Worker Asymmetric Information and Employment Distortions

Russell Cooper, Cowles Foundation for Research in Economics

I. Introduction

Drawing on the well-known capability of asymmetric information to provide insights into apparent ex post inefficiencies, recent research has focused on explanations of involuntary underemployment based on imperfect information. In an environment in which a stochastic shock to a firm's technology is unobservable to its workers, Grossman and Hart (1981, 1983), Azariadis (1983), Chari (1983), and Green and Kahn (1983) characterize the optimal labor contract. Azariadis provides an example of underemployment in a model allowing work sharing, while Green and Kahn show that overemployment may arise. Grossman and Hart consider a model with no work sharing and generate unemployment.

This paper represents an extension of some of the ideas contained in chap. 3 of my Ph.D. dissertation (Cooper 1982). I am grateful to Costas Azariadis, Sanford Grossman, Andrew Postlewaite, and Michael Riordan for helpful comments on my earlier research. Charles Kahn, Joe Tracy, and seminar participants at the University of Pennsylvania, Wesleyan University, and Rochester University provided helpful comments on this paper. Financial support from the Richard D. Irwin Foundation, CARESS at the University of Pennsylvania, the Cowles Foundation, and the NSF (SES 82-08899) is gratefully acknowledged.

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of workers. (The employment distortions in models of work sharing are characterized by a comparison of worker’s and firm’s marginal rates of substitution between consumption and employment, whereas in models with discrete hours the comparison is one of employment and unemployment states.)

As emphasized in Cooper (1983) and Hart (1983) the two key factors in determining the form of the employment distortion are the worker’s preferences over consumption and leisure and the firm’s degree of risk aversion. If the firm is risk neutral, then leisure must be an inferior good for underemployment to occur. When leisure is a normal good, there appears to be a minimal level of firm risk aversion necessary for underemployment.¹

These conditions for underemployment appear to be rather restrictive. In particular, there is no basis for the argument that leisure is an inferior good. Assuming that firms are somewhat risk averse is not unreasonable given the incompleteness of capital markets and managerial incentive problems. Nonetheless, it is not clear how risk averse firms must be in order to generate an underemployment result when leisure is a normal good for workers. In the end, we may be left with firms so risk averse that they are shedding risks to their workers. This seems quite contrary to our basic understanding of the role of labor contracts.

In response to these criticisms, this paper considers an alternative environment of worker asymmetric information in which workers’ preferences over consumption and leisure are not perfectly observed by the firm. There are a number of possible interpretations one can offer for this type of model. The uncertainty in worker’s preferences may arise from the random nature of outside opportunities as in Hart (1983) or Prescott and Townsend (1984). Alternatively, the uncertainty may be associated with the disutility of work on a certain job as discussed by Hall and Lilien (1979), Hashimoto and Yu (1980), and Carmichael (1983). Finally, workers can be thought of facing uncertainty in their opportunity sets because of stochastic prices and income other than wages. In each of these situations, an insurance motive for contracting exists, as discussed in Section II. To the extent that realizations of the random variables described above are observable only to the worker, a situation of imperfect information exists. This incentive problem, interacting with the worker’s demands for insurance, creates the employment distortions studied here. It should be noted that one could adapt the present model to focus on the bargaining problem between a firm and a worker (or a union) in which the worker’s preferences over the contract terms are not observable to the firm. Though the analysis would require

¹ This has not been formalized but is rather a conjecture based on the previous research in this area. See the related discussion of Stiglitz (1984).
some modification, the results (described below) will hold under this sorting interpretation as well.\(^2\)

The key question addressed in this paper is whether the conditions for underemployment are more reasonable when the informational asymmetry is on the worker's side of the contract. I pursue an answer in two ways.

First, following an overview of the model in Section 2, I consider the case of worker asymmetric information in Section 3. I show that when the firm is risk neutral, if leisure is normal in worker's preferences, then underemployment will arise. I also present results for the special case of perfect substitutability of income and leisure (which has been a leading example in the literature) and show that either underemployment or overemployment may arise.

Second, I consider a situation of bilateral asymmetric information in which both parties to a contract have private information. As discussed in Section 4, a general treatment of this problem is rather difficult. Hence I confine my attention to an extension of Grossman and Hart's (1981) model to a situation of bilateral asymmetric information. The results, which complement those obtained by Hall and Lazear (1984), indicate a tendency toward unemployment.

II. Worker Asymmetric Information and Implementation of the Full-Information Solution

To begin the analysis, I will carefully specify worker preferences and analyze the full-information solution. Risk neutral firms have profits (π) represented by

\[ \pi = l - \omega, \]

where \( l \) is the level of employment and \( \omega \) is total compensation. Since I assume a constant returns to scale technology I restrict my attention to the relationship between this firm and a single worker. Represent a worker's preferences over consumption (equal to \( \omega \) by assumption of one good) and labor by \( U(\omega, l, \theta) \) where \( \theta \) is a random disturbance to worker's preferences. Assume that, for given \( \theta \), preferences are convex and that \( U_\omega > 0, U_{\omega\omega} < 0, U_\theta < 0, U_{\theta\theta} < 0 \). Consumption is assumed to be a normal good. Define \( G(\omega, l, \theta) \) as the worker's marginal rate of substitution between compensation and labor; that is,

\[ G(\omega, l, \theta) = \frac{-U_\theta}{U_\omega}. \]

\(^2\) In the sorting model, we would use an ex post individual rationality constraint for each \( \theta \).
I initially assume that preferences satisfy $U_{\omega \theta} \geq 0$ as well as the single-crossing property of $G_\theta > 0$. That is, for given $(\omega, l)$, the worker's marginal rate of substitution is increasing in $\theta$ as shown in figure 1 so that indifference curves (for alternative $\theta$'s) cross only once. Assume that $\theta$ takes on values in the interval $[\bar{\theta}, \tilde{\theta}]$ with $\bar{\theta} > 0$ and denote by $\rho(\theta)$ the probability that $\theta$ occurs.

Ex ante, the firm and the worker sign a contract $\delta = [\omega(\theta), l(\theta)]$ specifying compensation and employment for each realization of $\theta$. When realizations of $\theta$ are observed by both parties, the optimal full-information contract, $\delta^* = [\omega^*(\theta), l^*(\theta)]$ solves

$$\max E[\pi + \lambda U(\omega, l, \theta)],$$

where the expectation is taken over $\theta$ and $\lambda > 0$ is an arbitrary weight to parameterize the expected utility possibility frontier. To be feasible, $l(\theta) \leq \bar{l}$ for all $\theta$ where $\bar{l}$ is the time endowment of the worker. The interior solution to (2) satisfies

$$U_{\omega}(\omega, l, \theta) = \frac{1}{\lambda}, \quad \text{for all } \theta$$

(3)

and

$$G(\omega, l, \theta) = 1, \quad \text{for all } \theta.$$  

(4)

Conditions (3) and (4) represent conditions for optimal risk sharing and productive efficiency, respectively.

To motivate our results on employment distortions in the next section, we can ask whether or not $\delta^*$ is implementable when $\theta$ is observed only by the worker. That is, faced with $\delta^*$, would the worker have an incentive to reveal $\theta$ truthfully? The answer can be obtained by using conditions (3) and (4) to define $\omega^*(\theta)$ implicitly as the level of compensation associated with $l$ units of employment time.

So consider an arbitrary $\theta$, $\bar{\theta}$, as shown in figure 2. The optimal compensation and employment levels associated with $\theta$ are determined at the point of tangency between the indifference curves of the two parties as in (3) and (4). I have drawn $\omega^*(l)$ flatter than $G(\omega^*(\theta), l^*(\theta), \bar{\theta})$ in the diagram to depict a situation where $\delta^*$ is not implementable. It is clear from figure 2 that the worker will have an incentive to

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3 Here and in the sequel, $G_\omega$ is the partial derivative of $G$ with respect to $x$. The assumption of $G_\theta > 0$ is crucial in models of self-selection (see Riley 1979 or Cooper 1985). For some of this analysis we need the additional assumption that $U_{\omega \theta} \geq 0$. I will make clear when this last restriction is necessary.

4 We assume that $l$ is large enough so that an interior solution exists.

5 For a discussion of implementation under asymmetric information see Myerson (1979) or Harris and Townsend (1981).
some modification, the results (described below) will hold under this sorting interpretation as well.²

The key question addressed in this paper is whether the conditions for underemployment are more reasonable when the informational asymmetry is on the worker's side of the contract. I pursue an answer in two ways.

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FIG. 1.—Single-crossing property of worker preferences. \( \theta_2 > \theta \),

FIG. 2 }
report a $\theta$ that will reduce the amount of employment relative to $l^*(\theta)$. It should be equally apparent that $\theta^*$ is implementable if and only if $\omega^*(l)$ is tangent to the worker’s indifference curve at $[\omega^*(\theta), l^*(\theta)]$ for all $\theta$ as in figure 3. Implementation requires

$$U_\omega \frac{d\omega^*(\theta)}{d\theta} = -U_l \frac{dl^*(\theta)}{d\theta}. \tag{5}$$

As in Cooper (1983), it will prove useful here to understand the conditions regarding the misrepresentation of $\theta$. In particular, are there interesting conditions under which the worker will lie about the true value of $\theta$ to reduce employment as in figure 2?

\[\text{This tangency condition is obviously necessary. It is also sufficient given the single-crossing property and monotonicity of worker's preferences. See App. A for a complete discussion of implementation.}\]
PROPOSITION 1. If leisure is a normal good, then a worker, faced with \( \delta^* \), will misrepresent \( \theta \) to reduce employment.

PROOF. The proposition clearly holds if the slope of \( \omega^*(l) \) is less than one for all employment levels. If consumption (as stated above) and leisure are both normal, then differentiation of (3) and (4) yields the result.

Hence we see that the normality of leisure is a sufficient condition for workers to want to reduce their employment under the first-best contract \( \delta^* \). It should be noted that proposition 1 requires \( G_0 > 0 \) but does not use the restriction that \( U_{u\theta} \geq 0 \). Other cases of lying to reduce work will be discussed in the next section for preferences where consumption and leisure are perfect substitutes. The implication of proposition 1 is that the optimal contract under asymmetric information must provide incentives for the worker to be honest about \( \theta \) and not attempt to reduce employment when leisure is a normal good. I show in the following section that these incentives for truth telling will create underemployment.

III. Optimal Contract with Worker Asymmetric Information

We now consider the optimal contract when realizations of \( \theta \) are observed only by the worker. Following standard practices, contracts under asymmetric information are implemented through a direct revelation game. Define underemployment as occurring when \( G < 1 \), since both parties could benefit ex post from increasing employment and overemployment occurs if \( G > 1 \) where \( G = -U/L \) (see fig. 4).

The optimal contract under worker asymmetric information solves

\[
\max E[\pi + \lambda U(\omega, l, \theta)]
\]

s. t. \( U(\omega(\theta), l(\theta), \theta) \geq U[\omega(\theta), l(\theta), \theta], \ \forall \theta, \ \theta \). \hspace{1cm} (6)

In this problem, I have simply appended incentive compatibility constraints to (1) to ensure truth telling by the worker.

PROPOSITION 2. If leisure is a normal good, \( G_0 > 0 \) and \( U_{u\theta} \geq 0 \), then the solution to (6) yields underemployment for \( \theta \in (\underline{\theta}, \bar{\theta}) \) and efficiency for \( \theta = \underline{\theta}, \bar{\theta} \).

PROOF. See Appendix A.

The proof of the proposition analyzes (6) by setting up the associated Hamiltonian problem. The proof uses the condition that \( U_{u\theta} \geq 0 \) as well as the single-crossing property of preferences.\(^7\)

There are two additional properties of the solution to (6) worth

\(^7\) I have been able to prove a version of proposition 2 without restrictions on \( U_{u\theta} \) when \( \theta \) takes on two values. It remains an open question whether or not \( U_{u\theta} \geq 0 \) is necessary for proposition 2.
mentioning. First, as is common in problems with self-selection features, the incentive compatibility conditions imply that \( \omega(\theta) \) and \( l(\theta) \) are monotonically decreasing functions of \( \theta \). Second, there is a simple intuitive mechanism for the implementation of the solution to (6). The firm announces a function \( \hat{\omega}(l) \) and the worker, having observed \( \theta \);
simply chooses the number of hours he or she wishes to work based on \( \hat{\ell}(l) \). Given this behavior by workers ex post, the firm selects \( \ell(l) \) to maximize the objective function in (6).

We have assumed that leisure is normal for all \((\omega, \ell, \theta)\) to prove proposition 2. If leisure is an inferior good, overemployment may arise. We could presumably have regions of overemployment and underemployment if leisure was a normal good for some \((\omega, \ell, \theta)\) and an inferior good for other \((\omega, \ell, \theta)\).

It may prove instructive at this point to analyze in more detail some special utility functions. The first of these examples will provide some intuition for proposition 2 and the second will be useful in our discussion of bilateral asymmetric information.

Example 1: Separable Preferences

Here we assume that the worker’s preferences are represented by

\[ V(\omega) - \theta g(l), \]  

(7)

where \( V(\cdot) \) is increasing and strictly concave and \( g(\cdot) \) is increasing and strictly convex. This case was initially examined in Cooper (1982). In the first-best contract, \( \omega(\theta) \) is constant and \( l(\theta) \) decreases with \( \theta \). The worker would have an obvious incentive to announce that \( \theta \) occurred, regardless of the true value of \( \theta \), to minimize employment.

In the solution to (6), we find that underemployment occurs because leisure is a normal good if preferences are separable. The reason is that the firm must alter \( \omega^*(\theta) \) and \( l^*(\theta) \) to induce the worker to announce the true \( \theta \) rather than \( \hat{\theta} \). Given \( l^*(\theta) \), there will exist a compensation schedule that will implement this first-best employment rule. However, it will leave the worker facing excessive risks, so that adjustments in the employment rule are necessary as well. The solution to (6) balances these adjustments and creates imperfect insurance (i.e., \( \omega \) is not independent of \( \theta \) when preferences are separable) and ex post underemployment of labor.

Example 2: Perfect Substitutes

Here we represent worker’s preferences by \( U(\omega - h(l, \theta)) \). This representation (with \( \theta \) degenerate) was used by Grossman and Hart (1981, 1983) and Azariadis (1983) to produce examples of unemployment when firms had privately observed shocks to their technology. With these preferences, leisure demand is independent of income, so that this case is not covered by proposition 2. In this specification \( h(l, \theta) \) represents the monetary value of supplying \( l \) units of labor when state \( \theta \) occurs. Moore (1982) and Hart (1983) analyze the cases of \( h(l, \theta) = \theta \) and \( h(l, \theta) = -(l - l)\theta \) where \( l - l \) is the leisure consumption. In the first special
case underemployment occurs, whereas overemployment arises for the second example.\footnote{The intuition behind these results is presented in the following sec. Also see the related discussion in Kahn (1982) on severance pay under asymmetric information about employees' outside opportunities. While these preferences are very special, they have played a historical role in the contracting literature.}

With the general $h(l, \theta)$ specification, we see that the full-information contract will stabilize $[\omega - h(l, \theta)]$ as $\theta$ varies and that $l$ will decrease with $\theta$. If the firm attempted to implement this contract under worker asymmetric information, the worker will have an incentive to misrepresent his preferences by overstating $\theta$ if and only if $h_0 > 0$ and by understating $\theta$ if and only if $h_0 < 0$.

We can analyze this specification of preferences without the assumption maintained in proposition 2 that $U_{o\theta} \geq 0$. The single-crossing property will hold as long as $h_0 > 0$, which we will continue to assume. Since the direction of lying under the full-information solution depends on $h_0$, it is not very surprising that the direction of the employment distortion will depend on this term as well. When $h_0$ is positive, then $U_{o\theta}$ exceeds zero as well and proposition 2 holds.

**PROPOSITION 3.** Suppose that worker's preferences are represented by $U(\omega - h(l, \theta))$. Then the optimal contract under asymmetric information has underemployment when $h_0 > 0$ and overemployment when $h_0 < 0$.

**PROOF.** See Appendix B.

When leisure demand is independent of income we can have either overemployment or underemployment depending on whether or not high $\theta$'s are "good" states. If $h_0$ can not be signed globally, we can have regions of overemployment and underemployment. Hence, we see from propositions 2 and 3 that, in contrast to the firm asymmetric information problem, it is possible to obtain underemployment with both firm risk neutrality and worker's preferences in which leisure is not inferior.

Having obtained a characterization of the conditions for underemployment in the worker asymmetric information problem we can now address questions concerning bilateral asymmetric information. For readers unacquainted with contracting problems of firm asymmetric information Azariadis and Stiglitz (1983) and Hart (1983) may be useful references.

**IV. Bilateral Asymmetric Information**

A natural contracting situation is that of bilateral asymmetric information, in which both parties to the contract face future uncertainty. This was the environment initially studied by Hall and Lilien (1979) though their main question concerned the implementation of the full-
information employment rule rather than characterizing the optimal incentive-compatible contract. In that paper, as well as in Hall and Lazear (1984), both the firm and the worker were assumed to be risk neutral. As shown in Riordan (1984), the full-information solution can generally be implemented when both parties to the contract are risk neutral. Green and Honkapohja (1981) also consider a model of exchange under bilateral asymmetric information but do not stress the risk-sharing aspects of the problem.

The motivation for studying the bilateral asymmetric information environment is twofold. First, it seems quite reasonable that both parties to a contract will have random valuations of the contract terms. Since firms will generally have superior information over technological factors and workers will be better informed about their own opportunities and job satisfaction, the assumption of bilateral asymmetric information is quite realistic. Second, in the search for a characterization of conditions for underemployment, an interesting question arises whether the bilateral nature of the asymmetry will strengthen the case for this type of employment distortion.

A general characterization of the optimal contract under bilateral asymmetric information is beyond the scope of this paper. That is, I will not present necessary and sufficient conditions for underemployment in a broad class of models. Instead, I will present the general problem, discuss the difficulties in solving it, and discuss some examples that I hope will be illuminating.

As in the previous sections of this paper, I represent workers’ preferences by \( U(\omega, l, \theta) \). Firms have profits \( \pi = \bar{s}(l) - \omega \), where \( \bar{s} \) represents a stochastic technology shock, \( f(l) \) is a concave production function, and an increasing concave function \( V(\cdot) \) defines their preferences over income. Under full information about the realizations of \( (s, \theta) \), the optimal contract \( \gamma^* = [\omega^*(s, \theta), l^*(s, \theta)] \) maximizes, for arbitrary \( \lambda > 0 \),

\[
E[V(\pi) + \lambda U(\omega, l, \theta)].
\]  

(8)

In (8), the expectation is taken over the joint distribution of \( (s, \theta) \).

When realizations of \( \bar{s} \) and \( \theta \) are the private information of the firm and worker, respectively, a number of important issues arise. First of all, we must choose between alternative equilibrium concepts of dominant strategy, ex post Nash and Bayesian, for the revelation game played between the parties. Since preferences of the parties depend only on their own shocks, it is well known that dominant strategy and ex post Nash yield identical solutions.\(^9\) Second, whether or not \( \bar{s} \) and \( \theta \) are

\(^9\)See the discussion in Dasgupta, Hammond, and Maskin (1979) and in Cremer and McLean (in press).
independent will be important in the Bayesian equilibrium though
irrelevant for dominant strategies.

Determining the optimal contract under bilateral asymmetric infor-
mation entails solving (2) subject to the incentive compatibility constraints
implied by one of the two equilibrium concepts. When \( \theta \) and \( s \) are
continuous random variables, the optimization problem is a very difficult
double-integral variational problem with partial differential equations as
constraints. Even in the case of discrete states, the problem is difficult
to solve unless further structure is assumed.

Instead of focusing on these general problems, I will present below
two interesting special cases. The simplifications include the use of a
specific worker-utility function and concentration on dominant strategy
implementation.

To begin, we need a more precise definition of dominant strategy
implementation. Let \( \pi(s_k, \theta_j|s) \) be the firm's profits under an arbitrary
contract when the parties announce \((s_k, \theta_j)\) and the firm's true state is \( s \).
Similarly \( W(s_j, \theta_i|\theta) = U[\omega(s_j, \theta_i), l(s_j, \theta_i), \theta_j] \) and is the worker's utility
in state \( \theta \) when the announced states are \((s_j, \theta_i)\).

**Definition 1.** A contract is **dominant strategy incentive compatible**
if (i) for each \((s_j, \theta_i)\), \( \pi(s_j, \theta_i|s) \geq \pi(s_k, \theta_j|s) \forall s_k \) and (ii) for each \((s, \theta_i)\),
\( W(s_j, \theta_i|\theta) \geq W(s_i, \theta_j|\theta) \forall \theta_j \).

Implementation in dominant strategies thus requires that each party
truthfully reveal its true state independent of the announcement of the
other party. This is viewed (see, e.g., the discussion in d'Aspremont
and Gerard-Varet [1979]) as applicable to situations of "complete ignorance"
about other parties to the agreement. In dominant-strategy incentive-
compatible contracts, agents need no information at all about the
strategies of other agents in the contract. In many circumstances this
simplicity can be quite appealing.

I consider a model which extends the analysis of Grossman and Hart
(1981) to the bilateral asymmetric information setting.\(^{10}\) In particular I
assume that worker's preferences are represented by \( U(\omega - \Theta l) \) so that
\( \Theta \) represents a continuous random disutility of work and takes values in
\([0, 1]\). I also assume that \( l \) only takes on the values 0 or 1 to correspond
to unemployment and employment states, respectively. Finally, I maintain
the assumption of constant returns to scale, so that \( \pi = \hat{s} l - \omega \) where \( \hat{s} \)
is a continuous random variable in \([0, 1]\). I assume that both \( \hat{s} \) and \( \Theta \)
have densities known to both parties.

Grossman and Hart (1981) show that when \( \theta \) is degenerate, unem-
ployment occurs in the resulting firm asymmetric problem. Incentive

\(^{10}\) I am grateful to Oliver Hart and an anonymous referee for suggesting that
I use these special preferences as an example. Preferences of this form play a
strong role in the incentives literature as well as in the contracting papers
mentioned earlier.
compatibility clearly requires that the contract specify only two wages: \( \omega^* \) if employment occurs and \( \omega^a \) if employment does not occur. This means that employment states are risky for the risk-averse firm, so that unemployment occurs as a means of avoiding this risk. When \( \tilde{s} \) is degenerate, a similar story arises in the worker asymmetric information problem. That is, there will again be two wages and the worker will bear the risk of \( \Theta \) in the employed states. To avoid this risk, unemployment arises.\(^{11}\) In each of these cases, the party with asymmetric information essentially chooses whether employment will occur.

When neither \( \tilde{s} \) nor \( \tilde{\Theta} \) are degenerate, the full-information contract stipulates that employment occurs if and only if \( s \geq \Theta \), as depicted in figure 5. The compensation schedule is set to efficiently share the remaining risks.

The optimal contract under bilateral asymmetric information takes a very intuitive form when dominant strategy implementation is utilized. As in the unilateral asymmetric information case, since \( \ell \in \{0, 1\} \), compensation can vary only with employment in an incentive-compatible contract. To see why, assume that compensation varies while \( \ell \) does not.

\(^{11}\) This case is not covered by proposition 2 since we have allowed the firm to be risk averse. However, the proof of this result follows Grossman and Hart (1981) and will not be presented.
Then one of the parties would have an incentive to misrepresent their true state (by the monotonicity of preferences) for a given announcement of the other party. This violates dominant strategy incentive compatibility. In particular, \( \omega' \) is independent of the cause of separation, so that severance pay is received whether the worker quits or is fired. Define \( d \) as the difference between the employment and unemployment wages: \( d = \omega' - \omega \).

I also impose ex post constraints on the problem so that parties to the agreement are not forced to work against their will. For the worker, this implies that in order for employment to occur \( (l = 1) \), \( d \geq \theta \). Similarly, from the firm's viewpoint, \( s \geq d \) is necessary for \( l = 1 \). In Grossman and Hart's analysis and in the worker asymmetric information problem discussed above, these ex post constraints were met.

At this stage, we could formally characterize the optimal contract by choosing over \( d \) and one of the wages. However, the unemployment result can be seen directly. In fact the optimal contract in this setting is very similar to one of the "simple contracts" described by Hall and Lazear. Ex ante, the contract sets \( d \). The firm observes \( s \) and decides whether it wants to employ workers or not. Similarly, workers observe \( \theta \) and decide whether or not they wish to work. From the ex post individual rationality constraints discussed above employment occurs if and only if \( s \geq d \geq \theta \). Comparing figure 6 with figure 5, we see that unemployment will definitely occur for \( s \geq \theta \geq d \) and \( d \geq s \geq \theta \). This is not surprising, since we had unemployment in both the firm and worker asymmetric information cases discussed above.

However, we do get a different and somewhat surprising result when worker's preferences are represented by \( U[\omega + (1 - l)\theta] \). In this specification, the value of leisure time is random rather than the random disutility of labor studied above. As discussed by Moore (1982) and Hart (1983) and shown in proposition 3, the optimal contract with worker asymmetric information yields overemployment because the employment states are less risky than the unemployment states. Again, employment occurs if and only if \( d \geq \theta \).

The argument concerning the role of \( d \) discussed above holds in the bilateral asymmetric information case as well. Hence, the contract yields unemployment, just as in the previous case. Intuitively, with dominant strategy implementation and the ex post constraints it takes only the "veto" of one party to set \( l = 0 \). Though workers may wish to create more employment states by increasing \( d \), this only makes it more likely that the firm will not wish to employ any workers.

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12 I am grateful to Lorne Carmichael for referring me to the paper by Hashimoto and Yu (1980), who use a similar diagram in understanding quits and dismissals in a related program.
Hence these two examples both lead to unemployment. Of course, the difference in preferences will lead to alternative \( d \)'s being chosen.

It should be made clear that this was a very special example in terms of the preferences of the workers, the use of a dominant strategy equilibrium, and the existence of ex post constraints. While one can argue that the ex post constraints are reasonable given the voluntary nature of labor exchanges, they do play a strong role in the analysis.

V. Conclusion

The main result of this paper is that the normality of leisure implies underemployment when worker's characteristics are unobservable by the firm. I have also demonstrated that when leisure demand is independent of income, either underemployment or overemployment can occur. The type of distortion that occurs when leisure is an inferior good remains an open question.

I have also discussed, on a preliminary basis, the problem of bilateral asymmetric information. The general solution to this problem is very difficult and will receive more attention in future research. The examples discussed in the paper showed that the case for unemployment may be strengthened if both parties can veto an employment state.

There are at least three additional problems to consider as extensions of this analysis. First, can we use models of worker and/or bilateral
asymmetric information to provide insights into macroeconomic phenomena? This, of course, requires an argument that aggregate shocks are private information to one of the parties to the contract. Second, in the bilateral asymmetric information extension of Grossman and Hart's model, we see that severance pay will be independent of the cause of the separation. Given that governments generally pay unemployment insurance to workers who are fired and not to those who quit, it would be interesting to add the government as a third party to the labor contract and investigate its role in the separation decision. Finally, the results obtained for the case of bilateral asymmetric information were based on specific preferences and dominant strategy implementation. Extensions to more general preferences and alternative equilibrium concepts seems warranted.

Appendix A

Proof of Proposition 2

To prove proposition 2, we solve the Hamiltonian problem associated with (6). Ex post, the worker announces $m(\Theta)$, when $\Theta$ is the true state of nature, where $m(\Theta)$ solves

$$\max_{m(\Theta)} U\{w[m(\Theta)], l[m(\Theta)], \Theta\}. \quad (A1)$$

Incentive compatibility requires that $m(\Theta) = \Theta$ for all $\Theta$. This implies (see the related discussion in Green and Kahn [1983]) that

$$U_w \frac{dw}{d\Theta} + U_l \frac{dl}{d\Theta} = 0 \quad \text{at} \quad m(\Theta) = \Theta \quad (A2)$$

and

$$\frac{dl}{d\Theta} \leq 0. \quad (A3)$$

(A3) follows from the second-order conditions to (A1) because of the single-crossing property of preferences. We will focus on separating solutions so that (A2) is the incentive compatibility constraint for the problem.\(^{13}\)

Following standard methodology (see, e.g., the presentation in Kamien and Schwartz [1981]), we write the Hamiltonian as

\(^{13}\) Here I follow Green and Kahn (1983) and assume that (A3) is a strict inequality. While some pooling may be optimal, it will not change the distortions arising in the optimal contracts.
\[ H(\omega, l, \theta, k) = p(\theta)\{(l(\theta) - \omega(\theta)) + \lambda U[\omega(\theta), l(\theta), \theta]\} \]
\[ + \phi(\theta)\left(\frac{U_l}{U_\omega}\right)k + \eta(\theta)k. \tag{A4} \]

Here \( k = dl/d\theta \) and is the control variable; \( \phi(\theta) \) and \( \eta(\theta) \) are the costate variables, and \( p(\theta) \) is the probability that \( \theta \) occurs. Constraint (A2) defines \( d\omega/d\theta \) as a function of \( k \), and the resulting substitution has been made.

The necessary conditions for optimality (where \( \dot{x} = dx/d\theta \)) are
\[ \phi \frac{U_l}{U_\omega} = \eta \tag{A5} \]
\[ \dot{\phi} = p(1 - \lambda U_\omega) - \frac{\phi k}{U_\omega} \left(U_{\omega\omega} \frac{U_l}{U_\omega} - U_{\omega l}\right) \tag{A6} \]
\[ -\dot{\eta} = p + \lambda U_l + \frac{\phi k}{U_\omega} \left(U_{\omega l} \frac{U_l}{U_\omega} - U_{ll}\right). \tag{A7} \]

Total differentiation of (A5) and substitution of (A6) and (A7) yields
\[ p(1 - G) = \phi G, \tag{A8} \]

where \( G = -U_l/U_\omega \) and \( G_0 > 0 \) by the single-crossing property. Hence if \( \phi > 0, \ G < 1 \) and we have underemployment, while the opposite holds for \( \phi < 0 \). So to prove the proposition we need to show that when leisure is normal, \( \phi > 0 \) for \( \theta \in (\theta_l, \theta_r) \).

Assume that leisure is normal, that is, \( U_{\omega\omega}(U_l/U_\omega) - U_{\omega l} > 0 \). Suppose though that \( \phi(\theta) < 0 \) for \( \theta \in (\theta_{\text{min}}, \theta_{\text{max}}) \) and \( \phi(\theta) \geq 0 \) for \( \theta \in (\theta_l, \theta_{\text{max}}) \). Since \( \phi(\theta) \) is a continuous function of \( \theta \) (again see Kamien and Schwartz), there will exist \( \theta^* \) such that \( \phi(\theta^*) < 0 \) and \( \phi(\theta^*) = 0 \), as shown, for example, in figure 7. From (A6), when \( \theta = \theta^*, \{1 - \lambda U_{\omega l}[\omega(\theta^*), l(\theta^*), \theta^*]\} > 0 \) as \( k < 0 \) by incentive compatibility.

The transversality conditions imply that \( \phi(\theta) = \phi(\bar{\theta}) = 0 \). If \( \theta = \theta_{\text{min}} \), then \( \phi(\theta) < 0 \). If \( \theta < \theta_{\text{min}} \), then \( \phi(\theta_{\text{min}}) < 0 \). From the construction of \( \theta_{\text{min}} \) we also know that \( \phi(\theta_{\text{min}}) = 0 \). From (A6), \( 1 - \lambda U_{\omega l}[\omega(\theta_{\text{min}}), l(\theta_{\text{min}}), \theta_{\text{min}}] < 0 \).

Hence,
\[ U_{\omega l}[\omega(\theta_{\text{min}}), l(\theta_{\text{min}}), \theta_{\text{min}}] > U_{\omega l}[\omega(\theta^*), l(\theta^*)]. \tag{A9} \]

The normality of leisure also implies that \( U_{\omega l}[\omega(\theta), l(\theta), \theta] \) must increase with \( \theta \). Total differentiation of \( U_\omega \) yields
\[ \frac{dU_\omega}{d\theta} = U_{\omega l} \frac{d\omega}{d\theta} + U_{\omega l} \frac{dl}{d\theta} + U_{\omega \theta} \]
\[ = k \left[U_{\omega l} \left(-\frac{U_l}{U_\omega}\right) + U_{\omega l}\right] + U_{\omega \theta} > 0, \tag{A10} \]
since $d\omega/dl = -U_l/U_m$ from (A2). With $k < 0$ from incentive compatibility and $U_{\omega\theta} \geq 0$ by assumption, when leisure is normal, $dU_m/d\theta > 0$.

Since $\theta_{\text{max}} < \theta^*$, (A9) and (A10) are inconsistent, so that if leisure is normal, $\phi$ cannot be negative for $\theta \in (\theta_{\text{min}}, \theta_{\text{max}})$ as we assumed. If $\theta_{\text{min}} = \theta_{\text{max}} > \theta$, we also get a contradiction of (A10). Hence $\phi(\theta) > 0$ for $\theta \in (\bar{\theta}, \theta)$ when leisure is normal, so that underemployment occurs. Since $\phi(\theta) = \phi(\bar{\theta}) = 0$, we have efficiency in these states.

Appendix B:

Proof of Proposition 3

To prove proposition 3, we follow the steps of Appendix A for the preferences of $U[\omega - h(l, \theta)]$. Using $p(\theta)$ as the density function for $\theta$, the optimal contract will

\[
\max_{\omega(\theta), \theta} \int_\theta \{\pi + \lambda U[\omega - h(l, \theta)]\} \\
\text{s.t. } \frac{d\omega}{d\theta} - h_l \frac{dl}{d\theta} = 0 \text{ for all } \theta.
\]  

(B1)

We can convert (B1) into a Hamiltonian by letting $\omega(\theta)$ and $l(\omega)$ be the state variables and defining $k = dl/d\theta$ as the control variable. We define the Hamiltonian as
\[ H(\omega, l, \theta, k) = p(\theta)(l(\theta) - \omega(\theta)) + \lambda U(\omega(\theta) - h[l(\theta), \theta]) + \phi(\theta)[h_{\theta}k] + \eta(\theta)k, \]  
\[ \text{(B2)} \]

where \( \phi(\theta) \) and \( \eta(\theta) \) are the costate variables.

The necessary conditions for an optimal solution are

\[ -\phi h_l = \eta \]  
\[ \dot{\phi} = p(1 - \lambda U') \]  
\[ \dot{\eta} = p(\lambda h_{ll} U' - 1) - \phi h_{lh}. \]

Here the arguments have been omitted for convenience and \( \dot{x} = dx/d\theta \).

Totally differentiating (B3) yields

\[ -\phi h_l - \phi h_{lh}k - \phi h_{\theta} = \dot{\eta}. \]  
\[ \text{(B6)} \]

Substitution of (B4) and (B5) implies that

\[ \phi h_{lh} = p(1 - h_l). \]  
\[ \text{(B7)} \]

For these preferences \( h_{\theta} > 0 \) by the single-crossing property. Furthermore, \( G = h_l \) so that \( h_l < 1 \) implies underemployment and \( h_l > 1 \) implies
overemployment. From (B7) we see that the sign of \( \phi \) will determine the direction of this distortion as in Appendix A.

Total differentiation of (B4) yields

\[
\dot{\phi} = \dot{r}(1 - \lambda u') + p\lambda u'b_0. \tag{B8}
\]

We will use (B8) to construct an argument similar to that used in Appendix A to connect the sign of \( b_0 \) to the sign of \( \phi \) as stated in the proposition. To do so we note that at \( \hat{\phi} = 0 = p(1 - \lambda u') \), \( \hat{\phi} \) has the opposite sign to \( b_0 \). If \( b_0 > 0 \) globally, then \( \hat{\phi} < 0 \) when \( \phi = 0 \) as shown in Figure 8. (Here we also use the transversality conditions of \( \phi[0] = \phi[\hat{\phi}] = 0 \).) Hence \( b_0 > 0 \) implies that \( \phi > 0 \), so that \( h_1 < 1 \) and underemployment occurs. When \( b_0 < 0 \), then \( \phi < 0 \) and overemployment occurs. Of course, if \( b_0 \) changes signs this argument will not work and we shall observe intervals of overemployment and underemployment.

References


