

PERFECT COMPETITION IN STRATEGIC MARKET GAMES WITH INTERLINKED PREFERENCES

Pradeep DUBEY and Martin SHUBIK

Cowles Foundation for Research in Economics, New Haven, CT 06520, USA

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When the competitive system is viewed as a strategic market game with a continuum of agents, concern for others need not influence outcomes. The generalization of preference conditions is noted and discussed.

This is a remark which, once made, is entirely obvious but merits being put on record. Consider a strategic market game with a continuum of traders T [see, e.g., Dubey, Mas-Colell and Shubik (1980)]. Then the map from strategy-selections to net trades satisfies the condition:

(a) If a single trader unilaterally changes his strategy, this does not affect the trades of any of the others.

We consider two games $\Gamma, \tilde{\Gamma}$ which differ only in the utilities u^t, \tilde{u}^t ascribed to traders $t \in T$. Let l be the number of commodities in the economy. Then $u^t: R_+^l \rightarrow R$ is the standard utility of personal consumption of the Walrasian model. On the other hand, we incorporate in \tilde{u}^t the possibility that t 's utility may depend also on others' consumption. Thus \tilde{u}^t is defined on pairs (α, x) , where $\alpha \in R_+^l$ is t 's consumption, and $x: T \setminus \{t\} \rightarrow R_+^l$ gives the consumption by others. Assume that u^t, \tilde{u}^t are 'compatible' for each t in the following sense:

(b) For any $x: T \setminus \{t\} \rightarrow R_+^l$, the function $\tilde{u}_x^t: R_+^l \rightarrow R$ given by $\tilde{u}_x^t(\alpha) = \tilde{u}^t(\alpha, x)$ induces the same preference relation on R_+^l as does u^t . (This says, in particular, that a trader never considers his own consumption as negligible when compared with that of others.)

Then it is immediate that the Nash Equilibria (N.E.), of Γ and $\tilde{\Gamma}$ coincide. In other words, benevolence or malevolence towards others is washed out by virtue of perfect competition, i.e., by the presence of a continuum of traders. This phenomenon does not appear in a natural way in the Walrasian model, but can be made explicit in the framework of strategic market games.

An asymptotic version of this remark can also be stated, but let us content ourselves with a simple example. Let Γ be a two-person strategic market game, Γ_r its r th replica (thus $\Gamma_1 = \Gamma$) and Γ_* the limit game with a continuum of each type of trader. As is well-known, the N.E. allocations of Γ tend to be inefficient [see Dubey and Rogawski (n.d.)]. One would think that a judicious choice of $\tilde{\Gamma}$, i.e. of \tilde{u}^t , might restore efficiency. Let us introduce for each player a coefficient of concern for others,¹ and suppose \tilde{u}^1, \tilde{u}^2 (the utilities of the two trader-types) to be each a 'social welfare function' of the standard linear separable form. Thus $\tilde{u}^1(\alpha, x, y) = \lambda^1 u^1(\alpha) + \lambda^2 u^1(x) + \lambda^3 u^2(y)$ where $\alpha \equiv$ personal

¹ Called by Edgeworth (1881) a 'coefficient of effective sympathy'

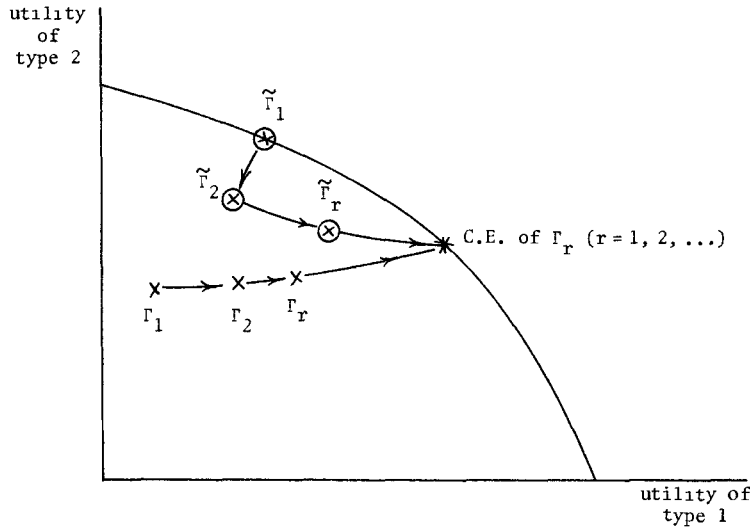


Fig. 1.

consumption of the trader of type 1, $x \equiv$ average consumption by others of type 1, $y \equiv$ average consumption by traders of type 2. (In $\tilde{\Gamma}_1$, since there is only one player of type 1, take the middle term to be zero.) Define \tilde{u}^2 similarly. If we choose the coefficients identically (so that $\tilde{u}^1 = \tilde{u}^2$) then $\tilde{\Gamma}_1$ will indeed have an efficient N.E. (simply maximize the common social welfare function on the feasible allocations of the game). But with replication this N.E. will, in general, move off the efficient frontier and converge to it again in the limit. Restricting ourselves to type-symmetric equilibria we have the picture as given in fig. 1.

As indicated by the result in the continuum, both sequences of N.E.'s (of $\Gamma_r, \tilde{\Gamma}_r$) converge to the same limit. Under appropriate conditions [e.g., see Dubey, Mas-Colell and Shubik (1980)] this common limit will be the C.E. of the economy.

References

- Dubey, P., A. Mas-Colell and M. Shubik, 1980, Efficiency properties of strategic market games: An axiomatic approach, *Journal of Economic Theory*.
- Dubey, P. and R.J. Rogawski, n.d., Inefficiency of Nash equilibria in a private goods economy, Cowles Foundation discussion paper no. 631.
- Edgeworth, F.Y., 1881, *Mathematical psychics* (Kegan Paul, London) 53.