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On Extending the Negishi Approach to Computing Equilibria: The Case of Government Price Support Policies*

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The theoretical foundation for the use of mathematical programming models in general equilibrium is a theorem of Negishi, establishing the equivalence between the Arrow–Debreu equilibrium problem and a specific mathematical program This equivalence offers an alternative to traditional fixed point methods for computing equilibria, which is especially attractive when the number of consumers is small relative to the number of commodities In this note, we show that the above equivalence extends to economies in which a government enforces real price rigidities through market operations financed by lump-sum taxes *Journal of Economic Literature* Classification Numbers 021, 213. © 1988 Academic Press, Inc

1. INTRODUCTION

The traditional approach for computing an Arrow–Debreu equilibrium (and also for proving its existence) is a fixed point calculation in the space of commodity prices The data for this calculation are demand and supply functions (or correspondences) expressed in terms of commodity prices A fixed point algorithm then determines a price vector at which supply exceeds or equals demand for each commodity (Scarf [13]).

A different approach to computing an equilibrium was suggested by Negishi [11]. The calculation of supply and demand functions is replaced

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with the solution of a mathematical program. This mathematical program maximizes a social welfare function, consisting of a weighted sum of consumer utility functions, subject to the material balance constraints. Negishi's theorem states that for given initial endowments, there is a choice of welfare weights such that the welfare optimizing solution is an Arrow–Debreu equilibrium. The computation thus involves a search for "equilibrium" welfare weights The Negishi theorem is proved by formulating this search as a fixed point problem in the space of welfare weights, and no longer, as in the traditional approach, in the space of commodity prices. Equilibrium welfare weights can be computed using "tâtonnement" procedure (Dixon [4], Ginsburgh and Waelbroeck [8]) See also Dantzig, Eaves and Gale [2]

The computational requirements of fixed point algorithms are quite substantial and severly limit the size of problems which can be solved numerically. When the number of commodities is large relative to the number of consumers a traditional fixed point calculation in commodity price space, even if feasible, may be inefficient. In such cases, the existence of efficient mathematical programming techniques favors a Negishi type calculation, the cumbersome fixed point search now being carried out in the relatively smaller space of consumer welfare weights. Moreover, in the special, but quite frequent case of an economy with a single consumer group, the equilibrium calculation reduces to solving a single mathematical program, whatever the number of commodities

We extend the Negishi approach to computing equilibria to a case of upper and lower bounds on prices. The special case of price rigidities considered is that of constraints enforced by a government (or agency) through market operations. Minimum real prices are maintained through government purchases at these prices. Maximum real prices are maintained through commodity sales from government stocks, as long as these stocks are not depleted. Once these stocks are depleted, the government is unable to constrain prices to stay below the declared maxima. The government budget is balanced purchases or sales are financed or redistributed through lump sum taxes or transfers to consumers. Issues related to the inventory of commodities purchased by the government are not considered in our model In particular, stocks acquired by the government in its attempt to support prices have no further economic value. This deficiency of the model in the case of storable commodities does not arise, however, in the case of downward price rigidities on labor, or on nonstorable factors of production like capital services.

Equilibrium prices in the Negishi approach arise as the dual prices of the material balance constraints in the welfare maximizing mathematical program The difficulty presented by price rigidities in this calculation is that the imputed nature of "shadow" prices does not allow them to verifany conditions other than the classical Karush–Kuhn–Tucker optimality conditions. The question posed by price rigidities in the Negishi approach is how to modify the social welfare function so that the imputed shadow prices verify the imposed constraints.

In this note we show that the addition of a single term to the social welfare function, representing government purchases and sales, suffices for the imputed prices to verify the price constraints. The government becomes an additional agent whose net expentidures are matched by lump-sum transfers from consumers and whose utility function is determined by the price constraints. Agents value purchases of government stocks at the lower bounds on prices, while they value sales of its stocks at the upper bounds. The resulting piecewise linear utility function is added to the social welfare function, multiplied with an appropriate weight. The number of welfare weights to be determined is thus increased by one, and is independent of the number of commodities submitted to price rigidities. The desirability of this approach over a fixed point calculation in commodity price space is thus maintained for the case where the number of consumers is small relative to the number of commodities.

Imam and Whalley [9] discuss a model, similar to ours, in which a government intervenes when prices fall below certain "trigger" prices. Government intervention is described in terms of functions giving the size of its purchases as a function of the wedge between "trigger" and market prices. In the Imam–Whalley model, prices are allowed to fall below their bounds so that the government is not fully effective in maintaining minimum prices. Another difference with this note is that Iman and Whalley suggest a traditional fixed point computation in commodity price space

The note is organized as follows. Section 2 formally introduces an equilibrium with government price support policies Section 3 contains our main propositions; the first proposition presents conditions under which the solution of a welfare optimizing program yields an equilibrium with price rigidities; the second proposition shows that such an equilibrium exists.

2. The Model

We consider an economy with n commodities $[j = 1, .., n]$ and m consumers $[i = 1, ..., m]$. The endowment of consumer i is denoted $w^i \in R^n_+$, his consumption $x^i \in R^n_+$, and his preferences are represented by a continuous, concave, and strictly increasing function, $U^i(x^i)$ Government endowment, purchases, and sales are denoted w^g, x^g, and z^g, all these vectors being in R^n_+ Total endowment is represented by $w = w^c + w^g$ with $w^c = \sum_{i=1}^m w^i$

The productive side of the economy is, for simplicity, described by an activity analysis matrix $A \in R^{n \times s}$: if $y \in R^s_+$ denotes the vector of intensities at which the activities are operated then $Ay \in R^n$ denotes net output.

A price system is a vector $p \in R^n_+$. We consider a price index $I(p)$ which is a continuous and homogeneous function of degree one in p. The real price of commodity j is subject to the real price constraints $\underline{p}_j \leqslant p_j/I(p) \leqslant \bar{p}_j$, $j = 1, ..., n$, where the upper bound is valid only as long as government stocks are not exhausted $[z^g_j < w^g_j]$. In the absence of price rigidity on commodity j we set $\underline{p}_j = 0$ and $\bar{p}_j = M$ with M sufficiently large.

The net cost of government market operations, $h = px^g - pz^g$, is covered by lump-sum taxes, any government surplus being returned to consumers as a lump-sum subsidy. These taxes and subsidies are described by continuous functions t^i: $R \to R$, $h \to t^i(h)$, $i = 1, ..., m$, verifying $\sum_{i=1}^m t^i(h) = h$.

We now define a market quasi-equilibrium with government price support policies.

DEFINITION 1. The consumption vectors $\hat{x}^1, ..., \hat{x}^m$, the vectors of government purchases $\hat{x}^g$, and government sales $\hat{z}^g$, and the activity vector $\hat{y}$, supported by the price vectors p and q, are a market quasi-equilibrium with government price support policies if

$$\sum_{i=1}^m \hat{x}^i + \hat{x}^g - \hat{z}^g - A\hat{y} \leqslant w^c,$$
$$p\left[\sum_{i=1}^m \hat{x}^i + \hat{x}^g - \hat{z}^g - A\hat{y} - w^c\right] = 0, \tag{1a}$$

$$\hat{z}^g \leqslant w^g, \qquad q[\hat{z}^g - w^g] = 0, \tag{1b}$$

$$p/I(p) \geqslant \underline{p}, \qquad [p/I(p) - \underline{p}]\,\hat{x}^g = 0, \tag{2a}$$

$$(p-q)/I(p) \leqslant \bar{p}, \qquad [(p-q)/I(p) - \bar{p}]\,\hat{z}^g = 0; \tag{2b}$$

either $\hat{x}^i$ maximizes $U^i(x^i)$ subject to
$$px^i \leqslant pw^i - t^i(p\hat{x}^g - p\hat{z}^g),\ x^i \geqslant 0 \tag{3a}$$

or

$$p\hat{x}^i = pw^i - t^i(p\hat{x}^g - p\hat{z}^g) = 0, \tag{3b}$$

$$pA \leqslant 0, \quad pA\hat{y} = 0, \tag{4}$$

$$p\hat{x}^g = p\hat{z}^g + \sum_{i=1}^m t^i(p\hat{x}^g - p\hat{z}^g). \tag{5}$$

Condition (1a) is the usual market clearing condition, except that the government may intervene by buying $\hat{x}^g$ or selling $\hat{z}^g$ to support prices; condition (1b) states that government stocks are limited, and that if stocks are depleted, the upper bound $\bar{p}$ on prices may be exceeded (see condition (2b)) Condition (2) requires the government to intervene only in support of maximum or minimum prices. Condition (3) requires each consumer to be either in equilibrium given his utility function and his budget constraint, or to be left without income after taxes; at a quasi-equilibrium a consumer without income is thus not necessarily maximizing utility Condition (4) requires the productive sector to be in equilibrium: activities cannot make positive profits, and activities in use make zero profits. Condition (5) requires the government to operate with a balanced budget. It states that government purchases, $p\hat{x}^g$, are equal to government sales, $p\hat{z}^g$, plus net transfers, $\sum t^i(p\hat{x}^g - p\hat{z}^g)$ This constraint will always be satisfied by definition of the tax functions t^i

3 Main Propositions

In this section, we prove that a market quasi-equilibrium with government price support policies can be generated as a solution of the following mathematical program

$$\mathrm{MP}(\alpha) \qquad \max \sum_{i=1}^{m} \alpha^i U^i(x^i) + \alpha^{m+1}(\underline{p}x^g - \bar{p}z^g)$$

subject to

$$\sum_{i=1}^{m} x^i + x^g - z^g - Ay \leqslant w^c$$
$$z^g \leqslant w^g$$
$$x^i,\ x^g,\ z^g,\ y \geqslant 0$$

where $\alpha = [\alpha^1, .., \alpha^{m+1}]$ is a vector of welfare weights belonging to $S^{m+1} = \{\alpha \in R_+^{m+1} \mid \sum_{i=1}^{m+1} \alpha^i = 1\}$

The objective function of $\mathrm{MP}(\alpha)$ is an augmented social welfare function; the augmenting term, representing government operations, is multiplied by an appropriate weight, α^{m+1}, which will, at equilibrium, be equal to the price index, $I(p)$ If p and q are the multipliers associated, at optimality, with the constraints it is easy to see, by duality, that at equilibrium they verify the price constraints In the absence of price rigidities ($\underline{p}_j = 0$ and $\bar{p}_j = M$, $j = 1, ..., m$), both x^g and z^g can be set equal to zero, and $\mathrm{MP}(\alpha)$ reduces to the standard Negishi welfare optimizing program

Note also that government price support policies introduce only one additional equilibrium weight, α^{m+1}, in the Negishi calculation This weight can be considered associated with a fictitious agent, the government, who maximizes the utility function $px^g - \bar{p}z^g$ subject to the budget constraint $px^g - pz^g \leqslant I^g$ and the nonnegativity constraints x^g, $z^g \geqslant 0$. At equilibrium, the income of this agent is equal to the total lump-sum transfer from the other agents, $I^g = \sum_{i=1}^m t^i(p\hat{x}^g - p\hat{z}^g)$ The presence of these transfers did not enable us to derive the existence of a qu... equilibrium with government price support policies as a straightforw... application of Negishi's theorem to an economy with an $(m+1)$st agent representing the government.

Our first proposition states that market equilibria with government p.... support policies can be generated as solutions of MP(α).

For each $\alpha \in S^{m+1}$, MP(α) is a convex programming problem It will be helpful to introduce notation to describe optimality in MP(α). We use 2^W to denote the set of subsets of a given set W. Let X $S^{m+1} \to 2^{R_+^{(m+2)n+s}}$, $\alpha \to X(\alpha) = \{x = [x^1, ..., x^m, x^g, z^g, y] \in R_+^{(m+2)n+s} \mid x$ is an optimal solution for MP(α)$\}$. Let D: $S^{m+1} \to 2^{R_+^{2n}}$, $\alpha \to D(\alpha) = \{d = [p, q] \in R_+^{2n} \mid d$ is a vector of shadow prices for MP(α)$\}$ and let P: $S^{m+1} \to 2^{R_+^n}$, $\alpha \to P(\alpha) = \{p \in R_+^n \mid [p, q] \in D(\alpha)$ for some $q \in R_+^n\}$.

Proposition 1 *Let $[x^1, .., x^m, x^g, z^g, y] \in X(\alpha)$ for some $\alpha \in S^{m+1}$ If there exist $[p, q] \in D(\alpha)$ such that $px^i \leqslant pw^i - t^i(px^g - pz^g)$ for $i = 1, ..., m$ and such that $\alpha^{m+1} = I(p) > 0$ then $[x^1, .., x^m; x^g, z^g; y; p, q]$ is a market quasi-equilibrium with government price support policies*

Proof Let $\partial U^i(x^i)$ denote the subdifferential of $U^i(x)$ at $x = x^i$ (see, e.g., Rockafellar [12]). The optimality conditions for MP(α) are

(i) $p \geqslant \alpha^i u^i$ with $u^i \in \partial U^i(x^i)$, $[p - \alpha^i u^i] x^i = 0$, $i = 1, \ldots, m$;

(ii) $p \geqslant \alpha^{m+1} \underline{p}$, $[p - \alpha^{m+1} \underline{p}]\, x^g = 0$,

(iii) $q - p \geqslant -\alpha^{m+1}\bar{p}$, $[q - p + \alpha^{m+1}\bar{p}]\, z^g = 0$,

(iv) $-pA \geqslant 0$, $pAy = 0$;

(v) $\sum_{i=1}^m x^i + x^g - z^g - Ay \leqslant w^c$, $p[\sum_{i=1}^m x^i + x^g - z^g - Ay - w^c] = 0$, $z^g \leqslant w^g$, $q[z^g - w^g] = 0$.

Using (iv) and the balanced government budget condition, the complementarity condition in (v) can be rewritten $0 = \sum_{i=1}^m px^i + px^g - pz^g - pw^c = \sum_{i=1}^m px^i + \sum_{i=1}^m t^i(px^g - pz^g) - pw^c$. The assumption that $px^i + t^i(px^g - pz^g) - pw^i \leqslant 0$ for $i = 1, ..., m$ then implies that all these inequalities are equalities This and condition (i) then verify statement (3) in definition 1.

Condition (iv) verifies statement (4) in Definition 1 The assumption $\alpha^{m+1} = I(p) > 0$ and conditions (ii) and (iii) verify statement (2) Finally condition (v) verifies statement (1) Statement (5) is satisfied by definition of the tax functions t^i ∎

Our second proposition states sufficient conditions for the existence of a market quasi-equilibrium with government price support policies These conditions are now listed and discussed.

ASSUMPTION 1 There exists an activity vector $y \in R^s_+$ such that $Ay + w > 0$

ASSUMPTION 2 The set $\{(x, y) \mid x - Ay \leqslant w, x \geqslant 0, y \geqslant 0\}$ is bounded.

ASSUMPTION 3 There exists a price p verifying $pA \leqslant 0$, $I(p) = 1$ and $p \geqslant \underline{p}$

ASSUMPTION 4 Let h denote the net expense of government market operations. If $h \leqslant pw^c$ with $p \in R^n_+$ then $t^i(h) \leqslant pw^i$ for $i = 1, \ldots, m$

ASSUMPTION 5. $I(p) > 0$ for all $p \in P(\alpha)$ with $\alpha^{m+1} = 0$.

ASSUMPTION 6. $I(p) < 1$ for all $p \in P(\alpha)$ with $\alpha = [0, \ldots, 0, 1]$.

The assumptions are minor or standard. Assumption 1 is the Slater condition for mathematical program MP(α). It states that it is feasible to bring to the market positive quantities of all goods The Slater condition implies certain regularity properties for the shadow prices of MP(α). Assumption 2 is standard and bounds the production set. Assumption 3 requires that the minimum prices be consistent with production equilibrium. It clearly needs to be verified for a market quasi-equilibrium with government price support policies to exist. Assumption 4 is a statement about the distribution of the tax burden. It states that the government, as long as its net expenses do not exceed total consumer income, will distribute the tax burden so as to force no consumer into bankruptcy, i.e., $pw^i - t^i(h) \geqslant 0$ for $i = 1, \ldots, m$ Assumption 5 requires the price index to be positive at any social welfare optimum It will clearly be verified in all applications Assumption 6 is discussed in the next paragraph.

The prices in MP(α^*) with $\alpha^* = [0, \ldots, 0, 1]$ are part of an optimal solution for the dual of MP(α^*), which can be written

$$\mathrm{DP}(\alpha^*) \min pw^c + qw^g$$

subject to

$$-pA \geqslant 0$$

$$p \geqslant \underline{p}$$

$$-p + q \geqslant -\bar{p}$$

$$p, q \geqslant 0.$$

Assumption 3 ensures the existence of a feasible solution $[p, q]$ for $\mathrm{DP}(\alpha^*)$ verifying $I(p)=1$ The nonnegativity of the vector $[w^c, w^g]$ then makes the requirement that $I(p)<1$ for all optimal solutions of $\mathrm{DP}(\alpha^*)$ minor

We now state our second proposition which we prove by extending Negishi's proof to the case of market quasi-equilibria with government price support policies

PROPOSITION 2 *Assumptions* 1–6 *ensure the existence of a market quasi-equilibrium with government price support policies.*

Proof. We set the stage for an application of the Katutani fixed point theorem and first discuss some properties of the mappings X and D

Let $\Phi=\{[x^1, ..., x^m, x^g, z^g, y]\in R_+^{(m+2)n+s} \mid \sum_{i=1}^m x^i + x^g - z^g - Ay \leqslant w^c, z^g \leqslant w^g\}$ Assumption 2 implies that the set Φ is bounded. Φ is also convex and closed. Assumption 2 and the continuity of the objective and constraint functions of $\mathrm{MP}(\alpha)$ make the mapping X upper semicontinuous and compact valued $\mathrm{MP}(\alpha)$ being a convex programming problem, X is also convex valued.

We now turn to the mapping D. Assumption 1 is the Slater condition for $\mathrm{MP}(\alpha)$ and implies that the mapping D is upper semicontinuous and compact valued (see, e.g., Cornet [1]). The upper semicontinuity of D ensures that the image set $D(S^{m+1})$ is compact as well. The convexity of $\mathrm{MP}(\alpha)$ yields that D is convex valued Let V be any convex compact set in R_+^{2n} containing the image $D(S^{m+1})$ and verifying $pA \leqslant 0$ for all $[p, q]\in V$

Consider $\alpha^*=[0, ., 0, 1]\in S^{m+1}$ $D(\alpha^*)$ is compact. By Assumption 6 and by continuity of the index function I it follows that there exists $\delta>0$ such that $I(p)<1-\delta$ for all $p\in P(\alpha^*)$ By the upper semicontinuity of D we can then find $\varepsilon>0$ such that $I(p)\leqslant I-\varepsilon$ for all $p\in P(\alpha)$ with $\alpha\in \{\alpha\in S^{m+1} \mid \alpha^{m+1}\geqslant 1-\varepsilon\}$

We now define a multivalued mapping F whose fixed point yields a market quasi-equilibrium with government price support policies Let $T=\{\alpha\in S^{m+1} \mid \alpha^{m+1}\leqslant 1-\varepsilon\}$ Let F: $T\times\Phi\times V\to T\times 2^{\Phi}\times 2^{V}$, $[\alpha, x, d]\to F(\alpha, x, d)=[A(\alpha, x, d), X(\alpha), D(\alpha)]$ The mapping A: $T\times\Phi\times V\to T$, $[\alpha, x, d]\to A(\alpha, x, d)=[A^1(\alpha, x, d), \;, A^{m+1}(\alpha, x, d)]$ is defined as follows

$$A^{m+1}(\alpha, x, d)=\min[I(p), 1-\varepsilon],$$

$$A^i(\alpha, x, d)=[\alpha^i + pw^i - t^i(px^g - pz^g) - px^i]^+ \, [1-A^{m+1}(\alpha, x, d)]/k \qquad \text{for} \quad i=1, ..., m$$

with $x=[x^1, .., x^m, x^g, z^g, y]$, $d=[p, q]$, and $k=\sum_{i=1}^m [\alpha^i + pw^i - t^i(px^g - pz^g) - px^i]^+$

The mapping A is well defined as we now show that $k\geqslant 1-\alpha^{m+1}\geqslant\varepsilon>0$

for all $[\alpha, x, d] \in T \times \Phi \times V$. The denominator k verifies $k \geqslant \sum_{i=1}^{m} [\alpha^i + pw^i - t^i(px^g - pz^g) - px^i] = 1 - \alpha^{m+1} + pw^c - px^g + pz^g - \sum_{i=1}^{m} px^i$. The fact that $x \in \Phi$ and $p \in V$ then yields $k \geqslant 1 - \alpha^{m+1} - pAy \geqslant 1 - \alpha^{m+1}$. The desired inequality, $k \geqslant \varepsilon$, now follows from $\alpha \in T$. The continuity of the tax and index functions imply the continuity of the funcion A.

The mapping F meets all the assumptions of the Kakutani fixed point theorem. This theorem asserts the existence of a fixed point $[\alpha, x, d]$ verifying $\alpha = A(\alpha, x, d)$, $x \in X(\alpha)$, $d \in D(\alpha)$. In view of Proposition 1 it now suffices, in order to establish the existence of a market quasi-equilibrium with government price support policies, to show that $\alpha^{m+1} = I(p) > 0$ and that $px^i \leqslant pw^i - t^i(px^g - pz^g)$ for $i = 1, ..., m$.

The fixed point $[\alpha, x, d]$ verifies $\alpha^{m+1} = A^{m+1}(\alpha, x, d) = \min[I(p), 1 - \varepsilon]$. Since $I(p) \leqslant 1 - \varepsilon$ for $\alpha^{m+1} = 1 - \varepsilon$, we easily conclude that $\alpha^{m+1} = I(p)$. Assumption 5 ensures that $I(p) > 0$.

For $i = 1, ..., m$ we have $\alpha^i = A^i(\alpha, x, d) = [\alpha^i + pw^i - t^i(px^g - pz^g) - px^i]^+ [1 - \alpha^{m+1}]/k$. If $\alpha^i = 0$ then $x \in X(\alpha)$ implies $px^i = 0$ and $pw^i - t^i(px^g - pz^g) \leqslant 0$. Assumption 4 then yields $pw^i - t^i(px^g - pz^g) = px^i = 0$. If $\alpha^i > 0$ then $[(k/(1 - \alpha^{m+1}) - 1]\alpha^i = pw^i - t^i(px^g - pz^g) - px^i \geqslant 0$, since we established earlier that $k \geqslant 1 - \alpha^{m+1}$. This completes our proof of Proposition 2. ∎

The above proof suggests calculating a market quasi-equilibrium by computing a fixed point of the mapping F in a space of dimension $(3 + n)m + 2n + s$. However, a more careful examination of the proof shows that a market quasi-equilibrium can be computed as a fixed point of the mapping H: $T \to T$, $\alpha \to H(\alpha) = \{h \mid h = A(\alpha, x, d), \ \alpha \in T, \ x \in X(\alpha), d \in D(\alpha)\}$. The latter calculation can be carried out in the space of welfare weights, which is of dimension m.

The existence of a fixed point for the mapping H cannot be asserted by applying the Kakutani fixed point theorem to H as this mapping is not convex valued. Convexity properties are obtained by extending the dimension and introducing the mapping F. A direct proof of the existence of a fixed point of the mapping H can be obtained by adapting Diewert's [3] proof of the Negishi theorem to the case considered in this paper. Diewert's proof uses the Eilenberg–Montgomery fixed point theorem which does not require the mapping to be convex valued. The more technical nature of such a proof prompted us to follow the indirect route of proving the existence of a fixed point of the mapping H by applying the Kakutani theorem to the mapping F.

4. Final Remarks

Positive prices for goods in excess supply are frequently observed for labor, and production capacities. Unemployment is a common phenomenon, so are idle capacities in industry. In both cases "government purchases" can be interpreted as supply rationing in the sense of Drèze [5], see Ginsburgh and Van der Heyden [7]. However, in the case of labor rationing, the interpretation requires that utility functions be separable in leisure For an application see Erlich, Ginsburgh and Van der Heyden [6].

In an economy with a single consumer (see also [10]) having utility function $U(x)$, the mathematical program $\mathrm{MP}(\alpha)$ reduces to

$$\max(1-\alpha)\, U(x) + \alpha(\underline{p}x^g - \bar{p}z^g)$$

subject to

$$x + x^g - z^g - Ay \leqslant w^c \qquad (p)$$

$$z^g \leqslant w^g \qquad (q)$$

$$x, x^g, z^g, y \geqslant 0.$$

The computation of a market quasi-equilibrium with government price support policies reduces to a simple one-dimensional search for a value of α verifying $\alpha = I(p)$, $p \in P(\alpha)$ This approach seems particularly attractive for problems in which income distribution is not an issue

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