cyclical growth models incorporate an explicit production technology and have particular solutions that correspond to the steady-state paths of the earlier neoclassical growth models of Solow and Swan. The Bergstrom models are therefore the synthesis of two earlier strands of research and are the first truly disequilibrium models of neoclassical growth.

The explicit form of these cyclical growth models as ordinary differential equations with a continuous-time parameter leads to econometric specifications as systems of stochastic differential equations. Part II contains seven papers that deal with the econometric methodology of stochastic differential equations. The first two were published originally by Econometrica (1966, 1983) and describe two alternate approaches to the estimation of continuous systems with discrete-time data. Chapter 3 exhibits an approximate discrete model that takes the form of a system of simultaneous equations and explores its consequences. This discrete approximation is especially interesting because it shows the precise sense in which a system of simultaneous interdependent equations may be regarded as an approximation to a recursive differential equation system. Chapter 4 is an extensive study of exact discrete-time models corresponding to stochastic differential equation systems, allowing for higher-order systems of equations and the presence of stock and flow variables. These two chapters provide the basis for much of the empirical work in this field, including the work discussed in Part III.

Chapters 5 and 6 address computational issues. The approach recommended in Chapter 4 involves the use of maximum-likelihood estimates based on a Gaussian likelihood (i.e. the procedure commonly known as Gaussian estimation). In practical work, the principal difficulty in the implementation of this method is the construction of the likelihood. The problem is acute when there are data irregularities such as mixed stock and flow variables, missing observations or initialization on a state vector with unknown elements. Bergstrom’s approach does not rely on the Kalman filter but makes explicit use of the exact discrete model for the observable variables, including the functional dependence of the error covariance matrix (Ω) on the system parameters. The likelihood can then be constructed readily from the conventional Gaussian form, given an initialization of the state vector, by using the Choleski factorization of Ω. The computations are of the order of Tn^3, where n is the dimension of the system and T the sample size. Chapter 6 extends the calculations to provide for open systems equations.
Chapter 7 is a fascinating and important new paper. Its subject is the development of practical statistical tests of the models and explicit parameterizations that arise in discrete versions of continuous systems. Discrete data generated from a continuous system with mixed stock and flow variables satisfy a vector autoregressive-moving average (VARMA) model with heavily restricted coefficient matrices. For a second-order system of stochastic differential equations with mixed stock and flow data, the VARMA model is of order (2, 2). A complication is that the MA coefficient matrices are temporarily dependent. The chapter gives recursive formulae for the computation of these matrices and shows that the recursion is stable in the sense that the coefficients converge (as \( t \to \infty \)) to fixed limit matrices. The statistical test procedures that are suggested involve three steps:

1. test the hypothesis that a VARMA (2, 2) is the data-generating mechanism;
2. test the form of the coefficient matrices in the VARMA (2, 2) that are implied by an underlying continuous system;
3. test the explicit parameterization of the coefficient matrices that are implied by the economic theory used in the construction of the continuous system.

Time-series model selection procedures are suggested for step 1 but are not discussed. (Note that these are not straightforward because of the possibility of degeneracies in the AR and MA polynomial matrices.) A likelihood ratio test is suggested for step 2 and a Wald test for step 3. Open systems of differential equations are also considered with essentially the same sequence of tests allowing for the presence of exogenous variables.

Chapters 8 and 9 conclude Part II with a treatment of forecasting and control of continuous systems. Optimal forecasts are shown to be generated from the discrete VARMA model in a simple way that utilizes the Gaussian estimates of the matrix Choleski factorization of the full error covariance matrix. Recursive formulae are provided for the generation of k-period forecasts. Chapter 9 deals with optimal control and gives the solution of a quadratic cost functional optimization problem given stochastic differential equations of motion for the system. This chapter extends earlier certainty equivalence theory and allows for weaker conditions on the innovation processes by relaxing the requirement that they be increments of Brownian motion.

Part III contains three papers. The first (Chapter 10, co-authored with C. R. Wymers) is the Bergstrom-Wymers model of the UK economy. This 13-equation continuous system is based on a formal one-sector model of a macroeconomy and is heavily constrained by cross-equation restrictions delivered from the underlying theory. The model is estimated with postwar quarterly data by Gaussian techniques, its long-run properties are examined analytically, a stability analysis is conducted and impressive out-of-sample forecasting performance is demonstrated. This model is distinguished from other empirical macro models in many ways but most notably because it has only one exogenous variable: a time trend. The Bergstrom-Wymers model is now a classic study in empirical econometric research. It is the prototype for many similar models that have been developed for other countries and is a showpiece of careful applied econometrics.

Chapter 11 reports some policy exercises with the Bergstrom-Wymers model. Monetary, fiscal and exchange rate policies are examined and their quantitative impact explored through various empirical re-specifications of the Bergstrom-Wymers model of Chapter 10. An interesting outcome of these experiments is that, whereas foreign sector leakages typically damp business cycles, exchange rate variations absorb balance of payments fluctuations and, therefore, in this model have a tendency to increase fluctuations in output.

Chapter 12 (joint with M. J. Chambers) develops a continuous-time model of the demand for durable goods. This chapter makes extensive use of the economic methodology in Part II and also allows for the fact that the stock of durables is unobservable variable. The model is applied to consumer durable expenditure over the period 1973-84 in the UK. Dynamic responses of consumer purchases to changes in disposable income are studied and the post-sample predictive performance of the model is tested for the period 1983-84 against simpler models. The results are to be a successful empirical implementation of the more sophisticated continuous econometric methodology.
There is no doubt that continuous-system methodology and application have become more common in recent years. There is, in fact, substantial interest in this approach in finance, where many of the data are now nearly continuously recorded. In macroeconometrics it is fair to say that there has been less popular interest. Macroeconometrics over the last 15 years has been heavily preoccupied with its own concerns, notably the Lucas critique, the new conservative macroeconomics and the empirical implementation of rational expectations and dynamic optimization models. At the empirical level, much of the research has concentrated on subsystem Euler equation estimation and the use of unrestricted atheoretical models like VARs. These concerns have created their own methodological needs, and the resulting developments have occurred more or less in parallel to the econometric methodology contained in this book (where, in fact, they receive no mention). However, there are important points of contact. For instance, the models discussed here are immune to the Lucas critique to the extent that the theory on which they are based involves only deep structural parameters. And empirical versions of atheoretical VAR models are usually very special cases of the VARMA systems studied in this book. In fact, one could argue that the dynamic processes of the models in this book are more complex than those of more popular applied econometric methodology, yet their reliance on economic theory is much more intensive, so that they typically involve much more parsimonious parameterizations. One can hope that during the 1990s some serious attempts will be made to evaluate empirically these different approaches to macroeconometric modelling. From the perspective of this reviewer, the Bergstrom methodology that is so well exemplified by the essays in this book offers much promise for the future.

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