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Growth and distribution: a neoclassical Kaldor–Robinson exercise

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Retrospect: Kaldorian distribution theory

In 1956 Nicholas Kaldor published his 'Keynesian' theory of the distribution of output between labour and property incomes, and in 1960 I published a short spoof of his article. I was a brash young American. In reprinting that note in a collection of my essays in 1971, I wrote:

> 'Chapter 7 is an irreverent spoof of a distribution theory advanced by Nicholas Kaldor and others . . . [It] is a footnote to the running controversy between neoclassical growth theory and its opponents. Neoclassical theory would have the division of full employment output between investment and consumption depend on the society's propensity to save. If property owners and wage earners differ in their saving behavior, the distribution of income between them would help to determine the share of investment in national output. The income distribution, in turn, depends, in neoclassical theory, on the marginal productivities of capital and labour. Kaldor rejected marginal productivity theory and needed an explanation of factor shares in its place. He regarded the investment share of total output as independently determined by technology and entrepreneurship—something to which the national saving propensity must adapt, rather than vice versa . . . I would like to record here my judgement, which the reading lists of my courses confirm, that Mr Kaldor has made many outstanding contributions to economic theory. He should be excused this aberration (1971, p. 3).

Nicky Kaldor was unperturbed by my note, although he did bother to reply (Kaldor, 1960). Fortunately, we subsequently became good friends and were usually on the same side of macroeconomic controversies. I had the opportunity to express my esteem for him in his presence both at the celebration of the centenary of Keynes's birth at Kings in 1983 and at Yale when Lord Kaldor gave the first set of Okun Memorial Lectures in 1983.[1]

For this symposium in his memory, I return to the subject of our disagreement three decades ago.

I was not criticising the proposition that saving propensities might differ for incomes of different types, as well as for incomes of different magnitudes. After all, one important strand of mainstream saving theory, the life cycle model, also focuses on the difference between human and nonhuman wealth. More important, businesses, especially corporations, may not be acting just as agents of convenience for individual shareowners when they plough back profits. They may be instead institutionalising that compulsion for

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[1] See my 'Comment' on Kaldor's paper 'Keynesian Economics after Fifty Years' at the 1983 conference, in D. Worswick and J. Trevithick, (eds) 1983. See also my Preface to Kaldor, 1985.

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accumulation which Marx—and Joan Robinson, Kaldor and other post-Keynesians—have regarded as central to capitalism.

In this institutional spirit, Kaldor himself applied his differential propensities to sources of income, labour or property, rather than to classes of persons, workers and capitalists. For this reason he could not get excited about the long-run implications of recognition that both classes save and accumulate wealth, the discussion triggered by the Pasinetti process.[1] Empirically, it has not been possible to prove that business saving is just a one-for-one substitute for household saving.

Kaldorian saving propensities can easily be built into Swan–Solow type neoclassical growth models where, like other savings functions, they help to determine a stable steady-state capital intensity and corresponding stable values of other variables. Indeed the classical saving function, a popular extreme form of the Kaldorian hypothesis—nothing is saved from wages and nothing is consumed from profits—leads to the 'Golden Rule' optimum, the steady state with maximum consumption per worker. In that equilibrium investment equal profits and, in a Swan–Solow model, the marginal productivity of capital is equal to the growth rate. However, the weight of evidence is against the view that national saving and investment are as large as capital incomes (Abel *et al.*, 1986).

Moreover, if Kaldorian saving propensities are built into a one-good neoclassical growth model, they will help to determine the distribution of income and wealth. The steady-state capital shock is endogenous, dependent on saving behaviour. Therefore the marginal productivities of capital and labour and, except in the special case of the Cobb-Douglas production function, the relative shares of labour and capital incomes are likewise endogenous and dependent on saving behaviour. But differential saving propensities are not, except in the special case of the Leontief production function, *necessary* to determine distributive shares. Almost any saving function, for example the primitive assumption that a constant fraction of income of all kinds is saved, will determine the steady-state capital/labour ratio and thus also marginal productivities and factor shares.

What did bother me thirty years ago? First, I found it hard to believe that factors' returns had nothing to do with their productivities, and my note made fun of some implications of that belief. Second, in relation to macroeconomic theory, my problem was this: If marginal productivity is dropped as an explanation of income shares and the consumption function is drafted to replace it, how is aggregate output to be explained? I was shocked to see a 'Keynesian' model that apparently assumed output to be independent of aggregate demand even in the short run. And given full employment, I thought, the role of the consumption function is to help to determine investment as equal to saving, which it cannot do if it is assigned the burden of determining wages and profits. To me, a model with investment wholly exogenous was both unKeynesian and unpalatable.

Of course, marginal productivities are indeterminate, within limits, when factors are fully employed and technology requires them to be used in constant proportions. Maybe differential savings propensities can help in these circumstances. What determines investment and output remains a problem. Animal spirits? Perhaps, in short-run business cycles. In those circumstances there is no mechanism to insure that capital capacity and

[1] Pasinetti, 1962, pp. 267–79. In the course of the debate provoked by this seminal article, Kaldor wrote . . . 'I have always regarded the high savings propensity out of profits as something which attaches to the nature of business income, and not to the wealth (or other peculiarities) of the individuals who own property. It is the enterprise, not the particular body of individuals owning it at any one time, which finds it necessary . . . to plough back a proportion of the profits earned . . . Hence the high savings propensity attaches to profits as such, not to capitalists as such' (1966).

labour supply stand in the correct proportions to each other. Whether capacity is and is expected to be short relative to labour supply or redundant is surely important in investment decisions. In the long run, capital capacity is adjusted to the requirements of exogenous growth in effective labour supply and of technology.

I take my cue from Joan Robinson:

> The rate of investment . . . can be accounted for in two ways which do not seem to be connected with each other. Investment is determined, in one sense, by profit expectations, the 'animal spirits' of entrepreneurs which incline them to take the risks of investment, and the state of supply of finance, which may be subsumed under the head of the level of interest rates.
>
> In another sense, the rate of investment that can be maintained over the long run depends on the technical conditions and the supply of labour. According to this view, the rate at which the effective supply of labour is growing . . . limits the rate at which capital can accumulate, because there would be no point in bringing capital goods into existence when there is not going to be labour to operate them (Robinson, 1960, p. 146).[1]

Factor shares and saving in a growth model with Leontief technologies

I provide here a simple example of a model in which the distribution of income between wages and returns to capital ownership cannot be explained by marginal productivities, because they are not determinate. The reason is that the technological input/output coefficients are constant *à la* Leontief. Kaldorian differential saving propensities are shown to be one way to close the model, to determine factor shares, and to equate aggregate saving to technologically required investment. Other saving functions, for example the primitive uniform constant propensity to save, cannot do these jobs. One workable neoclassical alternative is to equate the interest rate to a constant rate of time preference, possibly augmented by a constant term for decline in the marginal utility of growing per capita consumption. But the specification that saving is supplied perfectly elastically with respect to the interest rate depends on infinitely long horizons for consumption and saving decisions and other implausible assumptions.

A technology consists of two activities; one produces consumption goods, the other capital goods. The two goods are not the same; the price of capital goods in terms of consumption goods is endogenous. Each activity uses labour and capital services. I shall analyse steady states in which total quantities of labour and capital are fully employed in the two activities. The labour supply is exogenous, growing in effective units at a constant rate, determined by natural increase and/or Harrod-neutral progress. The steady-state capital stock, relative to the labour force, is determined by the technology. The output of capital goods—gross investment—is what is needed to offset depreciation and to equip the increment in labour supply. Capital goods are used in both activities. In production, the capital goods used in the consumption goods activity are the same as those used in the capital goods activity itself. In use, they are different, both in the labour required to operate them and in their speed of depreciation.

Available to the economy are two or more technologies, each defined by the four input/output coefficients describing the consumption and investment activities and by the two depreciation rates. The economy may choose one among whole technologies, but it cannot mix activities. That is, for example, the consumption activity of Technology I cannot co-exist with the investment goods activity of Technology II. The nature of the investment goods produced and used might determine the differential productivities of those goods and of labour in making the two kinds of goods. I believe this assumption is in the

[1] The author says that the paper is 'an amended version of a paper published in French in *Economie Appliqué*, Oct–Nov 1957.'

spirit of some of Joan Robinson's representations of technology.[1] The all-or-nothing choice of technology gives rise to the possibility that different technologies will be chosen at high and low profit rates.

I assume that the economy will be on its factor-price frontier, along which the activities in use break even and it is not possible to increase either wages or capital rentals, both measured in consumption goods, without reducing the other factor's share. The break-even conditions for the two activities determine two prices in terms of consumption goods: the wage and the price of capital goods. They determine these prices, *given the profit rate* (or interest rate), a pure number. The equations do not determine the profit rate. That is the essential indeterminacy, traceable to the fixed-coefficient technology, the vacuum that the Kaldorian saving function may fill. It cannot be filled by a function that says saving is a uniform fraction of all income.

The two factor prices are the wage and the rental of capital, net of depreciation. Both are measured in consumption goods. The dimension of the rental is consumption goods per unit of capital goods per unit of time. These are the prices which, if multiplied by the quantities of the factor inputs, give factor incomes that add up to the value of total output. And it is those factor incomes to which the Kaldorian consumption or saving coefficients apply.

The net rental is, of course, to be distinguished from the profit rate, a pure number per unit time. Within a technology, the net rental is the same in both activities, and so of course is the profit rate. The rental is the price of capital goods times the profit rate. The price varies endogenously with the profit rate, and with the technology. Therefore, the wage/profit frontier, which amounts to the same thing as the wage/rental frontier in a model where the relative price of capital goods is always the same, is not a proper factor-price frontier in the present model. While all capital in contemporaneous use must bear in the same profit rate in a steady state with constant prices, Technology I and II activities are alternatives never in simultaneous use. They imply different price systems. Nothing compels profit rates to be the same in the two technologies, even at switch points of common wages and rentals.

A neoclassical theorist might argue that the consumption/saving decision depends on the profit rate as well as, or even instead of, aggregate profits measured in consumption goods. This dependence would represent the intertemporal substitution effect, Irving Fisher's interest incentive for saving. This is certainly not what Kaldor had in mind. I have not allowed for it in the present model.

For each technology, the steady-state input balance equations determine outputs of consumption goods and investment goods (per effective worker). These are technologically determined, independent of wages, capital goods prices, and interest rates.

[1] See Robinson, 1966, especially Chapter 10, 'The Spectrum of Techniques'. Here she introduces discrete constant-proportion techniques for labour and fixed capital. However, she does not explicitly model technologies with distinct activities for consumption goods and investment goods.

Robinson and Eatwell, 1973, pp. 183–195 sets forth a multi-sector Sraffa type input/output model and derives from it a wage/profit-rate frontier. A uniform mark-up rate is applied to the cost of every intermediate input in the pricing of all intermediate and final outputs. Evidently this corresponds to a uniform one-period lag between inputs and proximate outputs. But no such lag and mark up apply to labour inputs and wages. In any case, this model does not handle fixed capital, or even inventories other than those implicit in the work in progress during the one-period lag.

The general model of alternative 'blueprints', each involving an indivisible technology using different kinds of capital goods that produce together with labour both the capital goods themselves and consumption goods has been discussed in, for example, Pasinetti, 1969, pp. 508–531. The general model is so complex that points of interest depend greatly on simple illustrations, like the one in my text.

'Reverse switching' is quite possible. That is, a lower-consumption technology can be on the factor-price frontier at higher wages and lower profits. Of course, the other direction of switching, which seems more normal, is also possible. However, this model allows no *re*switching, because the relevant factor-price frontier for any technology is linear.

I find this 'switching' implication preferable to the usual examples, which involve curiously rigid alternative sets of time lags between labour inputs and outputs.[1] In the present example, there are no such lags (although they could be added) and the emphasis is on fixed rather than working capital. Another advantage of the present model is that the steady-state output of investment goods is determined quite naturally to meet the requirements of technology and growth.

The formal model

Here is the model: first, the equations for a technology for the outputs of consumption goods C (activity a) and investment goods I (activity b):

$a_L C + b_L I = 1$ (Labour demand = supply, normalised to 1)

$a_K C = K_C, \quad b_K I = K_I$ (Capital in each industry)

$a_K(n+d)C + b_K(n+d+s)I = I$ (Steady state gross investment) (1)

$a_K(n+d)C + [b_K(n+d) - (1 - sb_K)]I = 0$ (Investment goods demand = supply) (2)

Here n is the growth rate, and d, $d+s$ (≥ 0) are the depreciation rates in the consumption goods and investment goods activities. Equations (1) and (2) may be solved for the steady-state outputs C and I. Let A be the determinant of the input/output coefficients, $a_L b_K - a_K b_L$, and let $v = (1 - sb_K)$. Then:

$$C = [b_K(n+d) - v]/[A(n+d) - va_L], \quad I = -a_K(n+d)/[A(n+d) - va_L] \qquad (3)$$

The factor-price frontier can be found from the 'dual' of the system (1)–(2). The price of consumption goods, the numéraire, is normalised to 1. The wage rate is w. The price of investment goods is p. The profit rate is r. Thus the gross rental cost of capital services is $p(r+d)$ in the consumption goods activity and $p(r+d+s)$ in investment goods production. The break-even equations for the two activities are:

$a_L w + a_K p(r+d) = 1$ (Consumption goods activity) (4)

$b_L w + [b_K(r+d) - (1 - sb_K)]p = 0$ (Investment goods activity) (5)

These two equations are to be solved for w and p, given r. The solutions are:

$$w = [b_K(r+d) - v]/[A(r+d) - va_L], \quad p = -b_L/[A(r+d) - va_L] \qquad (6)$$

By inspection, comparing (3) and (6) gives the standard 'Golden Rule' result that $w = C$ when $r = n$.

[1] In Chapter 10 of *The Accumulation of Capital*, 'The Spectrum of Techniques', Joan Robinson discusses switching as 'A Curiosum' (Robinson, 1966, pp. 109–113) See Samuelson, 1966 for a review of reswitching possibilities in models with input/output lags.

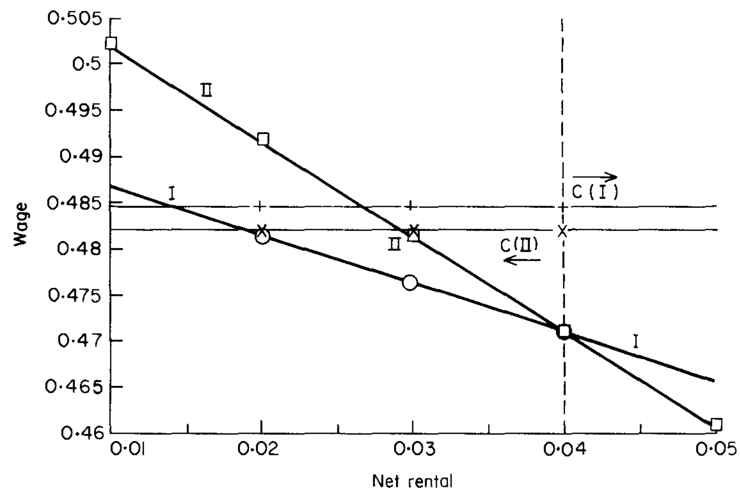

Fig. 1. *Factor prices and consumption: two technologies. Point I is the intersection of the downward sloping factor-price frontier for Technology I (○) with the constant steady-state consumption produced by Technology I. Point II is the similar intersection for Technology II (□). Both points correspond to a profit rate equal to the growth rate, assumed to be 0 03 and independent of technology. The net rental of capital at II is nonetheless higher than at I because the price of capital goods relative to consumption goods, which does depend on the technology, is higher.* + = *consumption I;* × = *consumption II.*

Table 1. *Assumed parameter values and key results in illustration (Fig. 1) (labour supply normalised to equal 1)*

	Technology I	Technology II
a_L	2	0·05
a_K	1	2
b_L	0·9	2
b_K	2	0·02
d	0·035	0·47
s	−0·025	0·1
n	0·03	0·03
C	0·485	0·482
I	0·034	0·488
K	0·50	24·64
Price when $r=n$	0·47	0·97
Rental when $r=n$	0·014	0·029
Switch point rental	0·04	0 04
Switch point wage	0·471	0·471

The factor-price frontier relates the wage w to net rental per unit of capital, $R=rp$. It can be derived algebraically from the above equations. It is linear, as follows:

$$w=(b_Kd-v)/(Ad-va_L)+va_KR/(Ad-va_L) \qquad (7)$$

When $R=np(n)$—where $p(n)$ is calculated from (6) with $r=n$—then $w=C$.

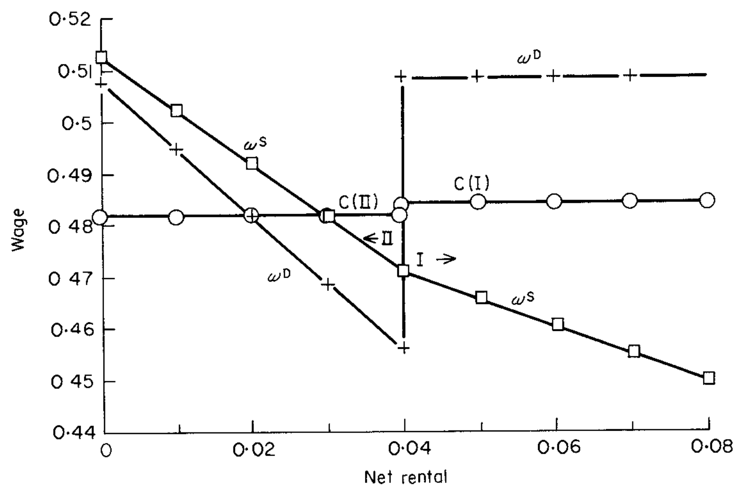

Fig. 2. *Determination of income distribution: equating consumption demand and supply. The switch to Technology I occurs, as in Fig. 1, at a net profit rate just below 0·04. Here the jump in consumption due to the switch is shown, also the kink in the wage-rental frontier ('supply wage', □) due to the switch. 'Demand wage' (+) is, for each value of net rental the wage which, given the two propensities to consume, will make consumption demand equal to the consumption available under the technology in use at that rental. Here the equilibrium is at the switch point.* ○ = *consumption.*

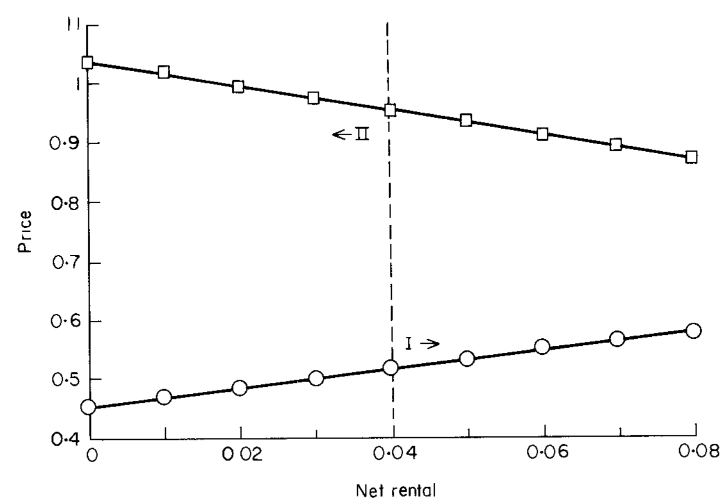

Fig. 3. *Capital goods prices and rentals: two technologies. Technology II (□), the more capital-intensive technology, implies much higher capital goods prices than Technology I (○). Technology II is used to the left of the vertical line, drawn at the switch point rental of 0 04.*

The slope of a wage–rental ($w-R$) frontier like (7) is supposed to be a measure of capital intensity. For example, in the Swan-Solow model—one product, two factors, etc.—the slope is the capital/labour ratio. Here the measure of capital intensity represented by the slope is more complex, but it is there. If d were zero, meaning no depreciation in the consumption goods activity, the slope would be the capital intensity of that activity, a_K/a_L.

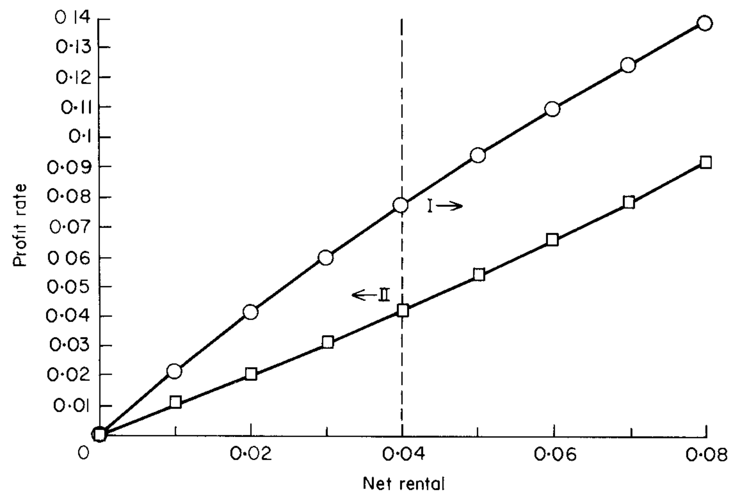

Fig. 4. *Profit rates and net rentals: two technologies. Technology II (□), the more capital intensive technology, implies lower profit rates than Technology I (○). Technology II is in use in the left half of the diagram, at rentals less than 0·04.*

The same calculations can be made for a second technology, indeed for every available technology, each one defined by values of a_L, a_K, b_L, b_K, d, and s. The growth rate n is also exogenous, but it is assumed to be independent of technology. Figure 1 shows illustrative factor-price frontiers for two technologies. Each is linear, with constant and slope dependent on six parameters. With so many degrees of freedom, even when respecting the obvious inequalities, many configurations are possible. The example has been contrived to exhibit reverse switching. Technology II has the lower steady-state consumption, but is used in preference to Technology I at lower net rentals and profit rates. Table 1 tells the numbers assumed in this illustration and reports some of the calculations.

Technology II is highly capital-intensive, in particular in the consumption goods activity, but the capital depreciates rapidly.

All that is needed now is to superimpose a Kaldor consumption function on Figure 1. If nothing is saved from wages and nothing is consumed from capital income, the profit rate is equal to the growth rate, the net rental is $np(n)$ for the technology in use at that profit rate (II in the example), and the wage is equal to the consumption afforded by that technology. In general, consumption demand C^D is equal to $c_L w + c_K RK$, where c_L and c_K are the propensities to consume from labour income and capital income respectively. In equilibrium consumption demand must equal consumption supply C^S, the consumption corresponding to the dominant technology (in the example, Technology I for the high profit rates, II for low).

Figure 2 continues the illustration of Figure 1. C^S is shown, jumping at the profit rate where the two factor-price frontiers cross in Figure 1. The downward-sloping broken line labelled 'supply wage' is the factor-price frontier, taking account of both technologies and the switch from one to the other. The other downward-sloping line, which jumps at the switch-point, is the 'demand wage', i.e. for each rental the wage that, given the propensities to consume, will make consumption demand C^D equal to supply C^S. This is given by:

$$w^D = C^S/c_L - c_K RK/c_L \quad (8)$$

In Figure 2 this is plotted for consumption propensities taken to be 0·95 and 0·05 for labour and capital incomes respectively. In equilibrium the 'demand wage' must equal the 'supply wage'. In the example depicted in Figure 2, this happens to occur only at the switch point. Whichever technology is used, the net rental will be 0·04 and the wage 0·47, on the 'supply wage' locus. Technology I gives the higher aggregate consumption.

Capital goods prices and profit rates vary with rental rates, as illustrated by Figures 3 and 4. Capital goods prices are higher and profit rates lower in Technology II, which is used at rentals less than 0·04, i.e. in the left half of Figures 2, 3, and 4.

In this exercise, I have tried to place certain insights of Nicholas Kaldor and Joan Robinson in a context where their purpose and relevance may be understood and appreciated by a wider audience. Are there morals to the story? Competitive markets are most likely to exist and to perform well when local incremental decisions are possible. However, societies frequently face all-or-nothing decisions, choices among lumpy alternatives, often difficult or impossible to reverse. The ordinary tools of neoclassical economics are much less useful for the second class of problems than for the first.

Bibliography

Abel, A., Mankiw, N., Summers, L. and Zeckhauser, R. 1986. 'Assessing Dynamic Efficiency: Theory and Evidence', National Bureau for Economic Research Working Paper No. 2097

Kaldor, N. 1956. Alternative theories of distribution, *Review of Economic Studies*, vol. XXIII, 83–100

Kaldor, N. 1960. A rejoinder to Mr Atsumi and Professor Tobin, *Review of Economic Studies*, vol. XXVII, 121–3

Kaldor, N. 1966. Marginal productivity and macroeconomic theories of distribution, *Review of Economic Studies*, vol. XXXIII, 309–19

Kaldor, N. 1985. *Economics Without Equilibrium* (The Okun Lectures), Armonk, NY, M.E. Sharpe

Pasinetti, L. L. 1962. Rate of profit and income distribution in relation to the rate of economic growth, *Review of Economic Studies*, vol. XXIX, 267–79

Pasinetti, L. L. 1969. Switches of technique and the 'rate of return' in capital theory, *Economic Journal*, vol. 79, 508–531

Robinson, J. 1960. *Collected Economic Essays*, vol II, Oxford, Blackwell

Robinson, J. 1966. *The Accumulation of Capital*, London, Macmillan

Robinson, J. and Eatwell, J. 1973. *An Introduction to Modern Economics*, London, McGraw-Hill

Samuelson, P. A. 1966. A summing up, *Quarterly Journal of Economics*, vol. 80, 568–583

Tobin, J. 1960. Towards a general Kaldorian theory of distribution: a note, *Review of Economic Studies*, vol. XXVII, 119–20, reprinted as ch. 7 in Tobin (1971)

Tobin, J. 1971. *Essays in Economics: Volume I, Macroeconomics*, Chicago, Markham Publishing Co.

Worswick, D. and Trevithick, J. (eds) 1983. *Keynes and the Modern World*, Cambridge, CUP