Assets, General Equilibrium and the Neutrality of Money

CHRISTOPHE CHAMLEY
Yale University and C.O.R.E.

and

HERAKLES POLEMARCHAKIS
Columbia University

When government liabilities (including money) are held in private portfolios only as stores of value and do not provide additional services (such as liquidity), real variables are not affected by changes in the money supply due to the government’s trading in real assets (open market operations). This neutrality of monetary policy fails if the government either trades in nominal assets, or it distributes subsidies and levies taxes.

1. INTRODUCTION

The ability of monetary policy to affect real variables is an important issue and has been the subject of numerous theoretical and empirical studies. The most useful theoretical framework within which to address the problem is a model of asset markets where the demand for money or other government liabilities is determined by rational portfolio choice (Tobin (1958)). These assets may, of course, provide additional services (such as transactions and liquidity). We assume here, however, that these services do not exist.

The standard argument for the effectiveness of monetary policy goes as follows: Changes in the supply of money affect the price level and hence the distribution of returns to money as an asset. As individuals realign their portfolios and prices adjust to maintain market clearing, the equilibrium holdings of physical assets change. In this case money is not neutral. We shall argue, however, that, as long as changes in the supply of money are effected through the government’s trading in real assets, this argument is, in general, not valid.

It is well known that the effectiveness of monetary policy depends on the method of injection of new money into the economy. Two procedures are used by the government to effect changes in the supply of money: It either trades in assets, nominal or real (open market operations), or it distributes subsidies and levies taxes. We show here that open market operations in real assets (capital, or indexed bonds) have no effects on real variables in a general equilibrium model with market clearing. The argument, which can also be applied to the management of the public debt, is presented in a most general framework in the next section; it does not depend on the existence of a complete system of contingent securities or on an operative bequest motive. Furthermore, it is compatible with incomplete and differential information.

The result is similar to the Modigliani–Miller theorem in corporate finance. That an argument along these lines can be employed to demonstrate the irrelevance of open
market operations between money and capital was first realized by Wallace (1981), who also provided an example to show the non-vacuousness of the argument. His setting is, however, unnecessarily restricted and obfuscates the result. In particular, Wallace works in a framework of complete contingent markets which prevents the creation of new assets; furthermore, he unnecessarily combines the open market operations with subsidies and taxes designed to shelter the goods price of money.

It is important to emphasize that, although government trading in existing real assets is neutral, the creation of new assets need not be so, provided that individuals in the same generation are sufficiently diverse. This can be illustrated easily in a model of heterogeneous overlapping generations.

The neutrality of changes in the money supply fails if the government trades in nominally denominated assets; this is illustrated by an example at the end of Section 2. In Section 3 we examine variations in the supply of money accompanied by subsidies or taxes. It is well known that under complete information an unanticipated jump of the money stock with lump sum subsidies to individuals proportional to their cash balances induces a proportional jump of the price level and does not affect the real quantity of money or other real variables. Concerning anticipated changes in the supply of money, the situation is different: In a non-stochastic framework no change in the money supply which leaves the real quantity of money unaffected is neutral; assets are perfect substitutes and a change in the return of any one asset (e.g. money) leads to a violation of the equilibrium conditions at the current allocation of real resources. In a stochastic framework, there do exist contingent, anticipated variations of the money supply, which do not alter the set of available assets and are neutral. In general, however, contingent monetary policies create new assets and are not non-neutral.

In the last section we discuss some implications of our results.

2. OPEN MARKET OPERATIONS

The government determines the composition of its portfolio by trading between different assets or liabilities on the corresponding markets. In order to analyse the effect of these open-market operations in general equilibrium, we consider trading in a capital input which produces a random return. In each period $t$, the economy is subject to random shocks which are denoted by the elements $s_t$ of sets $S_t$ with a finite number of elements $n_t$. An element $s_t$ determines among others the total return for each unit of capital $r_t$. This rate of return is always strictly positive. The state of nature at time $t$, $s_n$, may also include exogenous events prior to time $t$. We assume that factor productivities are independent of the level of inputs and are determined by the exogenous parameter $s_t$. This does not restrict the generality of our result since we show that the contingent path of aggregate capital accumulation is independent of the government's policy. All agents living at time $t$ know the probability of occurrence of the sequence $s_{t+k}$ for the periods in their own lifetime, which is finite or infinite, conditional on the state of nature $s_t$. They determine at time $t$ their own optimal intertemporal programme of consumption and portfolio choice for each future period, which is contingent on the realization of $s_{t+k}$ $(k \geq 1)$ in these periods.

We can assume, without loss of generality, that initially the government has no expenditure, raises no tax, and has created a fixed amount of fiat money which is held by private agents in their portfolios because of its return properties. The price of money with respect to the capital good is $p_t$. The prices of goods and other assets with respect to capital (the numeraire) remain invariant in the argument and need not be mentioned
explicitly. Denote by $K_t, K^e_t, M_t$ the amounts of total capital in the economy, the amount of capital owned by the government, and the (nominal) quantity of money, respectively, which are carried from period $t$ to period $t + 1$. The budget constraint of the government at time $t$ is given by

$$p_t(M_t - \bar{M}) = K^e_t; \quad p_t(M_t - M_{t-1}) + r_t K^e_{t-1} = K^e_t, \quad t > 1. \quad (1)$$

The origin of time can be chosen arbitrarily, it is fixed at period 1, and the initial money stock is fixed at $\bar{M}$. In the first period, the government announces an open-market policy which determines the level of $K^e_t$ (positive or negative) as a function of $s_t$ for all values of $t$ ($t \geq 1$). This policy is known by the private agents. The supply of money $M_t$, is determined in equation (1) for all periods.

We consider only allocations of resources in equilibrium. Since our purpose is to demonstrate the invariance of the equilibrium under different policies, it is not necessary to characterize completely an equilibrium, or to prove its existence.

An equilibrium in the economy is described by the functions $(x_t, K_t, M_t, p_t)$ of $s_t$, which give, respectively, the allocation of goods and other real assets to each individual, the aggregate capital stock, and the quantity and the price of money at time $t$. These functions satisfy the following restrictions: private programmes are supported by portfolio programmes which are state contingent; in each period, the sum of the amounts of capital and real quantity of money in all individual portfolios are equal to $K_t - K^e_t$ and $p_t M_t$, respectively. The equilibrium depends on the government's policy, and is represented in abbreviated form by the sequence

$$(x_t, K_t, M_t, p_t, K^e_t), \quad t \geq 1,$$

which is contingent on $s_t$.

Let us assume that the economy is in equilibrium under the following government policy: the quantity of money is fixed for all $t$ and equal to $\bar{M}$, and $K^e_t$ is equal to zero. In the first period the government announces a new policy described by the contingent sequence $(K^e_t)$. The following proposition shows that the effect of this policy on the real allocation of resources $(x_t, K_t)$ is nil, as long as money maintains a nonzero price.

**Proposition 1.** Assume that the sequence $(\tilde{x}_t, \tilde{K}_t, \bar{M}, \tilde{p}_t, 0)$ describes an equilibrium. Then the sequence $(\tilde{x}_t, \tilde{K}_t, M_t, p_t, K^e_t), \quad K^e_t \equiv 0$, describes an equilibrium where $M_t$ is determined by (1) and $p_t$ is given by:

$$p_t = \frac{p_{t+1}}{p_t} = (1 - \alpha_t) \frac{\tilde{p}_{t+1} + \alpha_t r_{t+1}}{\tilde{p}_t}, \quad t \geq 1, \quad (2)$$

with

$$\alpha_t = \frac{K^e_t}{K^e_t + \tilde{p}_t \bar{M}} = \frac{K^e_t}{p_t \bar{M}_t}.$$

**Proof.** To prove the proposition, we show that individuals "undo" the actions of the government and that the asset markets remain in equilibrium with no change in the allocation of goods or other assets to any individual. Consider an individual $i$ who chooses his portfolio for period $t$ contingent on $s_t$. Assume that in the first equilibrium $(K^e_t \equiv 0)$, his portfolio is given by $(\tilde{K}_i, \bar{M}_t)$. Construct now the portfolio $(K^e_t, M_t)$ as follows:

$$M_t = \frac{1}{1 - \alpha_t} \frac{p_t}{\bar{M}_t}.$$
\[ K_i^t = \bar{K}_i^t - \frac{\alpha_i}{1 - \alpha_i} \bar{p}_i \bar{M}_i^t, \]  

(4)

where \( p_i \) and \( \alpha_i \) are defined in (2).

The two portfolios are related as follows:

\[ p_i M_i^t + K_i^t = \bar{p}_i \bar{M}_i^t + \bar{K}_i^t, \]  

(5)

and using (2)

\[ p_{t+1} M_i^{t+1} + r_i K_i^t = \bar{p}_{t+1} \bar{M}_i^{t+1} + r_i \bar{K}_i^t. \]  

(6)

The first relation shows that the value of the portfolio \((\bar{K}_i^t, \bar{M}_i^t)\) at the price of money \(\bar{p}\) is identical to the value of \((K_i^t, M_i^t)\) at the price \(p\). From the second relation, we see that the real return of these two portfolios (at the prices \(\bar{p}\) and \(p\), respectively) are identical. It follows that any intertemporal programme chosen by individual \(i\) in the environment described by \(\bar{p}(s_i)\) is supported by a programme of portfolios \((\bar{K}_i^t, \bar{M}_i^t)\) as functions of \(s_i\) is also attainable in the environment described by \(p(s_i)\), and is sustained in this case by the portfolios \((K_i^t, M_i^t)\) as functions of \(s_i\). The reverse is also true. It follows that if \((\bar{K}_i^t, \bar{M}_i^t)\) is optimal in the first case, \((K_i^t, M_i^t)\) is optimal in the second.

The demand for money in the new equilibrium is determined by summing (3) over all individuals \(i\). Using the first equality in the definition of \(\alpha_i\) in (2), this demand \(M_i^D\) is given by

\[ p_i M_i^D = K_i^t + \bar{p}_i \bar{M}_i. \]  

(7)

The supply of money is given by equation (1):

\[ p_i M_i^s = p_i M_i^{s,t} - r_i K_i^{s,t} + K_i^t. \]  

(8)

For equilibrium we must have \(M_i^D = M_i^s\), which is equivalent to

\[ \bar{p}_i \bar{M}_i = p_i M_i^{s,t} - r_i K_i^{s,t}. \]  

(9)

This identity is proven by induction. If \(M_i^{D,t-1} = M_i^{s,t-1}\), from (7)

\[ \alpha_{t-1} = \frac{K_i^{s,t-1}}{K_i^{s,t-1} + \bar{p}_{t-1} \bar{M}_i} = \frac{K_i^{s,t-1}}{p_{t-1} M_i^{s,t-1}}. \]  

(10)

We now use the definition of \(p_i\) in (2) to transform the right hand side term in (9):

\[ p_i M_i^{s,t-1} - r_i K_i^{s,t-1} = (1 - \alpha_{t-1}) \frac{\bar{p}_i}{p_{t-1}} p_{t-1} M_i^{s,t-1} + \alpha_{t-1} r_i p_{t-1} M_i^{s,t-1} - r_i K_i^{s,t-1}. \]

Replacing \(\alpha_{t-1}\) by its value in (10), we see that this expression is equal to the left hand side term in (9).

Adding (4) over all individuals, we find that

\[ \sum_i K_i^t + K_i^t = \bar{K}_0, \]

which concludes the proof. ||

This result has a simple intuitive interpretation: when the government buys capital with money, the contingent price of money is altered from \(\bar{p}\) to \(p\), where \(p\) satisfies (2) and \(\alpha_i\) is equal to \(K_i^t/p_i M_i\). The "new" money is a composite asset consisting of "old" money and capital, where the share of capital in the determination of the rate of return on money is equal to the ratio between the government's capital and the real quantity
of money (in the new equilibrium). When the government buys capital with money, individuals maintain their real contingent consumption plans by reducing their capital and increasing proportionately their holding of money. This increase of the demand for money is equal to the increase of the supply and the real allocation of resources is not affected.

The above proposition calls for a few remarks. It should be emphasized that open-market operations do not affect the price of money in the first period. The result extends to operations announced in period 1 and implemented in period t. These policies do not alter the contingent price of money between periods 1 and period t included.

We have considered a very simple model with two assets. A similar proposition is valid for an economy with different types of government liabilities. It is clear that in this case open market operations affect only the prices of the liabilities traded by the government.

The price of money $p_t$ remains positive if $K_t^s$ and $r_t$ are positive. There is no upper limit on $K_t^s$, which could even be greater than the capital stock in the economy. There is a lower bound for the feasible values of $K_t^s$ such that the values of $p_t$ given by (2) are positive. This lower bound depends on the characteristics of the equilibrium without open-market operation.

We have assumed that there are no restrictions on individual portfolio choices. If short-sales are bounded below or not feasible (subject to $b_t^i$ for some $b_t^i$), the above result is clearly not valid.

The neutrality proposition fails if open market operations are carried out between money and assets denominated in nominal terms. To see this, consider an overlapping generations model with no uncertainty, as in Samuelson [1958]; for simplicity, suppose that there is no growth in population and let $(\bar{c}_1, \bar{c}_2, \bar{p}, \bar{M})$ denote the allocation of consumption between young and old, the price of money and the nominal money stock at the (monetary) steady state. A bond is introduced at time $t = 1$ with a one period maturity and a total nominal return at period $t = 2$ given by $r_2$, a random variable, which takes the value $r_2(s)$ for $s \in S$, a finite set, with probability $\pi_s$. The real price of the bond at $t = 1$ is $q$. First note that the mere availability of the bond need not affect the allocation; in the absence of intra-generational diversity, it suffices to set $q$ so that the agent(s) young at $t = 1$ finds it optimal not to trade in the bond market. Suppose, alternatively, that the government decides to contract the money supply at $t = 1$ issuing $B$ of the bonds and expands the money supply at $t = 2$ by issuing money to finance the interest payment. From the government’s budget constraint it follows that the price systems $(q, p_t)$ and the supply of money $M_t$ are linked by the following equations:

$$p_1(M_1 - \bar{M}) + qB = 0; \quad p_2(s)(M_2(s) - M_1) - p_2(s)r_2(s)B = 0. \quad (11)$$

If the original allocation $(\bar{c}_1, \bar{c}_2)$ is to be compatible with equilibrium as neutrality requires, the real quantity of money must remain unaffected at $t = 1, 2$ as well as subsequent periods:

$$p_1\bar{M} = \bar{p}\bar{M}; \quad p_2(s)\bar{M}_2(s) = \bar{p}\bar{M}. \quad (12)$$

The first equation is derived from the consumption of the old at $t = 1$ and the second from the consumption of the young at $t = 2$. Finally, the young at $t = 1$ must find it optimal to hold for the portfolio $(M_1, B)$:

$$p_1 = Ep_2(s); \quad q = Ep_2(s)r_2(s). \quad (13)$$

Notice that in the allocation $(\bar{c}_1, \bar{c}_2, \bar{p}, \bar{M})$, the ratio between the marginal utilities of first
and the second period consumption is constant and equal to one, and hence does not appear in the first order conditions (13); no assumption of risk-neutrality is involved.

A straightforward manipulation of the system of equations (11)–(13) implies that for neutrality to hold the price of the bond following the open market operation \( q \) must satisfy

\[
\bar{p} = E \frac{\bar{p}^{2} \bar{M}}{\bar{p}M - qB + \bar{p}Br_{2}(s)}; \quad q = E \frac{\bar{p}^{2} \bar{M}r_{2}(s)}{\bar{p}M - qB + \bar{p}Br_{2}(s)}
\]  

(14)

which need not be possible for a general distribution of returns \( r_{2} \). Hence, neutrality fails. Particular patterns of returns may of course allow for neutrality; an example is that of a nominally riskless bond, where \( r_{2} \) is independent of \( s \). Unlike the case of assets denominated in real terms, the change in the distribution of the price of money following an open market operation has a multiplicative effect on the returns of a nominally denominated asset, which invalidates the spanning argument.

It was argued earlier that the argument for the neutrality of open-market operations is compatible with incomplete and differential information. This is clear from the proof of Proposition 1, since the probabilistic beliefs of the various agents never entered the argument. We should point out however that if agents attempt to extract information from the price system itself, the argument may fail: the price system \( p_{t} \) derived from \( \bar{p} \), according to (2) may be more or less informative.

3. TRANSFERS

When the government expenditures are fixed, the revenues generated by the creation of money are used by the government either to purchase capital or other assets, or to lower taxes (or increase subsidies). We saw in the last section that the first type of operation has no effect on real variables. This neutrality surely does not apply in general for the second type of policy.

Consider, varying on Samuelson (1958), an economy with overlapping generations. The levels of government expenditures and taxes are initially equal to zero. In period one, the government announces a one-time increase of the money supply for the following period, combined with lump-sum subsidies. If this policy has no incidence on individual’s consumption, it should not alter the real quantity of money in period one or in period two. A variation of the nominal quantity of money in the second period would affect (inversely) the price of money, and the rate of return on money between the first two periods. In general, we can expect contingent monetary policies to create new assets and to be non-neutral.

We want to characterize neutral transfer policies in an economy with capital and money. Observe, however, that there are trivial policies which are neutral even though they do change the real quantity of money. An example is given in Samuelson’s model, by the combination of a lump-sum grant of money to the young and a lump-sum tax levied on the old. In the determination of neutral policies we thus restrict our attention to policies that do not affect the real quantity of money; we call them strongly neutral.

An equilibrium in the economy is described by the functions \( (x_{t}, K_{t}, M_{t}, p_{t}, w_{t}) \) of \( s_{t} \). The level of \( K_{t}^{e} \) is assumed without loss of generality to be identically equal to zero, and \( w_{t} \) is the vector of transfers from the government to all individuals in period \( t \). The government’s budget constraint at period \( t \) takes the form:

\[
p_{t}(M_{t} - \bar{M}) = W_{t}; \quad p_{t}(M_{t} - M_{t-1}) = W_{t}, \quad t > 1,
\]  

(15)
where \( W_t \) is the sum of all net lump-sum transfers from the government to individuals at period \( t \).

An important property of a neutral policy is to keep the set of available assets unchanged (we investigate below the necessity of this property for neutrality): the return of money under active policy is a linear combination of the returns to money in the absence of policy and the returns to capital. The following proposition establishes that any such given combination can be obtained by a strongly neutral policy.

**Proposition 2.** For any equilibrium \((\bar{s}_t, \bar{K}_t, \bar{M}_t, \bar{p}_t, \bar{\tilde{w}}_t)\) and any sequence \( \beta_t \), where \( \beta_t \) is independent of \( s_{t+k}, k \geq 1, t \geq 1 \), there exists a strongly neutral monetary policy accompanied by transfers.

The path of money under the policy is defined by

\[
p_t = \frac{p_t}{\bar{p}_t}; \quad \frac{p_{t+1}}{p_t} = (1 - \beta_t) \frac{\bar{p}_{t+1}}{\bar{p}_t} + \beta_t r_{t+1}, \quad t \geq 1,
\]

and aggregate transfers are given by

\[
W_{t+1} - \bar{W}_{t+1} = \beta_t \bar{p}_t \bar{M}_t \left( \frac{\bar{p}_{t+1}}{\bar{p}_t} - r_{t+1} \right), \quad t \geq 1.
\]

**Proof.** The method of proof is the same as for Proposition 1. Assume that at the equilibrium \((s_n, \bar{K}_n, \bar{M}_n, \bar{p}_n, \bar{\tilde{w}}_n)\) the levels of capital and money carried by an individual \( i \) from period \( t \) to \( t+1 \) are equal to \( \bar{K}_i^t \) and \( \bar{M}_i^t \), respectively. Define a new policy as follows: choose a sequence of values \( \beta_t \) \((t \geq 1)\), where \( \beta_t \) is independent of \( s_{t+k}, k \geq 1 \); they define the prices \( p_t \) in (16). Set \( M_i = \bar{p}_i \bar{M}_i / p_i \), \((i \geq 2)\). The portfolio of individual \( i \) in the new equilibrium \((K_i^t, M_i^t)\) is chosen such that he holds the same amount of capital and real balances:

\[
K_i^t = \bar{K}_i^t, \quad M_i^t = \bar{M}_i^t \bar{p}_i / p_i.
\]

Obviously, this portfolio has under prices \( p_t \) the same value as \((\bar{K}_i^t, \bar{M}_i^t)\) under prices \( \bar{p}_i \):

\[
\bar{K}_i^t + \bar{p}_i \bar{M}_i^t = K_i^t + p_i M_i^t.
\]

Choose now a contingent transfer policy \( w_i^t \), such that

\[
r_{t+1} \bar{K}_i^t + \bar{p}_{t+1} \bar{M}_i^t + \bar{\tilde{w}}_{t+1}^t = r_{t+1} K_i^t + p_{t+1} M_i^t + w_{t+1}^t, \quad t \geq 1.
\]

By the same argument as in the previous section, the portfolio \((K_i^t, M_i^t)\) is optimal with the new prices \( (p_t) \) and the transfers policy \( (w_i^t) \). It is a trivial exercise to show that the sum of the transfers \( w_i^t \) is equal to the variation of the quantity of money in period \( t \) (in real terms), and that the policy of transfers defined by (19) is feasible.

In (17), the left hand side is equal to the variation of the nominal quantity of money (measured in real term) in each period. The choice of \( \beta_t \) determines the distribution of returns to the "new" money as a combination of the returns to the "old" money and the returns to capital.

Using the results of this and the previous section, we can show that there are policies combining open market operations and transfers which are neutral and do not alter the price of money. Such policies have been analysed by Wallace (1981) in the case of models with two-period overlapping generations and complete markets in contingent claims. These policies can be decomposed in two parts: open market operations determine the path \((K_i^t)\) and are always neutral (provided that there is no restriction on short sales.
and \( K^p_t \geq 0 \). They affect the price of money according to equation (2) in Proposition 1. This effect can be neutralized by the second component of the policy, i.e. a strongly neutral policy described in Proposition 2. When \( \beta = -\alpha /(1 - \alpha) \), the total effect of these two policies on the price of money is nil.

Proposition 2 established that there exist strongly neutral policies for which the relation (16) and (17) are satisfied. To complete the argument, we also show that, if individuals are sufficiently diverse, these conditions are not only sufficient but also necessary for a strongly neutral policy.

Suppose that there are \( N \) individuals living in periods \( t \) and \( t + 1 \), and consider the matrices

\[
A_t(s_t) = [a^t_{u u}], \quad \text{with } a^t_{u u} = \frac{\pi^t u_{i u}^{t+1} (s)}{u''_u (s)}, \quad s \in S_{t+1}, \quad t \geq 1,
\]

where the terms \( u''_u (s) \) and \( u''_{i u} (s_{t+1}) \) represent the marginal utility of consumption of individual \( i \) in period \( t \) and \( t + 1 \), in state \( s_t \) and \( s_{t+1} \) respectively, considered at time \( t \); \( \pi^t \) is the subjective probability of state \( s_{t+1} \) for the individual \( i \) (it depends also on the state \( s_t \), but this relation is omitted from the notation, for simplicity). We assume that for all \( s \in S_{t+1}, \pi^t_i > 0 \) for some \( i \). Money and capital are said to be perfect substitutes in period \( t \) when the vectors \( (r_{t+1} (s), s \in S_{t+1}) \) and \( (p_{t+1} (s)/p_t (s), s \in S_{t+1}) \) are equal to each other.

**Proposition 3.** Consider the following properties:

(i) \((\bar{x}, \bar{K}, M, p, \bar{w})\) and \((\bar{x}, \bar{K}, M, p, \bar{w})\) describe two equilibria with

\[
M_t = \bar{M}_t, \quad p_t = \bar{p}_t, \quad w_t = \bar{w}_t, \quad \text{and} \quad p_t M_t = \bar{p}_t \bar{M}_t \quad \text{and} \quad t > 1.
\]

(ii) \( A_t(s_t) \) is of rank at least \( n_{t+1} - 1 \), \( s_t \in S_t, \) \( t \geq 1 \).

(iii) Money and capital are never perfect substitutes for any \( t \geq 1 \).

These properties imply that there is a sequence of numbers \( \beta_i \), where \( \beta_i \) is independent of \( s_{t+1}, k \geq 1, t \geq 1 \), such that the relations (16) and (17) are satisfied.

**Proof.** The proposition is proven by a simple spanning argument. Since the sequences \((\bar{x}, \bar{K}, M, \bar{p}, \bar{w})\) and \((\bar{x}, \bar{K}, M, p, \bar{w})\) describe two equilibria with the same real allocation of resources, the first order conditions between periods \( t \) and \( t + 1 \) for all relevant individuals, can be written as follows:

\[
e = r_{t+1} A_t(s_t), \quad e = \frac{\bar{p}_{t+1}}{p_t} A_t(s_t), \quad (20)
\]

where \( e \) is the \( N \)-vector \((1, \ldots, 1)\); and for the second equilibrium,

\[
e = r_{t+1} A_t(s_t), \quad e = \frac{p_{t+1}}{p_t} A_t(s_t). \quad (21)
\]

Because of (i), the marginal utilities in (20) and (21) are identical in both equilibria.

By taking differences between these equations we find that

\[
0 = \left( r_{t+1} \frac{\bar{p}_{t+1}}{p_t} \right) A_t(s_t), \quad 0 = \left( r_{t+1} \frac{p_{t+1}}{p_t} \right) A_t(s_t).
\]

By conditions (ii) and (iii), the vectors \( r_{t+1} - (\bar{p}_{t+1}/p_t) \) and \( r_{t+1} - (p_{t+1}/p_t) \) are proportional.
Therefore, there exists a number $\beta_t$ such that

$$\frac{p_{t+1}}{p_t} = (1 - \beta_t) \frac{\tilde{p}_{t+1}}{\tilde{p}_t} + \beta_t r_{t+1}, \quad s \in S_{t+1}. \quad (22)$$

Using the equality $p_{t+1}(s)M_{t+1}(s) = \tilde{p}_{t+1}(s)\tilde{M}_{t+1}(s)$ and the government budget constraint in period $t + 1$ for each equilibrium, we find that for each $s$ in $S_{t+1}$:

$$W_{t+1} - \tilde{W}_{t+1} = p_{t+1}(M_{t+1} - M_t) - \tilde{p}_{t+1}(\tilde{M}_{t+1} - \tilde{M}_t) = \tilde{p}_{t+1}\tilde{M}_t - p_{t+1}M_t.$$

Replacing $p_{t+1}$ by its value in (22):

$$W_{t+1} - \tilde{W}_{t+1} = \beta_t\tilde{p}_t\tilde{M}_t \left( \frac{\tilde{p}_{t+1}}{\tilde{p}_t} - r_{t+1} \right). \quad ||$$

The spanning argument applies only if generations are sufficiently nonhomogeneous. In particular, if all individuals living both in periods $t$ and $t + 1$ are identical, it is obvious that condition (16) is much too strong, and can be replaced by a weaker equality obtained from (21).

Observe that in the absence of uncertainty, money and capital are perfect substitutes; the identities $p_t = \tilde{p}_t$, $p_{t+1}/p_t = r_{t+1}$, and $p_tM_t = \tilde{p}_t\tilde{M}_t$ imply that $M_t = \tilde{M}_t$. No variation of the money supply is strongly neutral.

4. CONCLUSION

Our results have implications for theoretical and empirical studies on monetary policy and the management of the government debt. In a portfolio model of money and real assets, the expansion of the money supply has an effect on the current price level only if it is accompanied by current or future transfers from the government to individuals. Also, this paper is a reminder that models of assets demand may lead to misleading conclusions about the management of the government debt. In a Capital Asset Pricing Model of demand for government securities, for example, the covariance matrix is not fixed; it depends on the composition of the government debt because this composition affects the future tradings in assets and the deficits of the government. In the simple case of two types of assets, nominal and real, which are not perfectly substitutable, open market operations do not affect the current value of the debt nor the real allocation of resources. However, we have seen that the neutrality result does not hold when there are assets with returns tied to the value of another asset, e.g. when there is more than one nominal asset. At this point, it may be interesting to note that indexation shelters the return to other assets from the value of money and thus eliminates the effectiveness of open market operations. Finally, some components of the government debt provide liquidity services in addition to their speculative function. These issues should be addressed in subsequent studies.

First version received December 1981, final version accepted April 1983 (Eds.).

We wish to thank M. Bray, J. Geanakoplos, K. Hamdani, C. Lawrence, J. Tobin and L. Weiss and anonymous referees for many helpful discussions and comments.

This work was supported by National Science Foundation grants SES 80-01934 and SES 78-25910 and by a grant from the Center for the Study of Futures Markets at Columbia University.

An earlier version of this paper under the title "Asset Markets, General Equilibrium and the Neutrality of Money" appeared as Cowles Foundation Discussion Paper No 605.
REFERENCES