ON A SIMPLE RULE FOR THE OPTIMAL INFLATION RATE IN SECOND BEST TAXATION

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The optimal inflation rate is analyzed in the framework of dynamic second best with endogenous factor prices. It is shown that when the marginal excess burden of taxation is relatively small, the optimal inflation rate is approximated by a simple rule. The paper also analyzes the robustness of this rule to the specification of the model (money as an input in utility or production).

1. Introduction

The problem of the optimal inflation rate has been analyzed in numerous studies. The purpose of this paper is to reconsider the issue from the point of view of efficient second-best taxation. The inflation tax which is equivalent to money creation in an equilibrium framework, represents one instrument to be optimized in consideration of other distortionary taxes. In order to concentrate on efficiency, the standard assumption of a single private agent will be used here.

Among the studies which recently addressed this problem, we can single out the two main ones. In the first, Phelps (1973) derives a simple rule which is consistent with the standard results of Ramsey (1927). His method is to optimize the inflation rate together with the other distortionary tax rates, over the set of steady states (with exogenous factor prices). In general, this solution is different from the long-run (permanent) value of the optimal

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Tobin (1968) assumes imperfect intertemporal markets and an ad hoc savings function. Summers (1981) provides a quantitative evaluation of this argument. Drazen (1981) considered a framework of optimal taxation in a life-cycle model. This raises specific issues when the government debt cannot be optimized. Standard optimal taxation formulae do not apply [see Atkinson and Sandmo (1980)].

1The assumption of a single family is valid when individuals have finite lives, but have an operative bequest motive [Barro (1974)]. The framework used in the present paper represents an extension of the work of Sdrasuski (1967) to the second-best

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inflation rate in a dynamic model. Also, it does not take into account general equilibrium effects.\footnote{Drazen (1979) uses a dynamic model of general equilibrium. However, a close examination of the constraints taken into account reveals that his approach relies on a steady-state criterion [compare with the constraints in the problem (P) below].}

Turnovsky and Brock (1980) do not analyze the standard problem of second-best taxation because the initial level of the government debt is a fiscal instrument. This is equivalent to allowing a lump-sum capital levy. The solution is then obviously first best and generates the Friedman rule, as shown by Sidrauski (1967).\footnote{See Turnovsky and Brock (1980, p. 208). They consider also other second-best situations as the optimization of the inflation rate when the tax rate on labor income is exogenous.} Also, Turnovsky and Brock do not consider a model with capital — an important feature of the model presented here.

This paper relies on a stylized approach which highlights the theoretical arguments and evaluates their quantitative importance. The role of money is to free resources (produced goods and time) from transaction activities for production or consumption. An accurate description of these services can only be made in a specific context. The well-known inventory model of Baumol and Tobin is such an example. Although it has been extended by others, none of these models seems to be entirely satisfactory, given the complexity of transaction technologies.\footnote{See, for example, Barro (1972) and Jovanovic (1982)}

In view of these difficulties, the standard method in the second-best approach to inflation is to consider money as an argument in the utility function or in the production function. The rationale for this is clear; for example, a higher level of cash balances frees time for leisure and generates a higher level of utility (or it frees labor for other productive tasks). By leaving the form of the utility function or the production unspecified, we can hope to derive results which are sufficiently general. This is the approach taken here. It is validated by results which are expressed in terms of familiar parameters such as the interest elasticity of money or the marginal efficiency cost of taxation. The formulae do not appear to be too sensitive to the exact specification of the stylized model.

The paper is organized as follows. A simplified model where the utility function is separable between consumption, leisure and money is presented in section 2. The long-run value of the inflation rate is characterized by a simple formula. The requirements for time-consistency are also considered. The sensitivity of the basic formula to more general specifications of the model is analyzed in section 3. Further extensions are suggested in the concluding section.

2. A simple model of general equilibrium

2.1. The role of money

Fiat money is issued by the government which provides transaction
services. These services free resources (produced goods and time), for personal consumption or for production.

As an illustration, consider the Baumol–Tobin model where there is a fixed period, \( T \), between two consecutive payments. The average money holding during the payment period is equal to

\[
m = \frac{cT}{2n},
\]

where \( c \) and \( n \) represent the consumption level and the number of transfers between money and interest bearing assets during the period between two payments, respectively. When the cost of each of these transactions is equal to \( \theta \), the total cost per unit of time, is equal to

\[
A = \frac{\theta n}{T} = \frac{\theta c}{2m}.
\]

This cost can be incurred in different ways. Assume first that it is paid in time (trips to the bank). The level of leisure is then equal to \( 1 - l - \frac{\theta c}{2m} \), where \( l \) represents the quantity of time allocated to production and the time endowment per period is normalized to one. The current utility function per unit of time can be written in arguments of consumption, labor supply and money:

\[
u = u(c, 1 - l - \frac{\theta c}{2m}) = u(c, l, m). \tag{3}\]

As an alternative, assume now that the transaction costs reduce the effective level of consumption. As in the previous case, the utility level can also be written as a function of consumption, labor and cash balances:

\[
u = u(c - \frac{\theta c}{2m}, 1 - l) = u(c, l, m). \tag{4}\]

In the special logarithmic case,

\[
u = \gamma \log \left( c - \frac{\theta c}{2m} \right) + \log(1 - l)
\]

\[
= \gamma \log c + \log(1 - l) + \gamma \log \left( 1 - \frac{\theta}{2m} \right) \tag{5}\]

the function is additively separable in \( c, l \) and \( m \).
The above examples are obviously crude descriptions of transaction technologies and are given only as illustrations. In the rest of the paper, the level of utility per unit of time is a function of the consumption of produced goods $c$, of the labor devoted to the production of goods, and of the level of real cash balances.

We should also mention that money provides transaction services in the exchange of intermediary investment goods. Accordingly, money can also be considered as a productive input. Such an approach would lead to results very similar to those presented here. This is indicative of their robustness to the exact specification of the model.

For the sake of simplicity in this section, the utility function will be assumed to be additively separable in money:

$$u = u(c, l) + s(m).$$

This assumption will be relaxed in the next section.

2.2. The model

In order to address efficiency issues, all individuals have identical utility functions up to a proportionality factor. Also, they have an operative bequest motive à la Barro. They are aggregated into a single infinitely lived household with utility function equal to

$$U = \int_{0}^{\infty} e^{-(\rho - \rho_0) t} [u(c_t, l_t) + s(m_t)] dt,$$  

(6)

where $l$ and $g$ represent the discount rate and the growth rate, respectively ($\rho > g$). All quantities are measured per capita.

There is one good in the economy. The production technology is represented by a neoclassical function, $y = f(k, l)$, with the usual properties. The gross factor prices are determined by the marginal productivities:

$$r = \frac{\partial f}{\partial k},$$  

(7)

$$w = \frac{\partial f}{\partial l}.$$  

(8)

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6The Baumol–Tobin model is a special case of the more general models developed by Barro (1974, p. 981), and Jovanovic (1982, p. 567).
7See Fisher (1976).
8This method was used in an earlier version of the paper [Chamley (1982)].
9The functions $u(c, l)$ and $s(m)$ are concave.
10For simplicity, there is no technological change. The existence of an optimal balanced growth path with labor augmenting technological change requires strong assumptions (a unitary elasticity of substitution between consumption and leisure is a sufficient condition).
11The function $f$ is quasi-concave in $k$ and $l$. It does not need to have constant returns to scale, but it is not convex in $k$ and $l$. Also, for simplicity $\lim f'(k, l) = \infty$ when $k$ tends to zero, and $\lim f'(k, l) = \infty$, when $l$ tends to zero.
The representative household takes the endogenous factor prices as given and is endowed with perfect foresight.

The role of the government is to finance an exogenous constant stream of public consumption using a labor tax and the creation of fiat money.\textsuperscript{12} The problem is to determine at a given instant chosen arbitrarily as the origin of time, the policy of taxation and money creation which optimizes the private sector's utility subject to the government's budget constraint. It is a straightforward application of the standard efficient tax problem to the intertemporal framework. As in the static case, there is only one budget constraint which applies for the government over the policy horizon. Since the flow of receipts and expenditures may not coincide in the efficient solution, the government trades between different instants (as private individuals) at the interest rate $r$. For this, the government issues bonds which are perfectly substitutable with capital and have the same real return (there is no uncertainty).

The level of the debt at the origin of time $b_0$ must be taken by the government as exogenous. If this level were a fiscal instrument, the government could in effect implement a capital levy; this is excluded by assumption in the second-best framework.\textsuperscript{13}

The variation of the debt (per capita) is equal to \textsuperscript{14}

\[ b = (r - g)b + a - (w - \bar{w})l - Q, \]

where $\bar{w}$ represents the net wage rate (determined by the tax policy), $a$ is the level of public consumption, and $Q$ is the level of revenues generated by the creation of money. It is equal to

\[ Q = nm = \dot{m} + (g + \pi)m, \tag{9} \]

where $n$ is the nominal rate of growth of money (per capita) and $\pi$ is the inflation rate.

2.3. The problem of second best

As in the standard atemporal second-best problem, the policy-maker has to take into account the constraints imposed by the optimizing behavior of the private sector. In the dynamic framework, it is most convenient to

\textsuperscript{12}The labor tax is considered here to model a second-best tax which is an alternative to money creation. Other instruments could be introduced (a tax on consumption or capital income), but this would only complicate the analysis [for a comparison of these taxes, see Chamley (1983)].

\textsuperscript{13}This levy can be implemented by defaulting on the debt or by a decree that the private sector owes a given amount to the government (see also footnote 19).

\textsuperscript{14}A dot represents a time derivative. Whenever convenient, the time subscript will be omitted.
express the first-order conditions of this optimization as follows:\footnote{These constraints are derived immediately from the first-order conditions in the solution of the optimal control problem of the private sector}

\begin{align}
    u_1(c, l) &= q, \\
    u_2(c, l) &= -q\bar{w}, \\
    s'(m) &= q(r + \pi), \\
    \bar{q}/q + r &= \rho.
\end{align}

The variable $q$ is the marginal utility of accumulated assets (the sum of capital and bonds), for the private sector. The first two equations correspond to the intratemporal first-order conditions for consumption and leisure. The third equation determines the optimal level of cash balances, and the last equation corresponds to the intertemporal first order conditions.

Using the first two equations, $c$ and $l$ can be replaced as functions of $q$ and $\bar{w}$:

\begin{align}
    c &= c(q, \bar{w}), \\
    l &= l(q, \bar{w}), \\
    u &= u(c, l) = u(q, \bar{w}).
\end{align}

From eq. (12), the demand for the real quantity of money depends only on $q$ and on the nominal interest rate $i = r + \pi$:

\begin{equation}
    m = \psi(q, i).
\end{equation}

Since the real rate of return $r$ depends on the input levels $k$ and $l$, and the labor supply is a function of $q$ and $\bar{w}$ in (15), the demand for the real cash balances can also be expressed as a function of $k$, $q$, $\bar{w}$ and $\pi$:

\begin{equation}
    m = \frac{M}{P} = \varphi(k, q, \bar{w}, \pi),
\end{equation}

where $M$ and $P$ represent the nominal quantity of money and the price level, respectively. Note that $\partial \varphi/\partial \pi$ is equal to $\partial \psi/\partial i$.

The levels of the variables $P$, $M$, $q$, $\bar{w}$ and $\pi$ at time zero, are exogenous. This assumption is made only to simplify the exposition at this step. It will be re-examined below. The inflation rate at time zero is also exogenous and is determined implicitly in (18).
Differentiating eq. (18) with respect to time:

\[(n - \pi)m = \phi_1'k + \phi_2'\dot{q} + \phi_3'\dot{w} + \phi_4'\dot{\pi} \tag{19}\]

This relation shows that for given values of the other endogenous variables (levels and variations), there is a one-to-one relation between the nominal money growth rate, \(n\), and the variation of the inflation rate. Although the government controls \(n\), it is equivalent and more convenient to assume in the second-best problem that the government controls \(\dot{\pi}\).\(^{16}\) The other instrument is the variation of the net wage rate \(\dot{w}\). As mentioned above, the initial values \(\pi_0\) and \(w_0\) are given.

The second-best problem can now be formulated:

\[\text{(P)} \quad \text{maximize} \quad U = \int_0^\infty e^{-\rho t} e^{\alpha t} \left[ v(q, \bar{w}) + s(m) \right] dt, \]

subject to

\[\dot{k} = f(k, l) - gk - c - a, \tag{20}\]
\[\dot{b} = (r - g)b + a - (w - \bar{w})l - (g + \pi)m - \dot{m}, \tag{21}\]

with

\[\dot{m} = \phi_1'k + \phi_2'\dot{q} + \phi_3'\dot{x} + \phi_4'\dot{z}, \tag{22}\]
\[\dot{q} = q(p - r), \tag{23}\]
\[\dot{w} = x, \tag{24}\]
\[\dot{\pi} = z. \tag{25}\]

The variables \(c\) and \(l\) are functions of \(q\) and \(\bar{w}\) [in (14) and (15)], and the gross factor prices, \(w\) and \(r\), depend on \(k\) and \(l\).

It will be assumed that there is a unique dynamic path which satisfies the first-order conditions of the problem \(\text{(P)}\), and which converges to a stationary state.\(^{17}\) This path satisfies obviously the transversality conditions of the problem \(\text{(P)}\).

\(^{16}\)The second-best problem is also identical when the government levies directly a tax on money.

\(^{17}\)This property could be proven locally by considering the eigenvalues of the dynamic system linearized at the steady state. Since there are five state variables and an equal number of costate variables, there are 10 eigenvalues, and an algebraic analysis of their sign is beyond the scope of this paper. The local uniqueness of a convergent path is proven in a similar model without money when the excess-burden of taxation is not too large [Chamley (1983)].
Note also that the first-order conditions of the private sector are imbedded in the functional forms of $c, l, m$ and in eq. (22). The budget constraint of the private sector is satisfied when the dynamic path converges to a steady state, and does not have to be written explicitly. The same holds for the budget constraint of the government.

In the problem (P) the initial values $k_0, b_0, q_0, \bar{w}_0$ and $\pi_0$ are exogenous. The instruments of the problem are the paths of $x$ and $z$. As in similar studies, a distinction should be made between the mathematical formulation of the second-best problem (P) and its decentralization. For example, the variable $q$ is endogenous. However, its value is not controlled directly by the government. After the computation of the solution to (P), the government announces the programs of the wage tax and the inflation rate (or the money growth rate). The private sector then chooses the same program (and the same values of $q$), as in the solution to (P) because its optimizing behavior is taken into account in the second-best problem (P).

The solution is determined by using the current value Hamiltonian:

$$H = v(q, \bar{w}) + s(m) + \lambda\bar{k} - \mu\bar{b} + \xi\bar{q} + \alpha x + \beta z,$$

where $\lambda, \beta$ and $\xi$ are expressed in (20), (21) and (22).

By straightforward substitution, this expression can be rewritten:

$$H = v(q, \bar{w}) + s(m)$$

$$+ (\lambda + \mu \phi'_1)(f(l, k) - gk - c - a)$$

$$- \mu((r - g)b + a - (w - \bar{w})l - (g + \pi)m)$$

$$+ (\xi + \mu \phi'_2)q(\rho - r)$$

$$+ (\alpha + \mu \phi'_3)x$$

$$+ (\beta + \mu \phi'_4)z.$$  \hspace{1cm} (26)

The state variables of the problem are $k, b, q, \bar{w}$ and $\pi$. Their respective shadow prices are denoted by $\lambda, \mu, \xi, \alpha$ and $\beta$. The variable $\lambda$ represents also the social marginal value of the (unique) good in the economy. Because of the second-best nature of the problem, $\lambda$ is in general, different from the private marginal value of the good, $q$. The variable $-\mu$ represents the social marginal value of the public debt. It is also equal to the marginal excess
burden of taxation.\textsuperscript{18} (The first-best solution is attained when $\mu$ is equal to zero.)\textsuperscript{19}

2.4. The optimal inflation rate

Among the dynamic equations which define the solution to (P), two of them characterize more specifically the optimal inflation rate:

$$\frac{\partial H}{\partial \pi} = (\rho - g)\beta - \beta,$$

(27)

$$\frac{\partial H}{\partial z} = \beta + \mu \frac{\partial \phi}{\partial \pi} = 0,$$

(28)

Substituting for $\beta$ in (27):

$$\frac{\partial H}{\partial \pi} + (\rho - g) \mu \frac{\partial \phi}{\partial \pi} = -\beta.$$

(29)

In the steady state, this relation can be rewritten, after simple manipulations, as

$$\varepsilon = \frac{\nu}{1 + \nu},$$

(30)

where $\varepsilon$ is the interest elasticity of the demand for money,

$$\varepsilon = -\frac{(r + \pi)}{m} \frac{\partial}{\partial L} \psi(q, l),$$

and $\nu$ is the marginal efficiency cost of taxation expressed in consumption equivalent:

$$\nu = \mu / q.$$

\textsuperscript{18}The simplest method to see this is to introduce a term of lump-sum taxation, $\tau$, in the deficit equation (21) $b = (r - n)b + a - (w - \bar{w})l - (g + \pi)m - m - \tau$. When $\tau$ increases, other (distortionary) tax revenues can be decreased by the same amount. The gain is defined as the marginal efficiency cost of taxation and is equal to $\mu$. The marginal excess burden expressed in consumption equivalents is equal to $\mu / q$ and is constant over time [for a further discussion see Chamley (1983)].

\textsuperscript{19}Note also that if $b_0$ is an endogenous fiscal instrument, it is chosen such that $\mu$ is equal to zero. This is obvious since the situation corresponds to the case of a lump-sum capital levy.
In the interpretation of this simple formula, we should bear in mind that \( \varepsilon \) is the partial interest elasticity of the demand for money, keeping constant the marginal utility of consumption \( q \), and that the marginal excess burden is measured in equivalent of private consumption, \( v \). When the excess burden of other taxes tends to infinity, the government maximizes the revenues from money creation and \( \varepsilon \) is equal to one [Bailey (1956)]. When lump-sum taxation is feasible, \( v \) is equal to zero and \( \pi = -r \) [Friedman (1969)].

The optimal formula (30) expresses the optimal inflation rate in the long run as a function of \( q \) and \( v \). It is the steady-state solution of a dynamic problem. The interested reader could show as an exercise that a method using comparative statics between steady states would lead to a very similar result.

The initial levels of \( k_0 \), \( b_0 \), \( q_0 \), \( \bar{w}_0 \), and \( \pi_0 \) do not appear directly in the formula (29). However, they affect the marginal cost of taxation (which is constant over time), and therefore the long-run value of the optimal inflation rate (and also the other endogenous variables of the model). The constraints on \( q_0 \), \( \bar{w}_0 \), and \( \pi_0 \) (which are taken as an assumption in the second-best problem), are reconsidered in the next section.

2.5. Time-consistency and policy constraints

The issue of time consistency has been raised in numerous contexts of second best and is now familiar.\(^{20}\) The solution described in the previous section is time consistent because of the constraints on the initial values \( q_0 \), \( \bar{w}_0 \), and \( \pi_0 \) (the constraints of the initial value of the capital stock and the public debt are of a different type and were discussed in section 2.2). The purpose of this section is to discuss the minimum set of constraints which generates a time-consistent solution.

Let us consider the case where the government has been following an optimal policy described by the solution to (P), and that at some instant normalized at zero it faces the possibility of changing this policy. Assume that the government is honest in the sense of Auernheimer (1974), and takes the price level (inherited from past policy) as given. The previous assumption of fixed \( M_0 \) and \( b_0 \) is relaxed to admit the following operation: if a new policy is implemented, the supply of money is varied (by an open-market operation with the bonds), in order to maintain the value of \( P_0 \) invariant to the tax reform.

Consider now a tax reform with a marginal increase of \( \bar{w}_0 \), normalized at unity. This marginal change induces an increase of social welfare equal to the shadow price \( \lambda \). But at the same time the demand for the real quantity of money in eq. (18) increases by \( \psi' \) (algebraically). The government issues the

\(^{20}\)It is discussed for the inflation tax, the capital income tax and the wage tax by Auernheimer (1974) and Calvo (1978), Fisher (1974), and Chamley (1983), respectively.
same quantity of money and reduces the level of the debt by an equivalent quantity. The last operation increases the level of social welfare by an amount \( \mu \sigma^\gamma \). The total incidence on social welfare is equal to \( \alpha + \mu \sigma^\gamma \). This term is equal to zero because \( \partial H / \partial x = 0 \) [where \( H \) is defined in (26)]. No welfare gain is achieved by altering the level of \( \ddot{\omega} \) to its value prescribed in a previous optimal program, when the price level is constrained to vary continuously over time.

The same argument applies to the inflation rate. Therefore the continuity of \( P \) and \( q \) is sufficient to guarantee the time-consistency of the solution.

This condition is obviously also necessary. If \( P \) is not continuous, the government can raise capital levies by jumps in the nominal quantity of money. If \( q \) is not continuous, the government can induce an indirect capital levy because the incidence of the wage tax falls partially on capital when factor prices are endogenous [Chamley (1983)].

In conclusion, the solution of the second-best problem, where the variation of the price of money and the private marginal value of assets \( q \) are continuous, is identical to the solution of the problem (P) and is time-consistent. These conditions are also necessary for time-consistency.

3. Generalizations

3.1. Non-separable utility function

The assumption of additive separability was introduced in the previous section for the sake of simplicity. It is now relaxed, and the current utility level is expressed by a general concave function, \( u = u(c, l, m) \). The program of the representative household satisfies the first-order conditions:

\[
\begin{align*}
    u'_1(c, l, m) &= q, \\
    u'_2(c, l, m) &= -q\ddot{w}, \\
    u'_3(c, l, m) &= q(r + \pi), \\
    \dot{q}/q + r &= \rho.
\end{align*}
\]

Using the first two equations, the levels of consumption and labor supply can be expressed as functions of \( q, \ddot{w} \) and \( m \):

\[
\begin{align*}
    c &= c(q, \ddot{w}, m), \\
    l &= l(q, \ddot{w}, m).
\end{align*}
\]
Substituting in eq. (33), c and l by these functions, and noting that r is a function of k and l, eq. (33) defines implicitly the demand for real cash balances as a function of k, q, \( \bar{w} \) and \( \pi \):

\[
m = \phi(k, q, \bar{w}, \pi).
\] (37)

The problem of second best is then a straightforward extension of the problem analyzed in the previous section. Assuming constant returns to scale in the production technology, the optimal value of inflation in the steady state is given by the formula\(^{21}\)

\[
\varepsilon = \frac{\sqrt{1 - \delta}}{1 + \nu + A},
\] (38)

with the notation,

\[
\delta = \left[ \frac{m}{l} \frac{\partial l}{\partial m} \right] \left[ \frac{k}{m} \frac{\partial \phi}{\partial k} \right],
\]

\[
A = \frac{1}{(r + \pi) q} \left[ (\lambda - q + \mu \phi_k')(w_l' - c_m') + q + \mu(w - \bar{w})l_m' \right].
\]

Formula (38) is an extension of (30). In normal cases the quantitative difference between their respective right-hand sides is not very significant, as is illustrated by two examples.

3.2. Two examples

Consider first the case where transaction costs reduce only the quantity of 'effective' consumption which is then equal to \( cs(m) \) \( [s(m) < 1, \; s'(m) > 0] \). Furthermore, the utility function is additively separable between effective consumption and leisure, and inelastic in consumption:

\[
u = [cs(m)]^{1 - \gamma} + bh(l).
\]

After a simple transformation, formula (38) can be written:

\[
\varepsilon = \frac{\nu}{1 + \nu + \frac{1 - \gamma}{\gamma} \left( 1 - \frac{\lambda}{q} - \nu \phi_k' \right)}.
\] (39)

\(^{21}\)It is derived in the appendix.
when the marginal efficiency cost $v$ tends to zero, the solution tends to the first best, and the difference between the private value of capital, $q$, and its social value, $\lambda$, tends to zero. Therefore the first term of a Taylor expression of the right-hand side in (39) is equal to $v$. For relatively small values of the marginal excess burden of taxation, the optimal long-run inflation rate can be approximated by

$$\varepsilon = v.$$  \hspace{1cm} (40)

Assume now that the transaction costs reduce the quantity of available leisure (for a given level of labor in production), and that the utility function is of the form

$$u = h(c) + (1 - l + s(m))^{1 - \gamma} \quad (s(m) > 0).$$

The term $\delta$ is very small (it is of the order of magnitude of the ratio between money services and labor income net of taxes $(r + \pi) m / \bar{w})$.\footnote{The value of $\delta$ is obtained by differentiation of the first-order condition for labour, $(1 - \gamma) (1 - l + s(m))^{-\gamma} = q\bar{w}$, using the first-order condition for money, $(1 - \gamma) (1 - l + s(m))^{-\gamma} s(m) = q(r + \pi)$} The term $A$ is now equal to

$$A = \left[ \left( \frac{\lambda}{q} + v\phi_k \right) \frac{w}{\bar{w}} - 1 + v \left( \frac{w - \bar{w}}{\bar{w}} \right) \right].$$ \hspace{1cm} (41)

When $w$ tends to $\bar{w}$, the excess burden becomes small. The argument used in the previous example applies the optimal inflation rate is well approximated by (40).

3.3. Money as an input for production

In the model presented in the previous sections, the services of money depend only on the transactions in consumption and labor. While money may be especially useful as a medium of exchange for consumption goods, this framework omits the role of money for transaction in intermediate products or investment goods. This role can be represented by including money as an argument in the production function. The analysis is parallel to the one in this paper, and the results are very similar. The optimal value of the inflation rate in the steady state is determined by a formula analogous to (30):\footnote{For a full description, see Chamley (1982).}

$$\varepsilon = \frac{v}{1 + v + A}.$$ \hspace{1cm} (42)
The term \( A \) is positive when money is complementary with capital and labor, and it is of the same order of magnitude as the marginal efficiency cost, \( v \). This implies that the equality \( e = v \) provides a first-order approximation of the optimal inflation rate (in \( v \)), and in any case an upper bound for this value.\(^{24}\)  

4. Conclusion

The paper has analyzed the problem of the optimal inflation rate in a stylized dynamic model of second best with endogenous factor prices. The results show that the interaction between money on the one hand and consumption, labor and factor prices on the other matters only when the marginal efficiency cost of taxation is large. In this case, the formula which determines the revenue maximizing inflation is different from the partial equilibrium result [Bailey (1956)].

When the efficiency cost of taxation is relatively small, the partial equilibrium approach gives a good approximation of the optimal result, whatever the exact specification for the role of money in the model.\(^{25}\) This result is especially useful because there is more information on the empirical values of the interest elasticity of money and the efficiency cost of taxation than on the technologies of transactions.

The present model could be extended in at least two directions. The first would consider inside money. Calvo and Fernandez (1983) show that the introduction of commercial banks (with an interest paid on deposits) has the effect of decreasing the interest elasticity of the demand for high powered money, which is the inflation tax base. Other policies could also be analyzed in this context.

Another task would be to consider the transition effects in an economy with imperfect adjustments. For example, the existence of a short-term Phillips curve alters the optimal value of the long-run inflation rate. Feldstein (1976) has argued that with a low value of the discount rate, these transition effects are negligible with respect to the long run. Such policy debates rely in general on the simple framework of a steady state with exogenous factor prices. This may be a reasonable assumption in view of the results of this paper.

Appendix: The long-run inflation rate in the general case

The Hamiltonian associated with the optimization problem takes the same form as in (26), with two minor alterations: the term \( r(q, w) + s(m) \) is replaced

\(^{24}\)This assumes that the elasticity of income with the inflation rate, as in the models of Cagan (1956) or Barro (1972).  
\(^{25}\)In some cases (for example, when money is an input in production), the partial equilibrium value is greater than the value found in general equilibrium.
by the function \( v(q, \tilde{w}, m) = u(c(q, \tilde{w}, m), l(q, \tilde{w}, m), m) \), and the variables \( c \) and \( l \) are functions of \( q, \tilde{w} \) and \( m \).

The relation (29) becomes (with \( \beta = 0 \) in the steady state):

\[
\phi'_m [(\lambda + \mu \phi'_k)(w_l - c'_m) - \mu(br'_i(l^i - (w - \tilde{w}))l'_m
\]

\[-(\xi + \mu \phi'_q)qr'_i l'_m + (g + \pi)) + \mu m + (p - g) \mu \phi'_n = 0. \tag{A.1}
\]

In order to eliminate the shadow price \( \xi \) from this expression, we can consider another dynamic equation:

\[
\partial H / \partial k = (\rho - g) \lambda - \lambda, \tag{A.2}
\]

which is equivalent in the steady state, to

\[
(\lambda + \mu \phi'_k)(r - g) - \mu(br'_k - lw'_k) - q(\xi + \mu \phi'_q)r'_k
\]

\[+ \phi'_k [(\lambda + \mu \phi'_i)(w_l - c'_m) - \mu(br'_i(l^i - (w - \tilde{w}))l'_m
\]

\[-(\xi + \mu \phi'_q)qr'_i l'_m + \mu(g + \pi)]
\]

\[= \lambda(\rho - g). \tag{A.3}
\]

Reducing the terms \( \phi'_q r'_i \mu \) in (A.1) and \( \lambda(\rho - g) \) in (A.3) (since \( r = \rho \) in the steady state), these two equations can be rewritten:

\[
\phi'_k D + \mu m = q(\xi + \mu \phi'_q)r'_i l'_m \phi'_m. \tag{A.4}
\]

with

\[
D = \mu(r + \pi) + v'_m + (\lambda + \mu \phi'_k)(w_l - c'_m) - \mu(br'_i(l^i - (w - \tilde{w}))l'_m \tag{A.5}
\]

and

\[
\phi'_k D - \mu(br'_k - lw'_k) = q(\xi + \mu \phi'_q)(r'_k + r'_i l'_m \phi'_k). \tag{A.6}
\]

Taking the ratio between (A.6) and (A.4),

\[
\frac{\phi'_k D - \mu(br'_k - lw'_k)}{\phi'_k D + \mu m} = \frac{r'_i + r'_i l'_m \phi'_k}{r'_i l'_m \phi'_m},
\]

which is equivalent to

\[
r'_i(\phi'_k D + \mu m) + \mu mr'_i l'_m \phi'_k + \mu(br'_k - lw'_k)r'_i l'_m \phi'_k = 0. \tag{A.7}
\]
Since the production function has constant returns to scale, \( r'_1 = w'_1 = -(k/l)r'_k \). Also, \( v'_m = qe'_m - q\bar{w]'_m + q(r + \pi) \). Using these identities, a simple manipulation transforms (A.7) into the formula (38), presented in the text.

References


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