When managers cover their posteriors: making the decisions the market wants to see

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The stock market has opinions as to what choices firms should make. We show that concern for current share prices may lead managers to make these choices rather than those suggested by their own superior information. Even when arbitrarily many privately informed firms have to make a similar decision, the market's "prejudices" may still prevail. We compare the distortions that arise from share-price maximization with those due to herd behavior among profit-maximizing firms, and show that the former results in strictly less efficient use of information.

1. Introduction

This article examines what happens when managers aim to maximize the short-term share price of their firms rather than directly maximize expected future profits. We consider first a single manager faced with a decision and then a sequence of managers facing similar decisions. Although this context has attracted much recent attention, the article has wider relevance. It applies whenever someone cares about other people's current perceptions of his or her prospects for future success. For example, electoral or academic job market candidates often have such concerns. The following short story is an illustration.

Suppose that Mr. and Mrs. Bloom, being good parents, want nothing more than for their son Nathan to be happy in his future. To this end, they are not sure whether he should be a doctor or a lawyer. For some reason, they slightly favor his chances of finding happiness as a lawyer. But they recognize that Nathan, in the course of growing up, will acquire knowledge about himself that will make him more able than his parents to assess what is likely to make him happy. They thus leave the choice up to Nathan.

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Meanwhile Nathan, being a good son, wants nothing more than for his parents to be joyful today, and he knows what gives them joy today is the strength of their belief that he will one day be happy. Unfortunately, this apparent virtue will lead Nathan to become a lawyer regardless of what he learns about himself. Moreover, Mr. and Mrs. Bloom, being good game theorists, will realize this. Since they know that Nathan would be more likely to be happy were he to use his knowledge, they will not be as joyful as they might otherwise be.

Why is this? Suppose that Nathan planned to use his knowledge. Then if he chose the Law, his parents would realize that what Nathan learned must have agreed with their initial reasoning and they will be very joyful, with reinforced confidence that their son will one day be happy. On the other hand, if he chose Medicine, his parents would realize that what Nathan learned must have disagreed with their own initial reasoning. While they would realize that this was on balance the best decision for Nathan, they would not be so confident of his future happiness and hence would be only a little joyful. But Nathan, being a thoughtful son, will realize this and thus will always choose the Law. Even if Nathan could explain to his parents his reasons for favoring Medicine, he would always have an incentive to pretend that his reasons favored the Law and explain these instead, since they would make his parents more joyful.

Since Nathan does not use his knowledge, his course of action is inefficient. A less thoughtful son who acts to maximize his chance of future happiness, ignoring his parents' hopes for that future happiness, is more likely one day to be happy. Ironically, Nathan's parents, realizing this, would be made more joyful by such a thoughtless son. Perhaps Nathan should indulge in a wild adolescence to gain a reputation for thoughtlessness among his elders. But although this is a convenient ex post justification for the authors' youthful excesses, it is not a strategy easily available to managers of publicly traded companies.

Just as children sometimes feel constrained by their parents' beliefs and hopes, so managers also often complain that they feel pressured to make the decisions the stock market thinks is correct rather than the decisions they believe to be in the best interests of their firms. They find themselves looking over their shoulders at the market to see how today's share price responds to today's decisions. Against this, it has sometimes been argued that, provided the stock market is "efficient," share prices are unbiased predictors of future earnings. In this case, the argument concludes, maximizing current share prices and maximizing expected future profits are the same thing (see, for example, Jensen (1986)).

The Nathan Bloom story illustrates one reason why this argument is incorrect. Nathan's parents form an unbiased expectation of their son's future happiness. But maximizing someone else's expectation of your future happiness is not the same thing as maximizing your own, unless you happen to have the same information. Like Nathan's parents, market traders observe decisions taken by managers and update their beliefs about the state of the world accordingly. Share prices reflect the resulting posteriors of the traders. Therefore, managers who care about share prices will care about the posteriors generated by their actions and will act to mold the market's beliefs. In other words, share-price maximizers will be concerned not so much that their decisions are correct but that the market thinks these decisions are correct. Such decisions will not in general be those that maximize expected future earnings, and in an efficient market, share prices will reflect this fact.

The first big assumption that drives the results in this article is that managers are concerned with current share prices. There is some evidence that managers worry about current market valuations.¹ Many reasons have been advanced for this preoccupation.

¹ For example, The Economist (June 27, 1992, p. 89) reports that "the goal of American firms is to
Higher share prices may make it easier for firms to raise new capital and make them less vulnerable to hostile takeovers. If managers have varying abilities, then it may be necessary to relate their pay to some current indicator to compete with headhunters from other firms. Share-price-based compensation schemes may discourage managers from harvesting their businesses. That said, we do not discuss optimal contract design in this article; we simply assume that the objectives of the manager include current share price and proceed from there.

The other big assumptions are that managers have private information that they cannot commit to reveal and that they do not themselves trade in the stock market. These assumptions are likely to be satisfied where public disclosure and insider trading are subject to complex legal restrictions. It is often hard or expensive to verify information given to shareholders, and managers are often loath to disclose information about their firms for fear of giving their competitors an edge. Moreover, insider trading laws may make investment fund managers reluctant to receive information from managers of firms if this would undermine the liquidity of their holdings (Bhide, 1993). That said, our article may help explain why managers sometimes value devices such as external watchdogs or outside auditors that do enable them to commit to reveal information.

In applying this article, a key issue is how persistent are significant informational asymmetries compared with the time horizon over which managers are concerned with their share prices. If, for example, all uncertainty relevant to today’s decision will be resolved this week, then provided managers’ concern is for next week’s share price, the specific problem identified in this article will disappear. Our work becomes relevant in the opposite case, where the managers’ concern is for share prices formed before all uncertainty is resolved.

Section 2 first formalizes the intuition behind the Nathan Bloom story. We then extend the static story. First, we consider mixed as well as pure strategies. While all the manager’s private information is lost in pure-strategy equilibria, we show that part of the manager’s information can sometimes be salvaged in mixed-strategy equilibria. Some information, however, is still lost. Second, we relax most of the implicit symmetry and asymmetry assumptions of the model. The source of the problem in the original Nathan Bloom story arises from the skew in the parents’ prior beliefs favoring Law over Medicine. We show, however, that a skewed prior is not essential. All that matters is that there is an “imbalance” in the market posterior profit expectations that would result if the manager were to follow his or her information. These posteriors are functions not only of the priors but also of signal likelihoods and ex post payoffs. The

maximize shareholder value, as measured by the current share price.” Abegglen and Stalk (1985) note that high share prices ranked above product innovation, market share, streamlining production and distribution, and improving working conditions as an objective of surveyed American managers.

A referee points out, however, that the distortions caused by share-price maximization, such as those discussed in this article, may lead to more takeovers. If potential acquirers have the same information as managers, they may believe that the firm is undervalued. Moreover, takeovers may shift the market’s beliefs about what a firm should do.

None of these effects are explicitly modelled in this article. For further discussion, see Hirshleifer (1993).

The no-insider-trading assumption excludes all share transactions by the firm itself, including management buyouts and new stock issues. This assumption is clearly extreme, and extensions might relax it. Notice, however, that simple share-price maximization may not be a sensible objective to ascribe to managers if they trade in their firms’ shares.

“In a speech to a group of chief financial officers of Fortune 500 companies, I asked if they were forthright with stock research analysts, telling them all they would want to know. Uproarious laughter broke out. One participant summarized the views of the group: ‘We tell them as little as possible; they talk to all of our competitors.’” (Jacobs, 1991, p. 35). Bhattacharya (1992) and Chen (1994) explicitly model the decision to release information, alongside other decisions.
results in this section do not depend on the extreme assumption that the manager's
objective is simply to maximize the current share price. Even if managers also put
some weight on their own expectations of future profits, decisions will still be ineffi-
cient provided the weight put on current share prices is sufficiently large.

Section 3 considers a dynamic version of the model. We consider a situation in
which many firms have to make a similar decision, say the choice of which of two
technologies to adopt. The correct decision is the same for all firms, but their private
information may be different. In this case, the stock market's opinion as to which
decision is the correct one for a given firm may be affected by the choices of all the
firms. Moreover, each manager's own view may be affected by observing the choices
of managers who have gone before.

There are at least two reasons why these sequential decision problems are impor-
tant, other than that they occur in the real world. First, Banerjee (1992), Bikhchandani,
Hirshleifer, and Welch (1992) have shown that sequential choice contexts such as this
can lead to herd or cascade behavior. That is, even if managers ignore stock market
pressures, their choices will not result in efficient use of information. It is not enough,
therefore, to argue that stock market pressure leads firms to make inefficient choices.
Section 3 directly compares the distortions caused by concern for share prices with
those caused, in the absence of such concerns, by herd or cascade behavior. In our
dynamic model with share-price maximization, equilibrium choices are strictly less
efficient than herd behavior is in the corresponding model without concern for current
share prices. That is, share-price maximization exacerbates the distortions of herd or
cascade behavior.

Second, recall that in the one-firm model, there are sometimes mixed-strategy
equilibria in which part of a manager's information is reflected by his or her decision.
By analogy, one might reasonably conjecture that when there are many firms, each
firm might use part of its information. If there are many firms, this could aggregate to
a lot of information. We show, however, that equilibrium choices in the many-firm case
use no more information in total than is used in the one-firm model. Since there was
more information available with many firms, one could argue that the outcome is even
more inefficient.

The many-firm result does not rely on backward induction; that is, there need not
be a definite last firm in the sequence. Rather, we use a simple application of martingale
theory. There has been much recent interest both in share-price-based incentives and
in other areas where one agent's payoffs are a function of another agent's beliefs, such
as in career concerns. The method we use here may also be useful in extending these
models to dynamic settings.

The idea that managerial concern for current indicators of firm performance, such
as share prices, can lead to inefficient behavior is not new. Fuller recent surveys of the
topic include Hirshleifer (1992, 1993) and Grant, King, and Polak (1996). One reason
for the inefficiency is moral hazard. Narayanan (1985), Stein (1989), Paul (1991), and
Bebchuk and Stole (1992) each show that in the presence of moral hazard, concern for
current indicators can distort investment decisions. Paul (1992) argues that share prices
do not aggregate information efficiently for use as incentives: an optimal contract would
put most weight on information about the manager's hidden actions, whereas share
prices depend most on information about random shocks. Hagerty, Özer, and Siegel
(1992) show that optimal contracts under moral hazard may resemble call options. They
argue that such options are indeed more often used as incentives where managers have
more opportunity to "steal from the future."

The problem identified in this article is not one of hidden actions, however, but of
hidden information. Trueeman (1986), Hirshleifer and Chordia (1992), and Bebchuk and
Stole (1992) each show that adverse selection can also distort investment, though sometimes in a different direction from that caused by moral hazard. Myers and Majluf (1984) use a “lemons” argument adapted to share prices to explain underinvestment in equity-funded projects. Managers who believe that the market is undervaluing their firms may decide not to issue new equity so as to avoid diluting the value of previously issued shares. Bizjak, Brickley, and Coles (1993) argue that managerial compensation packages in fact put less weight on current stock prices where there are more insurmountable information asymmetries. In most of these models, the inefficiencies arise from costly attempts to signal. We could extend our model to allow for costly signaling, but we prefer to focus instead on stock market pressure.

Even in its static version, our model provides a rather different (complementary) reason why concern for current indicators may lead to inefficient decisions. In particular, it aims to capture the intuition behind managers’ complaints of market pressure. The problem we identify is not confined to investment decisions; it could affect managerial decisions on exit and entry, product positioning, hiring and firing, mergers and divestitures, and so on. For example, the model may help explain Kodak’s disastrous 1976 entry into the instant photography business then dominated by Polaroid. This decision appears to have been made in part due to pressure from Wall Street, possibly against the better judgement of management.

In addition to the herd and cascade articles already mentioned, Scharfstein and Stein (1990), Zwiebel (1995), Trueman (1994), and Graham (1995) each analyze what might be called reputational herding models. They consider the decisions of managers of varying ability as measured by the informativeness of their private signals. In contrast to our models, managers’ main concern is not the assessment of the quality of the decisions by the stock market, but the assessment of their ability by potential employers. Also in contrast, assessments are made after observing both the decisions and their outcome. In contrast to both our and the cascade models, most of the reputational herding models essentially involve just two managers, a leader and a follower, each of whom is of high or low ability. In our model, players do not vary in their intrinsic quality, but the effects we find reinforce and complement the effects of reputational herding. Bulow and Klemperer (1994) provide a separate explanation for apparent herd behavior. They show that in markets with sequential sales, small changes in price can produce large changes in the number of buyers currently willing to buy. In contrast to other herd or cascade stories, Bulow and Klemperer’s results can be obtained even when agents have independent private values.

2. One firm posterior

We start by setting up a simple model which we shall generalize below. Suppose that a firm has to make a decision to go left (L) or right (R). If the firm makes the correct decision it earns a profit of 1, whereas if it makes the incorrect decision it earns 0. Let λ be the state of the world in which the correct decision is L, and ρ be the state in which the correct decision is R. After the firm has made its decision but before the true state is revealed, players in the stock market assess the firm’s decision and value its shares accordingly. We assume that the firm’s objective is to maximize its expected share price.

There is some public information that causes both the firm and the stock market to assign a prior probability π to the true state being ρ. Without loss of generality, let π ≥ ½. In addition, the firm has access to private information about the true state.

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6 Hirshleifer and Chordia (1992) construct their model both with and without moral hazard.

7 For details see the “Polaroid-Kodak” case series (Harvard Business School Case Services, 376-266, 378-165, and 378-173 to 378-182).
Specifically, the firm receives a signal that takes on values \(l\) or \(r\). Let \(q\) be the likelihood that the signal is correct; that is, \(q \coloneqq \Pr(l|\lambda) = \Pr(r|\rho)\), where \(\frac{1}{2} < q < 1\). The manager of the firm chooses between \(L\) or \(R\) or some randomization between the two, contingent on the realization of the signal. Let \(\sigma(l)\) be the probability that the firm chooses \(R\) given the signal \(l\) and let \(\sigma(r)\) be the probability of choosing \(R\) given the signal \(r\), according to the strategy \(\sigma\). We shall call a strategy informative if \(\sigma(l) \neq \sigma(r)\).

Our standard of performance is the \textit{ex ante} expectation of the firm’s profit, denoted \(V(\sigma; \pi, q)\); this is equal to the \textit{ex ante} probability that the firm makes the correct decision given its strategy. Formally,

\[
V(\sigma; \pi, q) := (1 - \pi)[q(1 - \sigma(l)) + (1 - q)(1 - \sigma(r))] + \pi[(1 - q)\sigma(l) + q\sigma(r)].
\]

With slight abuse of terminology, we refer to a strategy that maximizes \(V(\sigma; \pi, q)\) as being optimal or efficient.

The strategy that maximizes the firm’s expected profit depends on the prior, \(\pi\), and the signal likelihood, \(q\). Recall that the public information weakly favors \(\rho\) as the true state. Clearly, if the firm receives the private signal \(r\), it should choose \(R\). But if the signal received is \(l\), then public and private information conflict. Formally, the posteriors are given by

\[
\Pr(\rho \mid r) = \frac{q\pi}{q\pi + (1 - q)(1 - \pi)}
\]

and

\[
\Pr(\lambda \mid l) = \frac{q(1 - \pi)}{q(1 - \pi) + (1 - q)\pi}.
\]

These are illustrated in Figure 1 both for the case \(\frac{1}{2} < \pi < q\) (solid lines) and for the case \(\frac{1}{2} < q < \pi\) (dotted lines). In the former case, \(\Pr(\rho \mid l) < \frac{1}{2}\), so the firm should choose \(L\) after receiving the signal \(l\). In the latter case, however, \(\Pr(\rho \mid l) > \frac{1}{2}\), so the firm should choose \(R\). In short, if \(\pi < q\), the firm should follow its signal, but if \(\pi > q\), the firm should choose \(R\) regardless. To make the problem interesting, therefore, assume \(\pi < q\). In this case, the \textit{ex ante} expected profit under the optimal strategy, which is for the firm to follow its signal, is just \(q\).
Now consider the game in which the firm's aim is not to maximize its expected profit but rather to maximize its current share price. There are at least two ways to formalize this game. One way, following Georgakopoulos, Pearce, and Stacchetti (1989), is to construct a "psychological game" in which the firm's payoffs are simply the stock market's posterior expectation of the firm's profit. A less direct but more conventional way is to construct a simple model of a stock market and let the firm's payoff be the share price. For example, suppose that there is a numeraire asset that pays $1 regardless of the state. After the firm has made its decision, an auctioneer sets the price of the firm, aiming to maximize the value of excess demand for the firm's shares. Simultaneously, traders announce net demands for the firm's shares and for the numeraire aiming to maximize expected future profit. Neither the auctioneer nor the traders are privy to the firm's information, but they can observe the firm's decision. The price, \( p_L \) or \( p_R \), can depend on that decision and can be positive or negative. Then the equilibrium price of the firm will equal the posterior probability formed in the market that the firm has made the correct decision, given the firm's equilibrium strategy.\(^{8}\)

Our first proposition confirms the intuition of the Nathan Bloom story. Unless the public information is neutral as to the true state, there is no pure-strategy equilibrium of the share-price maximization game in which the firm's strategy depends on its signal. In particular, the optimal strategy of always following the signal cannot be sustained in equilibrium.

**Proposition 1.** Suppose that the prior \( \pi > \frac{1}{2} \), and that the strategy that maximizes expected profit is for the firm always to follow its signal; that is, the signal likelihood \( q > \pi \). Then there is no pure-strategy equilibrium in the share-price maximization game in which the firm plays this or any other informative strategy. The pure-strategy equilibrium that yields the highest ex ante expected profit, \( V(\sigma; \pi, q) \), involves the firm's always choosing the action that maximizes its expected profit according to the market's prior beliefs; that is, choosing \( R \) regardless of its signal. In this case, the equilibrium share price \( p(R) = V(\sigma; \pi, q) = \pi \), which, by assumption, is less than \( q \), the maximum possible ex ante expected profit.

**Proof (by contradiction).** Suppose that there were a pure-strategy equilibrium in which the firm plays informatively. There are two cases to consider: when the firm follows its signal and when it always chooses the opposite of its signal. If the firm follows its signal in equilibrium, the market will interpret the choice \( R \) (respectively \( L \)) as an indication that the firm must have received the signal \( r \) (respectively \( l \)) and hence will assign the posterior probability given by equation (1) (respectively (2)) to the firm's having taken the correct action. Thus, in this putative equilibrium, equations (1) and (2) would also be the share prices assigned to the firm following \( R \) and \( L \) respectively. But a simple calculation shows that (1) is greater than (2) if (and only if) \( \pi > \frac{1}{2} \). It follows that the firm would deviate to playing \( R \) always. The case in which the firm always chooses the opposite of its signal is similar. Since the firm's equilibrium strategy does not use its private information, the best it can do is to maximize expected profit according to the prior, that is, to choose \( R \) regardless of its signal. Notice that this is what the market would choose if it were to make the decision, aiming to maximize expected revenue according to its inferior information. Q.E.D.

In equilibrium, the stock market is "efficient" in the sense that it assigns a value to the firm equal to its expected future profit given the equilibrium strategy. Nevertheless, the consequence of share-price maximization is clearly inefficient in that useful information is wasted. Everyone would be better off if the firm maximized its own

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\(^{8}\) We ignore equilibria in which the auctioneer randomizes.
expectation of future profit. There is also another equilibrium in which the firm always chooses \( L \) regardless of its signal, but this perverse equilibrium is even worse than that described above. The perverse equilibrium is less robust to equilibrium refinements, but restricting attention to refinements of Nash equilibrium will not yield efficiency: since our conclusions apply to all equilibria, they apply \textit{a fortiori} to any subset of equilibria.

The intuition behind Proposition 1 can be seen from Figure 1. If the prior \( \pi > \frac{1}{2} \), then the posteriors formed by the firm after observing its private signal, \( l \) or \( r \), are "unbalanced": they are not at equal distances from \( \frac{1}{2} \). If the firm's equilibrium choices fully revealed its signal, the market posteriors induced by those choices would inherit this imbalance and thus the share prices from choosing \( L \) or \( R \) would be unequal. But this gives the firm an incentive to deviate.

This intuition becomes clearer when we consider mixed strategies. Recall that share prices depend on the market's conjecture as to how the firm's observed choice depends on its unobserved signal. Restricting attention to pure strategies implicitly restricts attention to those particular market conjectures that are degenerate. Admitting mixed strategies extends the analysis to all conjectures. No informative pure-strategy equilibrium existed because, in fully transmitting the information contained in the signal, the firm's choices induced unbalanced market posteriors. But mixed strategies do not fully reveal the information: they can be thought of as "garblings" of the signal. If we can design them in such a way as to induce balanced market posteriors, then an informative mixed-strategy equilibrium may exist. With slight abuse of terminology, we can think of a mixed strategy garbling \( r \) more than it garbles \( l \) and so counteracting the initial skew in the prior toward \( p \).

Informative mixed-strategy equilibria, however, do not always exist. Figure 2 illustrates the three restrictions that a putative mixed-strategy equilibrium must satisfy, and thus provides the intuition for the next proposition. First, for the firm to be randomizing after receiving its signal, it must be that the firm is indifferent between choosing \( L \) or \( R \). That is, the share price induced by \( L \), namely \( \Pr(\lambda|L) \), must be the same as that induced by \( R \), \( \Pr(\rho|R) \). Second, the market's posterior probabilities \( \Pr(\rho|L) \)

![Figure 2](image-url)
and \( \Pr(\rho | R) \) must lie on opposite sides of \( \pi \): this is the martingale property of conditional probabilities. Third, since a mixed strategy is less informative than the signal itself, both these posteriors must lie in the interval between \( \Pr(\rho | l) \) and \( \Pr(\rho | r) \).

**Proposition 2.** Suppose that the prior \( \pi > \frac{1}{2} \), and that the signal likelihood \( q > \pi \). Then mixed-strategy equilibria exist in the share-price maximization game if and only if \( \pi'(1 - \pi)^2 < q/(1 - q) \). Even where mixed-strategy equilibria exist, the maximum *ex ante* expected profit, \( V(\sigma; \pi, q) \), that can be achieved in a mixed-strategy equilibrium is only equal to \( \Pr(\lambda | l) \); this is strictly less than \( q \), the maximum possible *ex ante* expected profit.

*Proof.* From the discussion above, we know that in any mixed-strategy equilibrium,\n\[ \Pr(\rho | l) \leq \min\{ \Pr(\rho | L), \Pr(\rho | R) \} < 1 - \pi. \]

A necessary condition for a mixed-strategy equilibrium to exist is then \( \Pr(\rho | l) < 1 - \pi \), which is easily shown to be equivalent to \( \pi'(1 - \pi)^2 < q/(1 - q) \). Next, notice that we can write *ex ante* expected profit as
\[ V(\sigma; \pi, q) = \Pr(\lambda | L)\Pr(L) + \Pr(\rho | R)\Pr(R), \]
where \( \Pr(L) \) (respectively \( \Pr(R) \)) is the probability that the firm chooses \( L \) (respectively \( R \)), given its strategy \( \sigma \), and \( \Pr(\lambda | L) \) and \( \Pr(\rho | R) \) are the corresponding posteriors induced by \( L \) and \( R \) respectively. We know that in a mixed-strategy equilibrium,
\[ \Pr(\lambda | L) = \Pr(\rho | R). \]

Therefore \( V(\sigma; \pi, q) = \Pr(\lambda | L) \). But \( \Pr(\lambda | L) \leq \Pr(\lambda | l) \). And since \( \pi > \frac{1}{2} \), we have \( \Pr(\lambda | l) < q \). To see that this bound is achievable provided \( \pi'(1 - \pi)^2 < q/(1 - q) \), let \( \sigma(r) = 1 \) and
\[ \sigma(l) = \frac{2q(1 - q)(\pi - \frac{1}{2})}{q^2(1 - \pi)^2 - (1 - q)^2\pi^2}. \]

It is simple to check that the latter is a well-defined probability if \( \pi'(1 - \pi)^2 < q/(1 - q) \) and that this strategy yields
\[ \Pr(\lambda | L) = \Pr(\rho | R) = q(1 - \pi)[q(1 - \pi) + (1 - q)\pi] = \Pr(\lambda | l). \]

*Q.E.D.*

Even when we admit mixed strategies, share-price maximization leads to inefficient choices. Notice that as the prior \( \pi \) moves closer to \( \frac{1}{2} \), the maximum equilibrium *ex ante* expected profit increases, approaching the efficient level, \( q \). Intuitively, if there is less skew in the prior, then the firm's own posteriors are less unbalanced. Therefore, less information has to be discarded from the signal \( r \) by the firm's equilibrium choices in order to balance the market posteriors. At \( \pi = \frac{1}{2} \), no information needs to be garbled, and an efficient pure-strategy equilibrium can be sustained.\(^9\)

Although the results above were produced by assuming a skewed prior, they do not depend on this assumption or on our specific assumptions about equal *ex post*

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\(^9\) That equilibrium strategies can approach efficiency as \( \pi \) approaches \( \frac{1}{2} \) is special to the one-firm case. See Section 3.
payoffs and equal signal likelihoods. The key is not that the prior is skewed but that the two decisions yield unequal posterior expected profit to the firm. To see this, suppose that $x(L, \lambda)$ is the ex post payoff to the firm from choosing $L$ when the state is $\lambda$. Similarly define $x(L, \rho), x(R, \lambda)$, and $x(R, \rho)$. We need make no special assumptions on how these payoffs are ordered. Suppose that $q_l := \Pr(l|\lambda)$ is the likelihood that the signal is correct when the state is $\lambda$. Similarly, let $q_r := \Pr(r|\rho)$. We can relax the assumption that these likelihoods are the same and that the prior $\pi > \frac{1}{2}$. To avoid triviality, however, we maintain the assumption that $q_l, q_r,$ and $\pi$ all lie strictly between zero and one. Formally,

**Proposition 3.** Generically (in the parameters), there is no pure-strategy equilibrium of the share-price maximization game in which the firm plays an informative strategy. In particular, if the optimal strategy involves the firm’s either always following its signal or always choosing the opposite of its signal, then this strategy cannot be sustained in equilibrium. The best pure-strategy equilibrium involves the firm’s always choosing the action that maximizes its expected profit according to the market’s prior expectations; that is, choosing the same action regardless of the signal.

**Proof.** It is optimal for the firm always to follow its signal if the following two inequalities hold.

$$\Pr(\rho|l)x(R, \rho) + (1 - \Pr(\rho|l))x(R, \lambda) \geq \Pr(\rho|l)x(L, \rho) + (1 - \Pr(\rho|l))x(L, \lambda) \quad (3)$$

and

$$\Pr(\lambda|l)x(L, \lambda) + (1 - \Pr(\lambda|l))x(L, \rho) \geq \Pr(\lambda|l)x(R, \lambda) + (1 - \Pr(\lambda|l))x(R, \rho). \quad (4)$$

where

$$\Pr(\rho|l) = \frac{\pi q_l}{\pi q_l + (1 - \pi)(1 - q_l)} \quad \text{and} \quad \Pr(\lambda|l) = \frac{(1 - \pi)q_l}{(1 - \pi)q_l + \pi(1 - q_l)}.$$

If both inequalities were reversed, then it would be optimal for the firm always to choose the opposite of its signal. If, contra hypothesis, the firm were always to follow its signal in equilibrium, then the share price following $R$, $p(R)$, would be given by the left side of inequality (3). And the share price following $L$, $p(L)$, would be given by the left side of inequality (4). Except in a knife-edge case, however, these two expressions are not equal. Therefore, the share-price-maximizing firm would deviate and always choose whichever was the greater. Similarly, unless the right side of inequality (3) is equal to the right side of inequality (4), it is not an equilibrium for the firm always to choose the opposite of its signal. Given that the firm’s equilibrium pure strategy cannot, in general, be conditional on its signal, the best it can do is to choose the action that maximizes the ex ante expectation of revenue given its prior information. That is, it will always choose $R$ if

$$\pi x(R, \rho) + (1 - \pi)x(R, \lambda) \geq \pi x(L, \rho) + (1 - \pi)x(L, \lambda) \quad (5)$$

and always choose $L$ otherwise. Notice that this is what the market would choose were it to make the decision, aiming to maximize expected revenue according to its own (weakly) inferior information. \textit{Q.E.D.}

Loosely speaking, unless the firm’s posterior expected revenue from following its signal is the same regardless of which signal is realized (which is unlikely to be the case), then following its signal is not an equilibrium. A verbal intuition for this more
general result is not as immediate as that for the Nathan Bloom story, in which we relied only on skew in the prior. For example, it is possible to choose parameters such that if the market believed (contrary to equilibrium) that the firm were following its signal, it would prefer to see \( L \), but the action it would prefer according to its own beliefs (and according to equilibrium beliefs about the firm’s strategy) is \( R \). That is, inequalities (3), (4), and (5) might all hold, but the left side of (4) might be greater than the left side of (3).\(^{10}\) Nevertheless, excluding knife-edge cases, the best pure-strategy equilibrium will still be always to choose the market’s \textit{ex ante} preferred action (which, in this case, would be \( R \)).

The assumption that a firm aims simply to maximize its share price can be relaxed somewhat while still maintaining the main conclusions. For example, suppose that the firm aims to maximize a convex combination of its current share price and future profit with weight \( \alpha \) assigned to future profit. Suppose that otherwise the structure of the game is unchanged. As in Proposition 1, suppose that the prior \( \pi \) \( > \frac{1}{2} \) and that the strategy that maximizes the firm’s expected profit is for it always to follow its signal, that is, the signal likelihood \( q \) \( \geq \pi \). Then a simple calculation shows that this efficient strategy can be sustained in equilibrium if and only if

\[
\frac{\alpha(2q - 1)}{q(1 - q)} \geq \frac{2\pi - 1}{\pi(1 - \pi)}.
\]

For intuition, notice that if the weight \( \alpha = 1 \), inequality (6) reduces to \( q \geq \pi \), the condition under which it is efficient for the firm to follow its signal. If \( \alpha = 0 \), inequality (6) is never satisfied, reproducing the conclusion of Proposition 1. We can think of the weight \( \alpha \) as alleviating the skew in the prior \( \pi \). In the case of mixed strategies, raising \( \alpha \) reduces the need to garble information, so it becomes easier to sustain an informative equilibrium, and the most efficient equilibrium moves closer to efficiency.

To summarize: If the firm is sufficiently concerned about its current share price, then there is often no equilibrium in which the firm uses its private information in making its decision. In particular, it is generally impossible to sustain an equilibrium in which the firm simply follows its signal, even when this is the efficient strategy. Where they exist, mixed-strategy equilibria can use more information than do pure-strategy equilibria, but they too are strictly less efficient than the first best. The best pure-strategy equilibrium involves the firm’s always choosing the action that the market would choose were it to make the decision, aiming to maximize expected revenue according to its own inferior information. In this sense, the firm makes the decision the market wants to see.

3. Many-firm posteriors

- We now consider what happens if there are many firms, each of which has to make a similar decision. The setup is similar to that of Proposition 1. The true state of the world is either \( \lambda \) or \( \rho \).\(^{11}\) A firm makes a profit of \( 1 \) if it makes the correct choice (\( L \) given \( \lambda \) or \( R \) given \( \rho \)), and \( 0 \) otherwise. That is, the same decision is correct for all the firms and each firm’s profit is independent of the choices of the other firms. Each firm receives a signal. For simplicity assume that, conditional on the true state, the signals are identically and independently distributed with \( \Pr(l | \lambda) = \Pr(r | \rho) = q \), where \( \frac{1}{2} < q < 1 \).\(^{12}\) The firms move sequentially, and each gets to observe the sequence of choices, but not the signals, of the preceding firms. In practice, firms might be able to

---

\(^{10}\) We thank an anonymous referee for this observation.

\(^{11}\) The assumption that there are only two states may not be innocuous when the number of firms is infinite.

\(^{12}\) The assumption of conditional independence is not crucial: similar results hold except in the special case where there is only one common signal.
observe how many firms precede them but are less likely to know how many firms will follow. Therefore, we treat cases with both a finite and an infinite number of firms, where the latter can be thought of as a metaphor for each firm’s (including the last) not knowing the number of subsequent firms.

The manager of the nth firm still chooses between L or R or some randomization of the two, but now the choice can be conditional on the choices of the first n – 1 firms as well as on the realization of the manager’s private signal. Formally, let \( \sigma_n(h_{n-1}, l) \) (respectively \( \sigma_n(h_{n-1}, r) \)) be the probability that the firm chooses R following the history \( h_{n-1} \) of earlier moves given the signal \( l \) (respectively \( r \)), according to the strategy \( \sigma_n \).

We call a strategy informative at \( h_{n-1} \) if \( \sigma_n(h_{n-1}, l) \neq \sigma_n(h_{n-1}, r) \), and we call firm n’s equilibrium strategy informative (without the qualifier) if it is informative at some \( h_{n-1} \) reached with positive probability in that equilibrium.

As before, we assume that there is some public information that results in everyone’s assigning a prior \( \pi \) to the true state being \( \rho \), where, without loss of generality, \( \pi \geq \frac{1}{2} \). By the time the nth firm gets to move, however, the prior may have evolved due to the moves of earlier firms. Formally, let \( \pi(h_0) = \pi \) and let \( \pi(h_{n-1}) \) be the preposterior (on \( \rho \)) of firm n after observing the history \( h_{n-1} \) but before observing its own signal. On an equilibrium path, \( \pi(h_{n-1}) \) is uniquely determined by updating from the prior \( \pi \). In this case, \( \pi(h_{n-1}) \) is also the probability assigned to \( \rho \) by all subsequent firms (and the stock market where relevant) after seeing \( h_{n-1} \).

Similar to the one-firm case, let \( V_n(\sigma; \pi, q) \) be the ex ante expectation of firm n’s profit given the strategy profile \( \sigma \). That is, \( V_n(\sigma; \pi, q) \) is the probability that firm n chooses correctly. If there are N firms, our standard of performance will be the average expected profit:

\[
W_n(\sigma; \pi, q) := \frac{1}{N} \sum_{n=1}^{N} V_n(\sigma; \pi, q).
\]

Where \( N \) is infinite, we take the limit of this expression (if it exists).

Recall that we want to compare the distortions caused by share-price maximization with those that would occur anyway due to herd or cascade behavior. Therefore, we first consider what would happen if the firms aimed directly to maximize their individual expected future profit. Our setup is close to those of Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), and Welch (1992), and the following proposition is in the spirit of more general results given in those articles.

**Proposition 4.** Suppose that the prior \( \pi > \frac{1}{2} \), that the signal likelihood \( q > \pi \), and that each firm’s objective is to maximize its own expected profit. Then for every number of firms \( N \) (possibly infinite), there is a unique equilibrium. The equilibrium average ex ante expected profit, \( W_N \), increases in \( N \) and is weakly between \( q \) and

\[
\lim_{N \to \infty} W_N = \frac{\pi q + (1 - \pi)q^2}{1 - q(1 - q)}.
\]

**Proof.** Consider Figure 3. As in the one-firm case, if firm n aims to maximize its expected profit, it will follow its signal if and only if \( (1 - q) \leq \pi(h_{n-1}) \leq q \). If

\( \pi(h_{n-1}) < (1 - q) \)

(respectively \( > q \)), then firm n will choose L (respectively R) regardless of its signal. Since \( \frac{1}{2} < \pi < q \), firm 1 will follow its signal. This is illustrated by the top fork in
Figure 3. From the one-firm case, we know that $W_1 = q$. If firm 1 receives the signal $r$ and hence chooses $R$, a simple calculation shows that firm 2's preposterior $\pi(R)$ exceeds $q$. In this event, firm 2 and all subsequent firms choose $R$. This is illustrated by the square terminal node following the initial branch $R$. A similar calculation reveals that $(1 - q) < \pi(L) < \frac{1}{2}$. Thus if firm 1 chooses $L$, firm 2 will follow its own signal. If firm 2 also receives a signal $l$ and hence chooses $L$, firm 3's preposterior $\pi(L, L)$ falls below $(1 - q)$. In this event, firm 3 and all subsequent firms choose $L$. This is illustrated by the square terminal node following $(L, L)$. Finally, if firm 3 chooses $L$ and firm 2 chooses $R$, the two signals exactly cancel, so $\pi(L, R) = \pi$ and firm 3 is in the same position as firm 1. This is illustrated by the circular node following $(L, R)$. This last event occurs with probability $q(1 - q)$. Using these facts, a calculation shows that $W_n$ increases in $N$ and that $\lim W_n = \frac{\sqrt{q} + (1 - \pi)q^2}{1 - q(1 - q)}$.

Q.E.D.

The outcome of this herding game is inefficient when $N$ is sufficiently large. The reason is that if more firms were to follow their signals, the information so revealed would assist later firms in their decision making. This externality is not taken into account by an individual profit-maximizing firm. If there are many subsequent firms, the benefit they gain from, say, firm 2 revealing its information after firm 1 has chosen $R$, outweighs the cost to firm 2 of having to follow its signal.

Now consider the corresponding game in which the firms aim not to maximize their expected profits but rather to maximize their expected share prices.\(^{13}\) Once all the

\(^{13}\) Unlike the one-firm case, the qualifier "expected" is not innocuous here: firms must form expectations over the signals and actions of subsequent firms.
firms have made their decisions but before the true state is revealed, a stock market
forms a share price for each firm. Without loss of generality, we can restrict the au-
tioneer to announce a single price for firms that chose $L$ and a single price for those
that chose $R$. In all other respects the stock market is analogous to that in the one-firm
case. On an equilibrium path, these prices must equal the market's probability that the
true state is $\lambda$ and $\rho$, respectively.

The following proposition extends both Propositions 1 and 2 to the game with a
(possibly infinite) sequence of share-price-maximizing firms. It is the main result of
this section.

**Proposition 5.** Suppose that the prior $\pi > \frac{1}{2}$, that the signal likelihood $q > \pi$, and
that each firm's objective is to maximize its own expected share price. Then, regardless
of the number of firms $N$ (including $N$ infinite), there is no pure-strategy equilibrium
in which any firm plays an informative strategy. The pure-strategy equilibrium that
yields the highest *ex ante* expected average profit, $W_N$, involves each firm's always
choosing $R$ regardless of its signal. In this case, $W_N = \pi$. Mixed-strategy equilibria
exist if and only if $[\pi(1 - \pi)]^2 < q(1 - q)$. Even where mixed-strategy equilibria
exist, at most one firm's strategy can be informative on any equilibrium path, and the
highest *ex ante* expected average profit that can be sustained in equilibrium is only
equal to $\Pr(\lambda | r)$, which is strictly less than $q$.

**Proof.** See the Appendix.

We shall discuss first the intuition, then the content of this result. Part of the
argument for the pure-strategy case can be gathered from Figure 3. Suppose, contra
hypothesis, that firm 1's equilibrium strategy is to follow its signal, and suppose that
it receives signal $r$ and hence chooses $R$. This move induces firm 2, all other subsequent
firms, and the stock market to update their beliefs to the preposterior $\pi(R) = \Pr(\rho | r)$,
as shown by the right branch of the solid-line fork in the top part of Figure 3. Suppose
that, according to the equilibrium strategy, firm 2 is also supposed to follow its signal
after observing firm 1 choose $R$, and suppose that it receives the signal $l$. If firm 2
chooses $L$, the market's new preposterior, $\pi(R, L)$, is the same as firm 2's own belief,
$\Pr(\rho | r, l)$, as shown by the left branch of the dashed-line fork in the bottom part of
Figure 3. What concerns firm 2, however, is not the immediate preposterior induced
by its choice, but rather where the market's posterior will end up after extracting any
information contained in the (equilibrium) moves of all the subsequent firms. This is
what will determine firm 2's share price. Since, in this case, the beliefs of firm 2 and
the market are the same, firm 2's expectation of what the market's eventual posterior
will be after the choices of the subsequent firms is equal to firm 2's expectation of
what its own posterior will be after observing those same choices: namely, firm 2's
own current belief. Formally, this is because conditional probabilities form a martingale.
Therefore, if firm 2 chooses $L$, its expected share price is $(1 - \Pr(\rho | r, l))$, which is
strictly less than $\frac{1}{2}$.

Now suppose that instead of following its signal, firm 2 deviates and chooses $R$.
In this case, the market's immediate preposterior, $\pi(R, R)$, is $\Pr(\rho | r, r)$, as shown by
the right branch of the dashed-line fork, whereas firm 2's own belief is $\Pr(\rho | r, l)$, as
before. Since these beliefs differ, we cannot use the previous reasoning, but a related
argument works. The market's expectation of its own posterior after observing the
choices of all the subsequent firms is equal to the market's current belief. Similarly,
firm 2's expectation of its own final posterior is equal to its own (lower) current belief.
Firm 2 reasons that if it observes more information, the market will "wise up"; that is,
firm 2 expects the market's posterior to be updated from the market's current belief
toward firm 2's own current belief. Therefore, firm 2's expectation of the market's final
posterior lies (weakly) between $\Pr(\rho | r, r)$ and $\Pr(\rho | r, l)$. Formally, this is because the market's posteriors form a supermartingale under firm 2's current posterior (as shown by Lemma A1 in the Appendix). Therefore, if firm 2 chooses $R$, its expected share price is (weakly) greater than $\Pr(\rho | r, l)$, which is weakly greater than $\frac{1}{2}$. Similar arguments rule out any other informative equilibrium strategy by firm 2 (or any subsequent firm) after firm 1 chooses $R$.

Now return to firm 1's putative equilibrium strategy of following its signal, and suppose that it receives the signal $l$. We have just argued that if it chooses $R$, no subsequent firm will move informatively. Therefore, choosing $R$ yields a share price equal to $\Pr(\rho | r)$, which is greater than $q$. If firm 1 chooses $L$, then the market's immediate preposterior, $\pi(L)$, is the same as firm 1's own belief, $\Pr(\rho | l)$, as shown by the left branch of the solid-line fork in the top part of Figure 3. But since these beliefs are the same, firm 1's expectation of the market's eventual posterior must equal its own current belief. That is, the expected share price from choosing $L$ after receiving signal $l$ is equal to $(1 - \Pr(\rho | l))$, which is less than $q$, a contradiction. Similar arguments rule out putative equilibria in which firm 1 always chooses the opposite of its signal. Since firm 1's equilibrium pure strategy cannot be informative, the same argument applies to all subsequent firms. Therefore, just as in the one-firm case, the best pure-strategy equilibrium is to follow the prior.

The argument for mixed strategies is more involved. In the one-firm case, mixed strategies can sometimes be informative. The tricky part of the proof in the many-firm case is to show that information cannot thus be divulged gradually, a little bit from each firm, by several firms playing mixed strategies. One might imagine that this would add up to a lot of information, but we show that at most one firm can play an informative mixed strategy in any equilibrium. The underlying idea is similar to the pure-strategy case. We use the fact that firms' expectations of market posteriors are well behaved (they form super- or submartingales) combined with our claim that if equilibrium preposteriors were ever to go above $q$ (or, analogously, below $(1 - q)$), then no subsequent firm would play informatively. Together, these arguments make it unnecessary to rely on there being a last firm.

The outcome of share-price maximization with many firms is strictly less efficient than that when the firms aim to maximize expected profits, even after allowing for herding. Whereas the ex ante expected average efficiency, $W_n$, with profit maximization is increasing in $N$ and weakly greater than $q$, the average efficiency with many share-price maximizing firms is no better than that with just one firm and strictly less than $q$. With revenue maximization, the equilibrium always uses the whole of the information of the first firm and, on average, part of the information of later firms. With share-price maximization, none of the firms' private information is used in any pure-strategy equilibrium, and at most only part of one firm's information is used in any mixed-strategy equilibrium. In this sense, concern for current share prices exacerbates the distortions of herds and cascades.

To summarize: Surprisingly, the outcome of the many-firm share-price-maximization game is essentially the same as that of the corresponding one-firm game. Regardless of the number of firms, the best pure-strategy equilibrium involves each firm's choosing the action that maximizes its expected profit according to the market's prior beliefs, regardless of all the information contained in the private signals.

Appendix

- This Appendix provides the proof for Proposition 5. We first establish our claim that the market's posteriors form a super- or submartingale under a firm's posterior.

Lemma A1. Consider a putative equilibrium profile of strategies $\sigma$. Suppose that firm $n$ receives the signal $l$ and chooses $R$ at $h_n$, that lies on an equilibrium path for $\sigma$. Suppose that $\sigma(h_{n_{-1}}, r) > 0$, so that firm
n's own posterior \( \Pr(\rho | h_{n-1}; l) \) is less than \( \pi(h_{n-1}, R) \), the market's preposterior on observing firm \( n \) choose \( R \). Then

(a) The sequence of market posteriors \( \{ \pi(h_{n-1}) \} \), where \( h_{n-1} \) lies on an equilibrium path for \( \sigma \) that continues from \( (h_{n-1}, R) \), is a (bounded) supermartingale under firm \( n \)'s current posterior \( \Pr(\rho | h_{n-1}; l) \). That is, for \( k = 1, 2, \ldots \),

\[
E[\pi(h_{n-k}) | h_{n-k}; l] \leq \pi(h_{n-k-1}).
\]  

(A1)

Inequality A1 is strict if and only if firm \( n + k \)'s equilibrium strategy, \( \sigma_{n+k} \), is informative at \( h_{n+k-1} \).

(b) Moreover, for any number of firms \( N \) (including \( N \) infinite), firm \( n \)'s current expectation of the eventual market posterior, \( \pi_n \), is well defined and satisfies

\[
\Pr(\rho | h_{n-1}; l) \leq E(\pi_n | h_{n-1}; l) \leq \pi(h_{n-1}, R).
\]  

(A2)

Finally, these conditions hold \textit{mutatis mutandis} (with submartingale replacing supermartingale where appropriate) for other signals and choices of firm \( n \).

\textbf{Proof.} Consider first the effect of firm \((n + 1)\)'s move at \( h_n = (h_{n+1}, R) \) on the posteriors of firm \( n \) and of the market. Prior to the move, firm \( n \)'s own belief \( \Pr(\rho | h_{n}; l) = \Pr(\rho | h_{n+1}; l) \), since firm \( n \) learned nothing from its own move. And, by assumption, \( \Pr(\rho | h_{n+1}; l) < \pi(h_{n+1}) \). Therefore, firm \( n \)'s own belief

\[
\Pr(\rho | h_{n+1}; l) < \pi(h_{n+1})
\]

for any \( h_{n+1} \) that lies on an equilibrium path continuing from \( h_n \). Let \( \mu := \sigma_{n+1}(h_n, R) \) and \( \nu := \sigma_{n+1}(h_n; l) \) denote firm \((n + 1)\)'s equilibrium strategy, and let \( \alpha \) (respectively \( \beta \)) be the probability that the market (respectively firm \( n \)) assigns to the event that firm \( n + 1 \) chooses \( R \) following \( h_n \). That is,

\[
\alpha = [q\mu + (1 - q)\nu] \pi(h_n) + [(q\nu + (1 - q)\mu)[1 - \pi(h_n)],
\]

\[
\beta = [q\mu + (1 - q)\nu] \Pr(\rho | h_{n}; l) + [(q\nu + (1 - q)\mu)[1 - \Pr(\rho | h_{n}; l)].
\]

Notice that \( \alpha > \beta \) if and only if \( \mu > \nu \). Likewise, the market posterior after firm \((n + 1)\)'s choice satisfies \( \pi(h_n, R) > \pi(h_n, L) \) if and only if \( \mu > \nu \). Now compute firm \( n \)'s expectation of this market posterior:

\[
E[\pi(h_{n+1}) | h_n; l] = \beta \pi(h_n, R) + (1 - \beta) \pi(h_n, L)
\]

\[
\leq \alpha \pi(h_n, R) + (1 - \alpha) \pi(h_n, L)
\]

\[
= \pi(h_n),
\]

where the inequality is strict if and only if \( \mu \neq \nu \). This establishes (a) for \( k = 1 \). The proof for general \( k \) is similar.

Since \( \Pr(\rho | h_{n+1}; l) < \pi(h_{n+1}) \) for any \( h_{n+1} \) that occurs with positive probability following \( h_n = (h_{n+1}, R) \),

\[
E[\pi(h_{n+1}) | h_{n+1}; l] = \beta \pi(h_n, R) + (1 - \beta) \pi(h_n, L)
\]

\[
> \beta \Pr(\rho | h_{n+1}; R; l) + (1 - \beta) \Pr(\rho | h_{n+1}; L; l)
\]

\[
= \Pr(\rho | h_{n+1}; l) = \Pr(\rho | h_{n+1}; l),
\]

where the inequality follows from the martingale property. Generalizing yields

\[
E[\pi(h_{n+k}) | h_{n-k}; l] > \Pr(\rho | h_{n-k}; l)
\]

for all \( k \).

Since the sequence of market posteriors is a bounded supermartingale, it converges almost surely to a limit. Therefore \( \pi_n \) is well defined even for infinite \( N \). The inequality \( E[\pi_n | h_{n+1}; l] \leq \pi(h_n) \) is an immediate consequence of the supermartingale property, while letting \( k \to \infty \) in the inequality

\[
E[\pi(h_{n+k}) | h_{n-k}; l] > \Pr(\rho | h_{n-k}; l)
\]

yields \( E(\pi_n | l) \geq \Pr(\rho | h_{n-k}; l) \). This establishes (b). \( Q.E.D. \)
Proof of Proposition 5. We first show that firm 1's strategy cannot be pure and informative even if subsequent firms have (possibly mixed) informative strategies. Consider a putative pure-strategy equilibrium \( \sigma \) and suppose that firm 1’s strategy \( \sigma_l \) is informative, in particular, \( \sigma_l(l) = 0 \) and \( \sigma_l(r) = 1 \). In this case,

\[
\pi(R) = \Pr(\rho|r) > q.
\]

We shall show that firm 2's strategy (pure or mixed) cannot be informative at \( h_l \) if \( \pi(h_l) > q \). Since

\[
\Pr(\rho|r, r) > \Pr(\rho|r, l) > \frac{q}{2},
\]

we know from the martingale property together with Lemma A1 that if firm 2 chooses \( R \), the lower bound on its expected share price exceeds \( \frac{q}{2} \). If firm 2 chooses \( L \), the upper bound on its expected share price is lower than \( \frac{q}{2} \). Hence, firm 2 will always choose \( R \) after \( h_l = R \). A similar argument implies that no firm \( n \)'s strategy, \( \sigma_n \), can be informative at \( h_{n-1} \) on an equilibrium path for \( \sigma \) such that \( \pi(h_{n-1}) > q \), or such that \( \pi(h_{n-1}) < q \). Hence no firm's strategy is informative after \( h_l = R \).

Now return to firm 1. If firm 1 receives the signal \( l \) and chooses \( L \), its expected share price will be \( \Pr(\lambda|l) \), using the martingale property. If firm 1 receives the signal \( l \) but deviates to \( R \), since no subsequent firm plays informatively, its expected share price will be precisely \( \Pr(\rho|l) \), which is greater than \( \Pr(\lambda|l) \). This establishes that firm 1’s equilibrium strategy cannot be to follow its signal. The case of choosing the opposite of its signal is similar. We have now shown that firm 1’s strategy, \( \sigma_l \), cannot be informative. Repeating the argument for subsequent firms establishes that no firm plays informatively in any pure-strategy equilibrium. Hence, just as in the one-firm case, the best equilibrium involves every firm's choosing \( R \).

Next consider mixed-strategy equilibria. We have to show that there is no mixed-strategy equilibrium in which more than one firm’s strategy is informative along an equilibrium path. We proceed case by case, always assuming, without loss of generality, that the first informative firm is firm 1. We have already shown that firm 1’s strategy cannot be both pure and informative. Next we show that \( \sigma_l(l) = 0 \) or \( 1 \), \( 0 < \sigma_l(r) < 1 \) is impossible. Suppose, for example, that \( \sigma_l(l) = 0 \). Then if firm 1 receives the signal \( r \) and chooses \( R \), its expected share price is \( \pi(R) = \Pr(\rho|r) \). If firm 1 receives the signal \( r \) and chooses \( L \), its expected share price is \( 1 - E(\pi_x|R; r) \), where \( E(\pi_x|R; r) \) is firm 1’s expectation of the market’s final posterior given that firm 1 receives the signal \( r \) and chooses \( L \). But by Lemma A1, \( E(\pi_x|R; r) \geq \pi(L) > \Pr(\rho|l) \), so

\[
1 - E(\pi_x|R; r) < \Pr(\lambda|l).
\]

Since \( \Pr(\lambda|l) < \Pr(\rho|l) \), strict randomization after \( r \) is impossible. The argument for \( \sigma_l(r) = 1 \) is similar.

Next we show that \( 0 < \sigma_l(l) < 1 \), \( \sigma_l(r) < 1 \) is impossible if any subsequent firm \( n \)'s strategy, \( \sigma_n \), is informative at \( h_{n-1} \) on an equilibrium path for \( \sigma \). To see this, use Lemma A1 to order the possible expectations that firm 1 can form of the eventual market posterior:

\[
E(\pi_x|R; l) \leq \pi(R) \leq E(\pi_x|R; r), \quad \text{(A3)}
\]

\[
E(\pi_x|L; l) \leq \pi(L) \leq E(\pi_x|L; r). \quad \text{(A4)}
\]

If firm 1 strictly randomizes after \( l \) (and at least sometimes) chooses \( L \) after \( r \), the expected share prices must satisfy

\[
E(\pi_x|R; l) - 1 - E(\pi_x|L; l), \quad \text{(A5)}
\]

\[
E(\pi_x|R; r) \leq 1 - E(\pi_x|L; r). \quad \text{(A6)}
\]

By Lemma A1(a), if some subsequent firm \( n \)'s strategy is informative, the inequalities in either (A3) or (A4) or both must be strict. Combining this with expressions (A5) and (A6) yields a contradiction.

The remaining candidate for an equilibrium \( \sigma \) in which more than one firm's strategy is informative (where, without loss of generality, we assume that the first such firm is firm 1 and the second is firm 2) involves \( 0 < \sigma_1(l) < 1 \) and \( \sigma_1(r) = 1 \). Define the parameter \( \theta \) by \( |\theta(1 - \theta)|^2 = q(1 - q) \), so that \( \Pr(\rho|l) = 1 - \theta \) if and only if \( \pi = \theta \). Notice that \( \theta \) is the critical value for \( \pi \) above which no mixed-strategy equilibrium could be supported in the one-firm case. There are two cases to consider: \( \frac{q}{2} < \pi < \theta \) and \( \pi \geq \theta \). First suppose \( \frac{q}{2} < \pi < \theta \). Observe that \( \sigma_1(r) = 1 \) implies that \( E(\pi_x|R; l) = \pi(L) = \Pr(\rho|l) \), and \( \pi < \theta \) implies \( \Pr(\rho|l) < 1 - \theta \). Given Lemma A1, strict randomization after \( l \) then implies \( \pi(R) \leq E(\pi_x|R; l) \), and strict randomization also implies \( E(\pi_x|R; l) = 1 - E(\pi_x|L; l) \). Hence, \( \pi(R) > \theta \).

Consider firm 2's following the choice \( R \) by firm 1. Since \( \pi(R) > \theta \), Proposition 2 implies that firm 2 cannot play informatively if it is the last firm to do so following \( h_l = R \). Repeating the argument made for firm 1, we see that if firm 2's strategy \( \sigma_2 \) is informative at \( h_l = R \), it must satisfy \( 0 < \sigma_2(R, l) < 1 \) and
\( \sigma(R, r) = 1 \). Moreover, since it cannot be the last informative firm, a subsequent firm must play informatively after \( h_2 = \langle R, R \rangle \). Notice that \( \pi(R, R) > \pi(R) \). Repeating this argument establishes that if firm 2's strategy is informative at \( h_1 = R \), then there must be an infinite sequence of firms (without loss of generality, the original sequence) such that for every firm \( n \) in the sequence, the strategy \( \sigma_n \) must satisfy \( 0 < \sigma_n(h_{n-1}, l) < 1 \) and \( \sigma_n(h_{n-1}, r) = 1 \) at \( h_{n+1} = \langle R, R, \ldots, R \rangle \). Also, \( \pi(R) < \pi(R, R) < \pi(R, R, R) < \ldots \). As a special case, if the number of firms, \( N \), is finite, we have reached a contradiction.

Next consider firm 2's following the choice \( L \) by firm 1. Using \( \pi(L) = 1 - \theta \), the argument of the preceding paragraph can be made (in symmetric fashion) to establish that if firm 2's strategy is informative at \( h_1 = L \), then there must be an infinite sequence of firms (without loss of generality, the original sequence) such that for every firm \( n \) in the sequence, the strategy \( \sigma_n \) must satisfy \( \sigma_n(h_{n-1}, l) = 0 \) and \( 0 < \sigma_n(h_{n-1}, r) < 1 \) at \( h_{n+1} = \langle L, L, \ldots, L \rangle \). Also \( \pi(L) > \pi(L, L) > \pi(L, L, L) > \ldots \). Again, as a special case, if the number of firms, \( N \), is finite, we have reached a contradiction.

The case \( \pi \geq \theta \) has already been treated when considering firm 2's following the choice \( R \) by firm 1.

Summarizing so far, the only remaining candidate for an equilibrium \( \sigma \) in which more than one firm's strategy is informative involves either (a) an infinite and increasing sequence of preposteriors \( \{ \pi(h_{n-1}) \} \) with \( \pi(h_{n-1}) > \pi \) for every \( n \) or (b) an infinite and decreasing sequence of preposteriors \( \{ \pi(h_{n-1}) \} \) with \( \pi(h_{n-1}) < 1 - \pi \) for every \( n \), or both. The proof concludes by showing that (a) is impossible (the proof that (b) is impossible is symmetric).

Notice that \( \pi(h_{n-1}) \leq q \) for every \( n \) (otherwise firm \( n \) could not play informatively). Since \( \{ \pi(h_{n-1}) \} \) is increasing, this implies that there is an \( n \) such that \( \pi_{n-1}(h_{n-1}, l) < \pi_{n-1}(h_{n-1}, l) \) on the path \( \langle R, R, \ldots \rangle \). There is no loss of generality (and considerable notational convenience) in taking firms \( n \) and \( n + 1 \) to be firms 1 and 2. Let \( \eta := \pi_{n-1}(l) \) and \( \zeta := \pi_{n-1}(R) \), so that \( \eta < \zeta \) and to avoid confusion, let us label the signals and the moves so that, for example, \( l \) refers to the signal \( l \) received by firm 1.

Recall that strict randomization by firm 2 after receiving the signal \( l \) requires \( E(\pi_n | R_1; l) \mid l \) = \( Pr(\lambda | l) \). Similarly, strict randomization by firm 2 at \( h_1 = R_1 \) after receiving the signal \( l \) requires that firm 2's expected signal price from choosing \( R_1 \), \( E_\pi(R_1, R_2; l) \) be equal to its expected signal price from choosing \( L_2 \), \( Pr(\lambda | l) \). But \( \pi(R_1) > \pi \) implies \( Pr(\lambda | R_1; l) < Pr(\lambda | l) \), so that

\[
E(\pi_n | R_1; l) > E(\pi_n | R_1, R_2; l).
\]  

(A7)

Write \( E(\pi_n | R_1, R_2; l) \) for firm 1's expectation of the eventual market posterior, recalculated after firm 1 sees firm 2 choose \( R_2 \). Since firm 2's strategy is informative and \( \sigma(R, r) > \sigma(R, l) \),

\[
E(\pi_n | R_2, R_2; l) > E(\pi_n | R_1, R_2; l).
\]  

(A8)

Finally, observe that the expectations \( E(\pi_n | R_1, R_2; l) \) and \( E(\pi_n | R_1, R_2; l) \) differ only in that the former is calculated using \( \eta \) while the latter uses \( \zeta \). Since \( \eta < \zeta \),

\[
E(\pi_n | R_1, R_2; l) > E(\pi_n | R_1, R_2; l).
\]  

(A9)

Combining inequalities (A7)-(A9) yields a contradiction.

Since only one firm can play an informative strategy on any equilibrium path, the conditions for a mixed-strategy equilibrium to exist and the bound on their efficiency are identical to those given in Proposition 2. The bound is achieved when the first firm plays the informative strategy described in the proof to that proposition. \( Q.E.D. \)

References


