Product warranties and double moral hazard

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This article explores a model of warranties in which moral hazard problems play a key role. The goal is to understand the important characteristics of warranties, including their provision of incomplete insurance and the relationship between product quality and coverage. We analyze a model in which buyers and sellers take actions that affect a product’s performance. Since these actions are not cooperatively determined, an incentives problem arises. We characterize the optimal warranty contract and undertake comparative statics to determine the predicted correlation of warranty coverage and product quality.

1. Introduction

- Warranties are prevalent in commodity markets.¹ Most durables, in particular, have some type of warranty promising payment from the producer, conditional on performance of the product. Three principal characteristics of these warranties interest us here:

(i) They provide less than full insurance against unsatisfactory performance.
(ii) They are supplied by the seller of the product rather than by independent insurance agencies.
(iii) The extent of warranty protection appears to bear no general relation to the overall performance of a product. That is, the sellers of more reliable brands of a particular product may offer more, equal, or even less warranty protection than sellers of less reliable brands.

Existing warranty theories have difficulty explaining these observations. If warranties are simply insurance policies sold by the seller to the buyer, Heal’s (1977) results suggest

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¹ Priest (1981, p. 1297) goes so far as to say that warranties may, in fact, be among the most common of written contracts.
that with risk-neutral sellers the insurance should be complete. This insurance theory also
does not explain why insurance companies are not providing the insurance, nor does it
speak to point (iii). Spence (1977) has argued for the signalling role of warranties. High
quality producers can afford to offer close to full insurance since their products are
unlikely to break. The signalling theory, however, also fails to explain (iii). In fact, it
allows only for a positive correlation between the extent of warranty protection and a
brand's reliability. While such positive correlations may indeed exist in some markets, in
many others there is little or even a negative relationship between the variables. 2 Signalling
theories also require the strong assumption that product qualities are exogenous.

In this article we explore a model of warranties in which moral hazard problems
play the central role. The model is consistent with all three of the phenomena listed
above. We are, of course, not the first to recognize the importance of moral hazard in
shaping warranty contracts. In fact, it can be argued that moral hazard problems lie at
the core of Priest's (1981) "Investment Theory" of warranties. Our approach here is both
more direct and more formal, however. 3

There are two types of incentive problems which we see as relevant. First, buyers
generally take actions which influence the performance of the product. To the extent that
these actions are not monitored or detectable by the seller, a moral hazard problem will
arise if warranties are present. Second, the actual qualities of many commodities are not
directly observable to buyers. In such a situation, the incentives to a seller to maintain
high quality are also embedded in the warranty. Hence, warranties may act as incentive
mechanisms for both sides of the product market. Here we focus on the resulting problem
of double moral hazard.

Recent work by Kamhbu (1982) and Mann and Wissink (1983) has analyzed models
which are structurally similar to ours. In contrast to their efforts, we focus on characterizing
levels of quality and consumer care in the optimal agreement between firms and customers
in the presence of double moral hazard. In Kamhbu's model there is no random variable
that affects the performance of the product: observed product quality is described as
being a deterministic function of buyer care and seller quality. Since performance is then
not state dependent, there is nothing quite like a warranty payment in his model. Kamhbu
gives little attention to the properties of the second-best problem, and derives his most
interesting results from an analysis of the role of informed third parties in restoring the
first-best outcome. Perhaps because introducing imperfectly informed third parties would
only compound the moral hazard problems, most warranty contracts involve only the
buyer and seller. For this reason, we have chosen to focus exclusively on such bilateral
agreements.

Mann and Wissink (1983) also consider bilateral arrangements to overcome the
moral hazard problem. They study a particular contracting mechanism (money-back
guarantees), which they find particularly appealing. In this essay we search over a broader
set of contracting possibilities. Mann and Wissink also give less attention to the properties
of the second-best solution. Their main interest is in the conditions under which firms
will choose to integrate to avoid the problems created by the presence of double moral
hazard.

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2 Garvin (1983) investigates the performance of air conditioners in the United States and Japan. He
comments on the uniformity of warranty contracts within each of these countries and notes the variance of
performance both within and between countries. Garvin finds that Japanese products perform better, but offer
shorter warranties. Priest (1981) also concludes that the signalling explanation for warranties is empirically
unfounded on the basis of his study of 62 consumer product warranties. His analysis can, at best, be described
as suggestive on this point, since there are no formal tests of the theory.

3 The referee has correctly pointed out to us that the recognition that moral hazard problems affect the
design of warranty contracts goes back at least as far as Oi's (1973) article on the economics of product safety.
For a related discussion of double moral hazard in the context of workplace safety, see Rea (1981).
Our results indicate, as one would expect, that owing to these incentive problems, warranties will offer only partial insurance. We are able to determine the effects of the imperfect information and resulting double moral hazard on the levels of care exercised by buyers and of quality built in by sellers. These distortions are shown to depend critically on whether care and quality are complements or substitutes in determining the probability that a product will work. We also study some comparative statics with respect to the costs of care and quality to the customers and firms, respectively. These results suggest that such heterogeneity may indeed explain why the observed correlation between quality and warranty protection is sometimes positive and sometimes negative.

2. Overview of the model

We consider a contract between a buyer and a seller. The parties to the contract may be viewed as two firms or as one consumer and one firm. In what follows we use the latter labels. To concentrate our attention on the incentive problems here, we remove any concerns about risk sharing by assuming that both the buyer and the seller are risk neutral.

The contract stipulates the price \( p \) to be paid by the buyer for a single unit of the commodity that will be traded. This good may or may not work after its purchase. The probability that it works is represented by \( \Pi \), thus the probability of breakdown is \((1 - \Pi)\). \( \Pi \) is a function of two variables: \( q \), the quality level chosen by the seller, and \( e \) the level of care or effort expended by the buyer. Letting subscripts denote partial derivatives, we assume that \( \Pi_q > 0, \Pi_e > 0, \Pi_{ee} < 0, \) and \( \Pi_{qq} < 0 \). This means that the inputs are productive though at a decreasing rate. The sign of \( \Pi_{ee} \) is left unspecified at this point, and it will prove crucial in the analysis that follows.\(^4\)

Each buyer's utility is represented by

\[
U(e, q, p, s) = y - p + \Pi z + (1 - \Pi)sz - g(e)
\]

with \( g'(\cdot) > 0, \ g'(0) = 0, \) and \( g''(\cdot) > 0 \). Each consumer's initial income is \( y \), and \( p \) is spent on the commodity studied here. If the product works (with probability \( \Pi \)), it is worth \( z \) dollars to the consumer. If the product fails, the warranty provides that the buyer receive \( sz \) as compensation from the seller. Thus, \( s \) is a measure of the degree of warranty protection in the contract. Product failure is always assumed to be a publicly observable phenomenon. The function \( g(e) \) measures the consumer's disutility of effort.

A firm's expected profits will be

\[
V(e, q, p, s) = p - C(q) - (1 - \Pi)sz,
\]

where \( C(q) \) is an increasing, convex cost of quality function, \( C'(\cdot) > 0, C'(0) = 0, \) and \( C''(\cdot) > 0 \). Hence, the firm's total cost is the sum of the cost of production, \( C(q) \), and the expected warranty payment.

The choices of \( e \) and \( q \) will affect the parties directly through the cost functions, \( g(e) \) and \( C(q) \), and indirectly through the probability function \( \Pi(e, q) \).

Before addressing the double moral hazard itself, we begin by stating the full-information, cooperative solution. Here, all elements of the contract \((p, s, e, q)\) are set cooperatively, and the contract is fully enforceable. This first-best contract will maximize \( U + V \) or total surplus. The elements \( p \) and \( s \) drop out of this sum and are therefore indeterminate. Then, the cooperative solution \((e^*, q^*)\) satisfies

\[
\Pi(e, q)z = g(e) \quad (1)
\]

\[
\Pi(e, q)z = C(q). \quad (2)
\]

\(^4\) Kambhu (1982) considers the case of \( \Pi_{ee} < 0 \).
We denote the solution to (1), for a given $q$, as $e^*(q)$, and the solution to (2), for a given $e$, as $q^*(e)$. The cooperative solution, also called first-best, is simply the combination of $(e, q)$ satisfying (1) and (2). Given the cooperative nature of the agreement, $e^*$ and $q^*$ are set so that marginal benefits (to the parties jointly) equal the marginal costs to each.

In general, there may be multiple or no solutions to these equations. We shall be more specific about the properties of the cooperative solution in the next two sections.

3. Warranties with unobservable effort and qualities: double moral hazard

The cooperative agreement, characterized by (1) and (2), requires that the input levels of the two parties be set at $(e^*, q^*)$ regardless of any incentives that may exist to alter these input levels. Hence, there must be an explicit enforcement mechanism for the implementation of the cooperative agreement. When these inputs are not costlessly observable to the parties to the contract or to the courts, an enforcement problem arises. In such a situation the agreement must be self-enforcing so that neither party has an incentive to deviate from the agreed-upon actions.

We suppose that the price of the product and the warranty level can be cooperatively set in an enforceable manner. In contrast to the previous section, we do not allow for consumer effort or producer quality levels to be determined cooperatively. Hence, the contract, through the choice of $p$ and $s$, must provide incentives for the parties to take appropriate actions.

To model this double moral hazard problem, we consider the two-stage game played by the parties to the contract. In the first (cooperative) stage, the parties sign a binding agreement with respect to $p$ and $s$. The second (noncooperative) stage takes $(p, s)$ as given, and the players choose their inputs, $e$ and $q$. We focus on the Nash equilibrium of this noncooperative game.

Payoffs are made after the condition of the product is determined. We begin by analyzing the second stage of the game for arbitrary $(p, s)$. In fact, given the linearity of the problem, the second stage equilibrium is independent of $p$, though it will depend crucially on $s$.

When $q$ and $e$ are chosen noncooperatively, we can analyze the problem by looking for reaction function equilibria. For given $s$ buyers choose effort to maximize $U$ with respect to $e$, given their conjecture about the level of $q$. The solution to this problem, $\hat{e}(q; s)$, will be independent of $p$ and will satisfy the first-order condition:

$$\Pi_e(e, q)(1 - s)z = g'(e).$$

Similarly, the firm chooses $q$, given $s$ and a conjecture about $e$, to maximize expected profits. The solution, $\hat{q}(e, s)$, satisfies

$$\Pi_q(e, q)sz = C'(q).$$

We assume that $\hat{e}(0; s)$ and $\hat{q}(0; s)$ are both positive for $0 < s < 1$. That is, if one party is providing no input at all, the best response of the other is to provide a strictly positive amount of input. Notice that the sign of the slopes of these reaction functions will depend on the the sign of $\Pi_q$. That is, the reactions functions will have positive slopes when $\Pi_q > 0$ and negative slopes when $\Pi_q < 0$. From (3) and (4) one can easily compute the partial effects of changes in $s$ on $e$ and $q$ (i.e., $\hat{e}$ and $\hat{q}$):

$$\hat{e} = \Pi_e z / [\Pi_e (1 - s)z - g'] < 0$$

$$\hat{q} = -\Pi_q sz / [\Pi_q sz - C'] > 0.$$
We can use (3) and (4) to compare the solutions to the full-information and the noncooperative equilibria. For given $s$ our model reproduces the results of Kambh (1982) and Mann and Wissink (1983) regarding the reaction curves of the two parties.\(^5\)

**Proposition 1.** For $0 < s < 1$, $\hat{q}(e, s) < q^*(e)$ and $\hat{q}(q, s) < e^*(q)$.

**Proof.** Direct from comparing (1) and (2) with (3) and (4).

Simply stated, with $0 < s < 1$, neither party receives the full benefit of exerting more effort (for the buyer) or increasing quality (for the seller). Hence, both parties have an incentive to shirk and to reduce their inputs into the II function.

To characterize the distortions in quality and effort in the second-best equilibrium, we relate the reaction functions in the first-best and second-best problems, i.e.,

$$e^*(q) = \hat{e}(q; 0)$$

and

$$q^*(e) = \hat{q}(e, 1).$$

Since $s$ cannot simultaneously equal 0 and 1, it is obvious that the full-information solution is not implementable as a noncooperative equilibrium.

In determining the distortions due to the double moral hazard, we differentiate among the cases of $\Pi_{eq} > 0$, $\Pi_{eq} = 0$, and $\Pi_{eq} < 0$. We begin with $\Pi_{eq} > 0$ so that quality and effort are complements. For the present discussion, we shall assume that both the first-best and second-best equilibria exist, and that they are unique and stable.\(^6\) We briefly consider the consequences of multiple equilibria later.

As shown in Figure 1, (5) and (6) allow us easily to compare the two solutions when $\Pi_{eq} > 0$. Point $K$ is assumed to be a unique solution to the full-information problem. Since $\hat{e}(q; s) < 0$, it is clear that the noncooperative reaction function for the choice of effort lies below the cooperative function $e^*(q)$ for $s > 0$. Similarly, $\hat{q}(e, s)$ lies above...

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\(^5\) Our results do not require the additional assumptions made by Kambh about the benefit and quality functions. That is, we do not require condition (9a) in Kambh (1982).

\(^6\) Although we have no explicit dynamic structure in the model, we take stability to mean that if either party deviates from an equilibrium, the optimal responses of the agents (acting alternately) brings $(e, q)$ back to the original equilibrium. A necessary and sufficient condition for this to hold locally is that the absolute value of $de/dq$ along the seller's reaction function is greater than the absolute value of $dq/de$ along the buyer's reaction function, at the equilibrium.
for \( s < 1 \). So if \( s \) is interior, the curves are as shown in the figure. Hence, point \( A \) is a Nash equilibrium for a given \( s \).

To compare quality and effort levels, it is clear from the figure that if there are unique equilibria for both the full-information and second-best economies, then quality and effort will be lower when the incentive problems are present.

**Proposition 2.** For \( 0 < s < 1 \), \( \Pi_{eq} > 0 \), and \((q^*, e^*)\) solving (1) and (2), there exists a \( q < q^* \) and \( e < e^* \) such that \((q, e)\) solves (3) and (4).

**Proof.** Since \( \hat{q}(e; s) \) is monotonically increasing in \( e \), we can define the inverse function \( \Phi(q; s) \) as the value of \( e \) such that \( \hat{q}(e; s) = q \). \( \Phi(q; s) \) is increasing in \( q \). We have assumed that \( \hat{e}(0; s) > 0 \) and \( \hat{q}(0; s) > 0 \) so that there exists \( \hat{q} > 0 \), where \( \Phi(\hat{q}, s) = 0 \). This is shown in Figure 1 as well. Finally, define \( D(q) \) by

\[
D(q) = \hat{e}(q; s) - \Phi(q; s).
\]

Clearly, \( D(\hat{q}) > 0 \). At \( q^* \), \( \hat{e}(q^*; s) < e^*(q^*) < \Phi(q^*; s) \) from Proposition 1. Hence \( D(q^*) < 0 \). Since all of the reaction functions are continuous, there exists \( \hat{q} \) in \((\hat{q}, q^*)\) such that \( D(\hat{q}) = 0 \). This \( q \) is a Nash equilibrium point. Since \( \hat{e}(q; s) \) is monotonically increasing in \( q \), \( e < e^* \).

Figure 2 makes it clear that quality and effort will again be lower than their first-best levels when \( \Pi_{eq} = 0 \). When \( \Pi_{eq} < 0 \), we can say very little in terms of comparisons. Figures 3a and 3b show the ambiguity. We do know that both \( e \) and \( q \) cannot be higher in the noncooperative solution than in the first-best since, as stated before, \( e^*(q) > \hat{e}(q; s) \) and \( q^*(e) > \hat{q}(e; s) \) for \( 0 < s < 1 \). It can be the case, however, that either \( e \) or \( q \) will be higher in the second-best, as we see from the figures.

Intuitively, the ambiguity is not surprising. Suppose we start at a cooperative solution. Then both the seller and the buyer would want to reduce their inputs while taking the other party's choice as given. This was also the case for \( \Pi_{eq} \geq 0 \). Now when \( \Pi_{eq} < 0 \), there are some offsetting effects. When the firm reduces \( q \), this makes the consumer's effort more productive so that \( e \) will be increased. A similar effect occurs in the firm's choice of \( q \). As Figures 3a and 3b show, it is possible for either of these two effects to dominate for one of the parties. It is clearly impossible for both \( e \) and \( q \) to increase.7

Thus far we have concentrated on the noncooperative game played between the seller and the buyer. Our analysis holds for arbitrary \( s \in (0, 1) \), though the magnitude of \( s \) will clearly affect the actual levels chosen in equilibrium.

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7 It is possible to show that for \( s \) close to 0, quality will be set lower than \( q^* \), and for \( s \) close to 1, effort will be lower than \( e^* \).
We can denote the unique Nash equilibrium of the noncooperative game, for given \( s \), by \( \hat{e}(s) \) and \( \hat{q}(s) \). Since the choice of \( p \) does not affect the Nash equilibria, we ignore this variable. The first stage of the problem is then to choose \( s \), given \( \hat{e}(s) \) and \( \hat{q}(s) \). We characterize the level of \( s \) that maximizes joint profits by solving:

\[
\max_s \hat{L} = \Pi z - g(e) - C(q)
\]

subject to

\[ e = \hat{e}(s) \quad \text{and} \quad q = \hat{q}(s). \]

Substituting the constraints into the problem and differentiating with respect to \( s \), we obtain

\[
\frac{\partial \hat{L}}{\partial s} = (\Pi z - C')\hat{q}_s + (\Pi z - g')\hat{e}_s = 0.
\]

Using (3) and (4), we can rewrite (8) as

\[
\hat{q}_s \Pi z (1 - s) + \hat{e}_s \Pi z s = 0
\]

with \( \hat{q}_s = \frac{d\hat{q}}{ds} \) and \( \hat{e}_s = \frac{d\hat{e}}{ds} \). At \( s = 0 \), \( \frac{\partial \hat{L}}{\partial s} = \hat{q}_s \Pi z > 0 \), and at \( s = 1 \), \( \frac{\partial \hat{L}}{\partial s} = e_1 \Pi z < 0 \), so that the solution to (9) will be \( s^* \in (0, 1) \). Thus, the optimal second-best level of warranty protection will be interior. Since \( C'(0) = g'(0) = 0 \) while \( \Pi z > 0 \), \( \Pi z > 0 \) for all \( e \) and \( q \), it will be to the parties' mutual advantage to set \( 0 < s^* < 1 \) to provide incentives for the provision of positive levels of both \( e \) and \( q \).

We turn now to a consideration of comparative statics on the second-best equilibrium. We are particularly interested in understanding what happens to the values of the observables, \( s \) and \( q \), when various parameters of the model change.

The signalling literature, as we described earlier, suggests that, within markets, \( s \) and \( q \) should be positively correlated. The story there relies on imperfect buyer information and says simply that firms with high-quality goods will attempt to signal this quality by offering a more complete warranty. Since warranties are less costly to provide when the product breaks less often, high-quality firms will be able to signal more cheaply, therefore a signalling equilibrium is possible.

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4 The properties of \( \hat{q}(s) \) and \( \hat{e}(s) \) are derived from (3) and (4) simultaneously. One can then see that \( \hat{q}_s(0) > 0 \) and \( \hat{e}_s(1) < 0 \).

5 The determination of \( p \) depends upon how the surplus of the consumer-firm relationship is split between the parties. In a competitive environment \( p \) is set so that the expected profits of the firm just equal zero.
Though positive correlations between $s$ and $q$ are undoubtedly observed in many markets, negative correlations are found as well. An obvious example here is the automobile market in which the Japanese manufacturers sell small cars of higher (by most accounts) quality than the domestic makers, but with inferior warranty protection.

Study of the comparative statics of this model reveals conditions under which both positive and negative correlations will be observed. It would seem plausible that certain differences among buyers could generate positive correlations. For example, more risk-averse customers might be expected to demand more protection from breakdown, and this protection might, in general, involve higher levels of both $s$ and $q$. With the risk-neutral buyers in this model, a similar story can be told with reference to each buyer's cost of effort.

On the other hand, certain differences among firms in the market may lead them to make different choices regarding how they protect their customers. Some firms (e.g., Japanese auto makers) may have cost advantages in building quality, but suffer cost disadvantages in providing warranty protection.\(^{10}\)

The analysis of the comparative statics of this model is complicated by the fact that the signs of most of the derivatives depend upon the sizes (i.e., absolute value), as well as signs, of the second derivatives of the $\Pi(\cdot)$ function. For this reason we have chosen to focus on a special case in which $\Pi(e, q)$ is linear, i.e., we define

$$\Pi(e, q) = \alpha e + \beta q$$

while the costs of quality and effort are simple quadratics:

$$C(q) = \gamma q^2 \quad \text{and} \quad g(e) = \delta e^2.$$ 

Its simplicity allows us to derive explicit solutions for the endogenous variables; yet the model remains rich enough to provide formal support for the intuition discussed earlier.

From conditions (3) and (4) we find the second-best levels of $e$ and $q$, given $s$, now to be:

$$\hat{e} = \frac{\alpha (1 - s) z}{2\delta} \quad \text{(10)}$$

and

$$\hat{q} = \frac{\beta s z}{2\gamma}. \quad \text{(11)}$$

Since $\Pi_{eq} = 0$, it will be true that $\hat{e}_s = \hat{e}$, and $\hat{q}_s = \hat{q}$, which will, in turn, be equal to:

$$\hat{e}_s = \frac{-\alpha z}{2\delta} < 0$$

$$\hat{q}_s = \frac{\beta z}{2\gamma} > 0.$$ 

Finally, the optimal $s$ is determined by (9):

$$\frac{(\beta z)^2}{2\gamma} (1 - s) - \frac{(\alpha z)^2 s}{2\delta} = 0$$

or

$$s^* = \frac{\beta^2 \delta}{\beta^2 \delta + \alpha^2 \gamma}. \quad \text{(12)}$$

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\(^{10}\) Smaller parts inventories and dealer networks raise the costs to the Japanese car makers of providing any given level of warranty protection, at least in the short run.
We consider first the effects of increasing the marginal disutility of effort. We are, therefore, interested in the effects of increases in \( \delta \) on the equilibrium levels of \( e \), \( q \), and \( s \). Changes in \( \delta \) will affect \( e \) directly and both \( e \) and \( q \) indirectly through \( s \). With effort more costly, the buyer, via (10), reduces his level of care. The optimal warranty also adjusts to shift more of the burden of care to the seller. Thus we see in (12) that \( ds^*/d\delta > 0 \).

The higher level of warranty protection causes the buyer to lower \( e \) further \( (\delta^* < 0) \), and the seller to increase \( q \) \((\delta^* > 0) \). We finish with a higher level of \( q \), a lower level of \( e \), and a larger \( s \) (fuller warranty). Thus, we observe a positive correlation between \( s \) and \( q \) when the variance is in the buyer’s costs of care.\(^{11}\)

To consider differences across firms, we first add a new element to the cost of the warranty. Now the firm’s expected costs are written: \( C(q) + (1 - \Pi)(sz + x) \), where \( x \) represents some cost of the warranty that does not go to the buyer. It could simply be the cost of verifying that the product is broken and of processing the claim.\(^{12}\)

This has no effect on the expression for \( \delta^* \), but (11) and (12) become

\[
\dot{q} = \frac{\beta(sz + x)}{2\gamma} \\
s^* = \frac{\beta^2\delta - \alpha^2\gamma(x/z)}{\beta^2\delta + \alpha^2\gamma}.
\]

Increasing \( x \) will raise \( q \) for unchanged \( s \), but then lower \( s^* \) according to (11a) and (12a). The falling \( s^* \) will tend to reduce \( q \), somewhat and to raise \( e \). The interested reader will have no difficulty proving that the full derivative of \( \dot{q} \) with respect to \( x \) is still positive. Thus, with variance in the cost of providing warranties, we observe a negative correlation between \( s \) and \( q \). In this case (and only this case) we can determine an unambiguous effect on the equilibrium level of \( \Pi \). Since both \( e \) and \( q \) are rising, \( \Pi \) must increase as well.

Finally, we consider the effect of increasing the marginal cost of quality by raising \( \gamma \). This case is analogous to the cost of effort case already discussed. The direct effect of the higher \( \gamma \) is to lower \( q \). The optimal warranty calls for a lower \( s \) to shift the burden of care marginally toward the buyer. This reduces \( q \) further and raises \( e \). Again we observe positive correlations between \( s \) and \( q \), this time as the costs of quality vary.

If consumers were risk averse, so that the warranty was also serving an insurance function, there would be reason to suspect that a rising \( \gamma \) might lead to a higher \( s \). The increase in the costs of quality would have two effects, analogous to substitution and income effects. With rising \( \gamma \) the seller would choose to switch to a lower \( q \) and higher \( s \) if he wanted to provide the same insurance protection. As we have found here, however, it will also be important for the contract to encourage further effort on the part of the buyer, so that \( s \) and the amount of insurance provided may still be reduced.

Combining the results from this analysis of the model’s comparative statics, in this example we find at least the seed of an explanation for observed positive and negative correlations between \( s \) and \( q \). Buyers with higher costs of effort will seek out sellers willing to carry more of the burden of preventing breakdowns by choosing a higher level of \( q \). These incentives are accommodated through the selection of a higher level of \( s \). When some sellers have a cost disadvantage in providing warranty protection, they may opt to

\(^{11}\) We have set aside any consideration of the additional problem of adverse selection that may arise in these markets when customers differ.

\(^{12}\) Actually, \( x \) may even be negative. The firms may not need to spend \( sz \) to get the buyer \( sz \) dollars of benefit out of the product. They may be able to repair it perfectly and cheaply, so that the uninsured portion \((1 - sz)\), represents only the buyer’s inconvenience cost.
reduce their warranty offerings and raise \( q \) (so that they have to pay off on the warranty less often).\(^\text{13}\)

5. Conclusions

- Our general interest here lies in trying to understand the role moral hazard plays in shaping sale-warranty contracts. To this end, we have solved for the optimal second-best equilibria that obtain under the conditions of double moral hazard. We were able to describe the directions of the second-best distortions, and to show the dependence of these distortions on the properties of the \( \Pi(e, q) \) function.

We also explored the sensitivity of these equilibria to changes in the values of certain parameters of the model. The results suggested conditions under which we could expect either positive or negative correlations between the extent of warranty protection offered with a particular brand of a product and the level of quality built into that brand.

When there are multiple equilibria, it is not so obvious how to make comparisons between the first-best and second-best equilibria as in Proposition 2. One cannot even ensure that the number of noncooperative equilibria bears any relation to the number of solutions to the full information problem. Moreover, as \( s \) varies, the number of noncooperative equilibria may vary as well, thereby complicating (7). Using the insights of the literature on smooth economies (Debreu, 1970; Balasko, 1978), if we assume that \( \Pi(e, q) \) is smooth (so that the reaction functions are continuously differentiable), then we know that there will generically be an odd number of locally unique equilibria.\(^\text{14}\) Hence, we can view \( \hat{e}(s) \) and \( \hat{q}(s) \) in (7) as a selection of an equilibrium. This local uniqueness provides a basis for the comparative statics results we reported.

It is also possible to compare levels of effort and quality in the first-best and second-best problems, if we restrict attention to stable Nash equilibria of the noncooperative game. When \( \Pi_{\text{NR}} > 0 \), there will be a stable Nash equilibrium with lower levels of both \( e \) and \( q \) associated with each full-information solution.

References


\(^{13}\) It will probably be clear to the careful reader that these comparative static results are not of the "knife-edge" variety. That is, they will continue to hold if the absolute values of \( \Pi_{\text{NR}}, \Pi_{\text{NR}}, \) and \( \Pi_{\text{NR}} \) are small enough.

\(^{14}\) It is not difficult to show, for example, that all these results will continue to hold if \( \Pi_{\text{NR}} = 0 \) and the following two conditions hold: (i) \( (\Pi_{\text{NR}} / \Pi_{\text{NR}}) + (g'/g) > 0 \) and (ii) \( (\Pi_{\text{NR}} / \Pi_{\text{NR}}) + (C'/C) > 0 \). These last two conditions, which essentially require that the II function exhibit less curvature than the cost functions, are also sufficient conditions for guaranteeing that the \( s^* \) that solves (9) satisfies the second-order conditions for a maximum.

\(^{15}\) To be precise, we need to assume that \( \lim_{e \to 0} D(q) < 0 \) for all \( s \).


