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Why equilibrium? A note on the noncooperative equilibria of some matrix games

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Abstract

Several examples are presented which raise questions concerning the virtue of the noncooperative equilibrium as a solution concept. It is observed that an optimum response cycle may payoff dominate the unique noncooperative equilibrium. This work builds on an earlier example provided by Shapley.

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Some years ago Lloyd Shapley suggested that the 3×3 matrix game shown in Table 1 has a unique mixed strategy noncooperative equilibrium which yields payoffs worse than either player could expect to obtain by openly moving first. The game in Table 1 has a unique mixed strategy NE with (1/3, 1/3, 1/3) for each player and an expected payoff of $1/3(4) + 1/3(5) = 3$, while if one player moved first he could expect 4 with the other obtaining 5.

Table 1

	1	2	3
1	0,0	↑5,4 →	4,5 ↓
2	4,5 ↓	↑0,0 ←	← 5,4
3	5,4 →	↑4,5	0,0

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Table 2

	1	2	3	4	…	n
1	$(1+\epsilon/n, 1-\epsilon/n)$	$(1-\epsilon/n, 1+\epsilon/n)$	0,0	0,0	…	0,0
2	0,0	$(1+\epsilon/n, 1-\epsilon/n)$	$(1-\epsilon/n, 1+\epsilon/n)$	0,0	…	
3	0,0	0,0	$(1+\epsilon/n, 1-\epsilon/n)$	$(1-\epsilon/n, 1+\epsilon/n)$	…	⋮
4	0,0	0,0	0,0	$(1+\epsilon/n, 1-\epsilon/n)$	…	
5	0,0	0,0	0,0	0,0	…	0,0
⋮	⋮	⋮	⋮	⋮		⋮
n	$(1-\epsilon/n, 1+\epsilon/n)$	0,0	0,0	0,0	…	$(1+\epsilon/n, 1-\epsilon/n)$

If they followed an optimal response pattern they would cycle on the six cells with positive payoffs and on the average each could expect 4,5.

The game itself is not particularly pathological but it does raise some questions concerning why a payoff maximizing individual should be interested in playing an equilibrium strategy.

In this note two further simple matrix games are given which extend this basic observation. We can construct an $n \times n$ matrix where the unique noncooperative equilibrium has a payoff to each player of $2/n$, the maxmin is $2/n$ and if one player can move first he can expect to obtain $1 - \epsilon$. We merely enlarge the Shapley example by constructing an $n \times n$ matrix where $(n-2)^2$ entries are 0 and n are $1 - \epsilon/n$ or $1 + \epsilon/n$.

As can be seen from Table 2 as n becomes large the NE remains unique and the NE payoff approaches 0 while coordination offers at least $1 - \epsilon/n$. The positive payoffs in the game in the limit approach 1 and the game in the limit would have $2n + 1$ NEs ($2n$ pure, 1 mixed).

As game n grows large the problem of *coordination* becomes more important and the difference in payoffs is less important. This game may be regarded as a formal representation of Schelling's intuitive concern for the role of coordination.

The two examples above make use of a liberal sprinkling of zeros in the matrix. The third example is a 3×3 game which is strictly ordinal. Let the best payoff be 9 for each. We may then consider the square matrix in Table 3. Every row payoff for Player 1 and column payoff for Player 2 sums to 15. If we were to give the matrix entries a cardinal interpretation the unique mixed strategy equilibrium would be at $(1/3,1/3,1/3)$ for each and yield 5.

If the matrix entries are interpreted as ordinal there is no mixed strategy solution, we note however that whoever could signal his first move could expect to get item 6 in his order. Player 1 by selecting strategy 3, player 2 by selecting strategy 1.

If both players are 'dumb' enough to reason only one move ahead and paranoid enough to believe that their move will be seen before the other has moved then player 1 will select his third strategy (0,0,1) and player 2 his first (1,0,0) giving payoffs of (6,7).

How would a game theorist advise his 'client' to play either the one shot game in these three examples, where pregame communication is free and costless; or where there is no communication, or where the players will play a fixed but large number of times?

Do equilibrium concerns dominate payoff concerns?

Table 3

	1	2	3
1	8,6 ↑	3,3 →	4,9 ↓
2	1,2 ↑	5,8 ↓	← 9,5
3	6,7 ↑	← 7,4	2,1