

THE OPTIMALITY OF REGULATED PRICING:
A GENERAL EQUILIBRIUM ANALYSIS*

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1. INTRODUCTION

Consider a world where consumers, with diminishing marginal rates of substitution, maximize utility subject to their budget constraints and firms, with constant or decreasing returns to scale technologies, maximize profits subject to the prevailing prices. Then every competitive allocation is Pareto optimal and every Pareto optimal allocation can, with lump sum redistribution of endowments and share holdings, be supported as a competitive equilibrium. These two propositions, the first and second welfare theorems, form the foundation of neoclassical welfare economics. Unfortunately, in a world where some firms with increasing returns to scale technologies are price-setting profit maximizers, both of these theorems fail to be true. In this case, there is a need for government intervention, which may take the form of regulated pricing of firms with increasing returns to scale technologies.

In this paper, we use the two sector general equilibrium model to investigate the optimality of several pricing schemes that have been proposed for the regulation of a public monopoly. Here a public monopoly is taken to be a multiproduct firm having a nonconvex production set.

A recurring concern in the literature on public utility pricing is the merit of cross-subsidization. That is, if a public enterprise produces two or more products, each of which can be priced separately, is it in the public interest to allow this firm to satisfy only a single break even constraint over its total menu of outputs? Under this single constraint, some product lines may be making a profit while others may be sold at a loss. Moreover, the prices need not reflect the marginal cost to society of producing these outputs. Simply put: Should I have to pay for your consumption?

Invoking the benefit principle, some economists have argued against

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cross-subsidization. Other economists have argued that the government can use commodity taxes and cross-subsidization to improve social welfare. The prices which emerge from this approach are the so-called Ramsey prices. A recent criticism of Ramsey pricing is that it takes the number of firms as fixed and that cross-subsidization may induce socially undesirable entry, i.e., the public enterprise is a natural monopoly (see the illuminating discussion in Faulhaber [4]). If one considers the conditions for total second best optimality which includes the optimal number of firms, then the government should require the regulated prices to be sustainable, i.e., prices at which no other firm with the same technology can profitably enter the markets of the public enterprise. In the best of all possible worlds, one might hope that the Ramsey prices are sustainable. In which case, the government, using commodity taxation, has obtained total second best optimality.

Suppose the public enterprise produces only two products, say electricity and grain, from two inputs, capital and labor, which are inelastically supplied. The production function for electricity and the production function for grain both exhibit nonincreasing average costs. Hence, the production possibility set for this firm will, in general, be nonconvex. Assume also that this is the only active firm in the economy, but there are potential entrants. We assume that there are two consumers, each endowed only with capital and labor, who consume only electricity and grain.

Clearly, in this model, Ramsey pricing and sustainable pricing are incompatible, without some special assumption on the technology of the public enterprise. This can be seen as follows.

In order that the consumer prices of electricity and grain are sustainable, they must equal the average costs of producing the equilibrium outputs. Moreover, in this world sustainable prices require the factor price ratio facing consumers be the factor price ratio at which the equilibrium outputs are produced at minimum cost. Given the prices of electricity, grain, capital and labor, we lose one degree of freedom by normalization. The remaining degrees of freedom are consumed by the sustainable conditions. Hence, there is no freedom to trade off the elasticities of demand through cross-subsidization to obtain the Ramsey optimum¹.

Can we use excise taxes to both improve social welfare and prohibit socially undesirable entry? The answer is yes, if we remember that con-

¹ The incompatibility of Ramsey pricing and sustainable pricing, for separable cost functions, has been independently noted by Mirman-Tauman-Zang [7].

sumer prices and producer prices can be chosen independently in a world where there are no profits in equilibrium. This is the fundamental insight in the paper of Diamond and Mirrlees (D-M, [3]). Let q and p be consumer and producer prices, respectively. In D-M, the intended interpretation of p is that it is the vector of shadow prices at the social optimum. Indeed, this has been the prevailing interpretation throughout the optimal taxation literature and the related literature on public sector pricing, as for example, in Baumol-Bradford [1].

We suggest that in the case of a natural monopoly, the p in the D-M analysis should be interpreted as supporting prices² rather than shadow prices. Of course, no such p need exist. If it does exist (a sufficient condition in this model is that each product line is produced with nonincreasing average costs) then the excise tax $t = q - p$ is to be interpreted as the optimal excise tax imposed by the government to prevent socially undesirable entry. That is, q are the socially optimal prices prevailing in the market, i.e., Ramsey prices, but the prices facing the public utility and potential entrants is q net of t or p , which are supporting prices. It is easy to see that the net tax revenue is zero. Hence, there is no surplus to dispose of. If each product line is produced with nonincreasing average costs, then p is simply average cost pricing, i.e., each product is sold at average cost and produced at minimum cost. If the Ramsey prices happen to be sustainable, then we let $q = p$ and $t = 0$.

It may seem strange at first reading that the government should use a per-unit tax to prevent entry rather than simply forbidding entry or requiring a licensing fee, once it has determined that the public utility is a natural monopoly. But without the threat of potential entry, average cost pricing is not incentive compatible. Even if the public enterprise is breaking even, it need not be producing at minimum cost. Also, the government wishes to encourage innovation. If some entrepreneur discovers a more efficient technology for producing the outputs of the natural monopolist, we want her to enter the market. Finally, a license fee acts as a fixed charge for entry, hence innovations whose profitability does not exceed this fee will be lost to society.

We shall assume that the reader is familiar with the two-sector model as a general equilibrium model of increasing returns to scale, say, as in the paper of Brown-Heal [2]. In fact, the present paper

² A cost function C is supportable at y_0 , if there exists a price p such that (1) $p \cdot y_0 = C(y_0)$ and (2) $p \cdot z \leq C(z)$ for all $z \in \mathbb{R}_+^n$ such that $z \leq y_0$. The prices p are said to be supporting prices. Sustainable prices are supporting, but the converse need not hold, see Sharkey-Tesler [9] for a discussion of supportable cost functions.

should be viewed as a continuation of the investigations initiated there.

Before proving our major theorem concerning the use of commodity taxes to improve social welfare, we first discuss first-best situations where lump sum taxation is feasible. In the final section of the paper, we extend our theorem on supporting Ramsey optima to a larger class of general equilibrium models.

2. THE MODEL

We consider an economy with two households, and a single firm producing two products from two factors which are inelastically supplied by the households. The factors are capital (K) and labor (L). The products are grain (G) and electricity (E). Households have utility functions denoted by U_x and U_y , respectively. Endowments and shareholdings in the firm are given by (K_x, L_x) , (K_y, L_y) ; θ_x and θ_y . The production function of the firm is separable, where the production functions for the products are F_G and F_E or equivalently, cost functions C_G and C_E . Let $K = K_x + K_y$ and $L = L_x + L_y$.

We make the standard assumptions regarding households and production functions, with the following important exception:

Although we assume that the factor markets are competitive, i.e., that the production functions for both products exhibit diminishing marginal rates of technical substitution, we do not assume constant or decreasing returns to scale.

Under these assumptions, we construct the Edgeworth-Bowley box for production and the associated production possibility set for the firm. In general, the production set for the firm (public monopoly) is non-convex.

Let P_G and P_E denote the prices of grain and electricity, and w and r denote the prices of labor and capital.

A point (G, E) is said to be *production efficient* if it lies on the frontier of the firm's production possibility set. Each point on the frontier uniquely determines the factor price ratio at which these outputs are produced at minimum cost. Moreover, the marginal rate of transformation (MRT) at this point is equal to the ratio of the marginal costs.

The income distribution is said to be fixed if $(K_x, L_x) = \theta_x(K, L)$ and $(K_y, L_y) = \theta_y(K, L)$.

A *marginal cost pricing (MCP) equilibrium* is a family of consumption plans, production plans, prices, and lump sum taxes such that households are maximizing utility subject to after-tax income; the public monopoly is producing at minimum cost and selling at marginal cost, where losses are covered by the lump sum taxes; and all markets clear.

An *average cost pricing (ACP) equilibrium* is a family of consumption plans, production plans and prices such that households are maximizing utility subject to their budget constraint; the public monopoly is producing at minimum cost and breaking even; and all markets clear.

In general, neither MCP nor ACP equilibria will be Pareto optimal. ACP equilibria violate the first order conditions necessary for Pareto optimality and MCP equilibria need not satisfy the sufficient conditions for Pareto optimality, if the production possibility set is nonconvex.

3. WELFARE THEOREMS

PROPOSITION 1. *If the income distribution is fixed, then every Pareto optimal allocation, where both households consume a positive amount of each good, can be supported as a MCP equilibrium with lump sum transfers to households³.*

PROOF. If $(\bar{E}_X, \bar{G}_X), (\bar{E}_Y, \bar{G}_Y)$ is the Pareto optimal allocation, let $\bar{E} = \bar{E}_X + \bar{E}_Y, \bar{G} = \bar{G}_X + \bar{G}_Y$ and

$$A = \{ (E, G) : E = E_X + E_Y, G = G_X + G_Y \text{ where } U_X(E_X, G_X) \geq U_X(\bar{E}_X, \bar{G}_X) \text{ and } U_Y(E_Y, G_Y) \geq U_Y(\bar{E}_Y, \bar{G}_Y) \}.$$

Then (\bar{E}, \bar{G}) is on the production possibility frontier and A is a convex closed set which is tangent to the production possibility set at (\bar{E}, \bar{G}) . Let $w/r, P_E/r, P_G/r$ be the factor price ratio at which \bar{E} and \bar{G} are produced at minimum cost, the marginal cost of producing \bar{E} , and the marginal cost of producing \bar{G} . These prices define the budget constraint for households as:

$$\frac{P_E}{r} E_j + \frac{P_G}{r} G_j = \frac{w}{r} L_j + K_j + \theta_j \Pi + T_j,$$

where $j = x$ or y , where Π is the firm's profit and T_j is the lump sum transfer to household j . The firm is required to produce (\bar{E}, \bar{G}) at minimum cost and sell at marginal cost. Since the income distribution is fixed, the right hand side of the budget constraint reduces to:

$$\theta_j \left(\frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G} \right) + T_j.$$

³ This is a special case of Theorem 4 in Guesnerie [5].

Let

$$T_j = \frac{P_E}{r} \bar{E}_j + \frac{P_G}{r} \bar{G}_j - \theta_j \left(\frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G} \right) .$$

Recalling that the MRT at (\bar{E}, \bar{G}) is the ratio of the marginal costs, i.e., P_E/P_G , we see that the set A is supported by the price line with slope P_E/P_G . Hence, (\bar{E}_x, \bar{G}_x) and (\bar{E}_y, \bar{G}_y) maximize U_x and U_y , subject to the budget constraints as defined above. ■

PROPOSITION 2. *If the income distribution is fixed and the utility functions are concave and homogeneous of degree one, then there exists a Pareto optimal MCP equilibrium.*

PROOF. Under the assumptions on preferences and endowments, it follows from Eisenberg's theorem (see Theorem 3 in [8]) that the market demand function is generated by a utility function $U(E, G)$, where

$$U(E, G) = \max[U_x(E_x, G_x)]^{\theta_x} [U_y(E_y, G_y)]^{\theta_y}$$

such that $E_x + E_y = E$ and $G_x + G_y = G$.

Suppose U achieves its maximum over the production possibility set at (\bar{E}, \bar{G}) . Then, assuming monotonicity of U_x and U_y , (\bar{E}, \bar{G}) is on the production possibility frontier. Let (\bar{E}_x, \bar{G}_x) and (\bar{E}_y, \bar{G}_y) be the Pareto optimal allocation, corresponding to $U(\bar{E}, \bar{G})$. That is

$$U(\bar{E}, \bar{G}) = \max[U_x(\bar{E}_x, \bar{G}_x)]^{\theta_x} [U_y(\bar{E}_y, \bar{G}_y)]^{\theta_y} .$$

If w/r , P_E/r , and P_G/r is the factor price ratio at which \bar{E} and \bar{G} are produced at minimum cost, the marginal cost of producing \bar{E} and the marginal cost of producing \bar{G} , then the optimal solution of $\max U(E, G)$ such that

$$\frac{P_E}{r} E + \frac{P_G}{r} G = \frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}$$

is (\bar{E}, \bar{G}) , since U is a concave function of (E, G) . Now note that

$$\begin{aligned} U(\bar{E}, \bar{G}) &= \max\{[U_x(E_x, G_x)]^{\theta_x} [U_y(E_y, G_y)]^{\theta_y} : E_x + E_y = \bar{E} \text{ and } G_x + G_y = \bar{G}\} \\ &\leq \max\{[U_x(E_x, G_x)]^{\theta_x} [U_y(E_y, G_y)]^{\theta_y} : E_x + E_y = E, G_x + G_y = G \text{ and } \frac{P_E}{r} E + \frac{P_G}{r} G = \frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}\} \\ &\leq \max\{U(E, G) : \frac{P_E}{r} E + \frac{P_G}{r} G = \frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G}\} \\ &\leq U(\bar{E}, \bar{G}) . \end{aligned}$$

Hence, (\bar{E}_x, \bar{G}_x) and (\bar{E}_y, \bar{G}_y) is the optimal solution of

$$\max\{ [U_x(E_x, G_x)]^{\theta_x} [U_y(E_y, G_y)]^{\theta_y} : \frac{P_E}{r}(E_x + E_y) + \frac{P_G}{r}(G_x + G_y) = \frac{P_E}{r}\bar{E} + \frac{P_G}{r}\bar{G} \}.$$

Therefore, by Eisenberg's theorem the optimal solutions to

$$\max U_j(E_j, G_j)$$

$$\text{such that } \frac{P_E}{r} E_j + \frac{P_G}{r} G_j = \theta_j \left(\frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G} \right), \quad j = x \text{ or } y$$

are (\bar{E}_x, \bar{G}_x) and (\bar{E}_y, \bar{G}_y) . Recalling the fixed income distribution, we see that the right hand side of each household's budget constraint can be expressed as

$$\frac{w}{r} L_x + K_x + \theta_x \Pi \quad \text{and} \quad \frac{w}{r} L_y + K_y + \theta_y \Pi,$$

where Π is the profit of the natural monopoly that is producing (\bar{E}, \bar{G}) at minimum cost and selling at marginal cost. ■

PROPOSITION 3. *Every Pareto optimal allocation, where both households consume a positive amount of each good, can be supported as an ACP equilibrium with lump sum transfers to households⁴.*

PROOF. If (\bar{E}_x, \bar{G}_x) , (\bar{E}_y, \bar{G}_y) is the Pareto optimal allocation, let $\bar{E} = \bar{E}_x + \bar{E}_y$ and $\bar{G} = \bar{G}_x + \bar{G}_y$. Let w/r be the factor price ratio at which (\bar{E}, \bar{G}) is produced at minimum cost. Cross-subsidization gives us one degree of freedom in determining the relative prices for products, i.e., we only have the single break-even constraint

$$\frac{P_E}{r} \bar{E} + \frac{P_G}{r} \bar{G} = \frac{w}{r} L + K.$$

Hence, we can require that the product price ratio equals MRT at (\bar{E}, \bar{G}) , i.e., $\frac{P_E}{r} = \text{MRT} \frac{P_G}{r}$. These two equations uniquely determine the values of $\frac{P_E}{r}$ and $\frac{P_G}{r}$, where

$$\frac{P_G}{r} = \frac{\frac{w}{r} L + K}{(\text{MRT}) \bar{E} + \bar{G}}.$$

These prices define the following budget constraint for households as:

$$\frac{P_E}{r} E_j + \frac{P_G}{r} G_j = \frac{w}{r} L_j + K_j + T_j,$$

where T_j is the lump sum transfer to household j and $j = x$ or y . Since the firm breaks even, the profit $\Pi = 0$. Let

⁴ This and the next proposition do not generalize beyond the two-sector model.

$$T_j = \frac{P_E}{r} \bar{E}_j + \frac{P_G}{r} \bar{G}_j - \frac{w}{r} L_j - K_j .$$

Then (\bar{E}_j, \bar{G}_j) maximizes $U_j(E_j, G_j)$ subject to the budget constraint. ■

Proposition 3 was suggested to us by T. N. Srinivasan.

PROPOSITION 4. *If the income distribution is fixed and the utility functions are concave and homogeneous of degree one, then there exists a Pareto optimal ACP equilibrium.*

PROOF. As in the proof of Proposition 2, we invoke the Eisenberg aggregation theorem to obtain $U(E, G)$. We then maximize U over the production possibility set, obtaining a maximum at (\bar{E}, \bar{G}) on the production possibility frontier. Let w/r be the factor price ratio at which (\bar{E}, \bar{G}) is produced at minimum cost given the MRT at (\bar{E}, \bar{G}) , we set

$$\frac{P_E}{r} = \text{MRT} \frac{P_G}{r} \quad \text{and} \quad \frac{P_G}{r} = \frac{\frac{w}{r} L + K}{(\text{MRT}) \bar{E} + \bar{G}} .$$

As in the proof of Proposition 3, these relative product prices are chosen to make the firm break even and the product price ratio equal the MRT. The rest of the argument is the same as the proof of Proposition 2, noting that in the family of maximization problems the budget constraints depend only on the MRT at (\bar{E}, \bar{G}) . ■

4. SUPPORTING RAMSEY OPTIMA

THEOREM 1. *If the social welfare function is individualistic and C_E and C_G exhibit nonincreasing average costs, then every socially optimal allocation can be supported.*

PROOF. We shall find it convenient to represent the production possibility set for the firm by a smooth transformation function $H(E, G)$. The production possibility set is the set of (E, G) such that $H(E, G) \leq 0$ and the set's frontier is characterized by $H(E, G) = 0$. Let Δ be the price simplex in \mathbb{R}_+^4 and $q = (q_E, q_G, q_L, q_K)$ be the normalized household prices for electricity, grain, labor and capital. Since households are only endowed with factors which they supply inelastically, let $X(q)$ be the market demand for electricity and grain. Let

$$D = \{q \in \Delta : H(X(q)) \leq 0\} .$$

If $V(q)$ is an individualistic (indirect) social welfare function, then

the government's problem is to maximize $V(q)$ subject to $q \in D$. The set D is clearly compact, hence we only need to show that $D \neq \emptyset$ in order for the government's problem to have a solution. Moreover, it follows from Lemma 1 in D-M [3], that the social optimum if it exists will be on the frontier of the production possibility set, i.e., households are endowed with labor but do not consume leisure. Unlike D-M, we cannot show that $D \neq \emptyset$ by invoking the existence theorem for exchange economies since households are not endowed with electricity or grain. Instead, we choose q_L and q_K sufficiently small such that $X(q)$ is close to zero and therefore inside the production possibility set. Hence, $D \neq \emptyset$. Let \hat{q} be an optimal solution of $\max\{V(q) : q \in D\}$, then $X(\hat{q})$ is on the frontier of the production possibility set. Denote $X(\hat{q})$ as (\hat{E}, \hat{G}) and let \hat{P}_L/\hat{P}_K be the factor price ratio at which (\hat{E}, \hat{G}) is produced at minimum cost and $\hat{P}_E/\hat{P}_K, \hat{P}_G/\hat{P}_K$ be the average costs of producing \hat{E} and \hat{G} , respectively. Renormalizing q so that capital is the numeraire good, we have \hat{q}_L/\hat{q}_K as the factor price ratio facing households and $\hat{q}_E/\hat{q}_K, \hat{q}_G/\hat{q}_K$ as the prices which households face in the product markets. The optimal excise taxes are

$$\hat{t}_E = \hat{q}_E/\hat{q}_K - \hat{P}_E/\hat{P}_K, \quad \hat{t}_G = \hat{q}_G/\hat{q}_K - \hat{P}_G/\hat{P}_K \quad \text{and} \quad \hat{t}_L = \hat{q}_L/\hat{q}_K - \hat{P}_L/\hat{P}_K ;$$

capital is untaxed. Applying Walras' law and the break-even condition, we see that the tax revenue is zero.

It is clear that with nonincreasing average costs for both electricity and grain no part of (\hat{E}, \hat{G}) can be produced for profit at product prices less than \hat{P}_E/\hat{P}_K and \hat{P}_G/\hat{P}_K , with a factor price ratio of \hat{P}_L/\hat{P}_K . This completes the proof. ■

5. EXTENSIONS

In this section, we extend our theorem on supporting Ramsey optima in the two-sector model to general equilibrium models where factors may be elastically supplied; the aggregate production possibility set is characterized by a cost function; and this cost function need not be separable.

We shall follow McFadden in [6]. The economy has n commodities where k commodities are factors or inputs and l commodities are products or outputs. The k inputs will be denoted by v and the l commodities by y . We treat factors and products as distinct commodities. Consumer prices are denoted as q where $q = (q_v, q_y)$ and producer prices

are denoted as p where $p = (p_v, p_y)$.

The production possibility set Y is the set of pairs (v, y) which are technologically feasible. $V(y)$, the input requirement set, is the set of all v which can produce y , i.e., $V(y) = \{v: (v, y) \in Y\}$. We make the conventional assumptions on Y , i.e., Y is non-empty and closed, with non-zero outputs requiring non-zero inputs; the input requirement sets are strictly convex from below and admit free disposal.

If producer factor prices, p_v , are strictly positive and factor markets are competitive, then the cost function is

$$C(y, p_v) = \min_u \{p_v \cdot u: u \in V(y)\}.$$

In our discussion, it will be convenient to characterize the production possibility set in terms of transformation functions, as we did in Theorem 1. The distance function is defined by

$$F(y, v) = \text{Max}\{\lambda > 0: \frac{1}{\lambda}v \in V(y)\}$$

for v strictly positive. Let

$$G(y, v) = F(y, v) - 1.$$

Then $G(y, v) = 0$ is a transformation function characterizing the efficient points or "frontier" of Y . Now

$$C(y, p_v) = \min_u \{p_v \cdot u: F(y, u) = 1\}.$$

We say that $C(y, p_v)$ is supportable if for every fixed \bar{p}_v , $C(y, \bar{p}_v)$ is supportable for all y .

Suppose there is a finite number of households in the economy, where each household is defined by her utility function and her endowment of products and factors. We make the standard assumptions on tastes and endowments such that demand functions are well defined, continuous, and satisfy Walras' law.

THEOREM 2. *If the social welfare function is individualistic and the cost function $C(y, p_v)$ is supportable, then every efficient socially optimal allocation can be supported.*

PROOF. Let $X(\hat{q})$ be the efficient socially optimal allocation, i.e., $G(X(\hat{q})) = 0$ or equivalently $F(X(\hat{q})) = 1$. Define \hat{p}_v as the normal of the unique hyperplane which supports the input requirement set $V(\hat{y})$ at \hat{v} , where $X(\hat{q}) = (\hat{y}, \hat{v})$. Let \hat{p}_v be supporting prices at \hat{y} for the cost function $C(\hat{y}, \hat{p}_v)$.

The optimal excise taxes are $\hat{t}_y = \hat{q}_y - \hat{p}_y$ and $\hat{t}_v = \hat{q}_v - \hat{p}_v$. Ap-

plying Walras' law and the break-even condition: $\hat{p}_Y \cdot \hat{y} = C(\hat{y}, \hat{p}_Y)$, we see that the net tax revenue on net demands is zero. This completes the proof. ■

Sufficient conditions for $C(y, p_Y)$ to be supportable are given in Sharkey-Telser [9]. One such condition is cost complementarity, i.e.,

$$\frac{\partial C}{\partial y_i \partial y_j} \leq 0$$

for all i and j , as in the two-sector model with nonincreasing average costs.

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REFERENCES

1. BAUMOL AND BRADFORD, "Optimal Departures from Marginal Cost Pricing," *AER* 60, 1970, 265-283.
2. BROWN AND HEAL, "Marginal vs. Average Cost Pricing in the Presence of a Public Monopoly," *AEA Papers and Proceedings* 73, 1983, 189-193.
3. DIAMOND AND MIRRLEES, "Optimal Taxation and Public Production, I: Production Efficiency," *AER* 61, 1961, 8-27.
4. FAULHABER, "Cross-Subsidization: Pricing in Public Enterprise," *AER* 65, 1975, 966-977.

5. GUESNERIE, " Pareto Optimality in Non-Convex Economies," *Econometrica* 43, 1975, 1-29.
6. McFADDEN, " Cost, Revenue, and Profit Functions " in *Production Functions* edited by Fuss/McFadden, North-Holland, 1971.
7. MIRMAN, TAUMAN, AND ZANG, " Ramsey Prices, Average Cost Prices, and Price Sustainability," *Northwestern Discussion Paper* No. 561, May 1983.
8. SHAFER AND SONNENSCHNEIN, " Market Demand and Excess Demand Functions," Chapter 14 in *Handbook of Mathematical Economics, Vol. II*, North-Holland, 1983, 671-692.
9. SHARKEY AND TELSER, " Supportable Cost Functions of a Multiproduct Firm " *JET* 18, 1978, 23-37.

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