Marginal vs. Average Cost Pricing in the Presence of a Public Monopoly

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The Arrow-Debreu analysis of decentralized resource allocation in a Walrasian economy assumes constant or decreasing returns to scale in production. Recently, several authors have extended this analysis to economies with a public monopoly, that is, a firm with increasing returns to scale. In this literature, the salient feature is the characterization of increasing returns to scale technologies as nonconvex production sets, so that under this definition both single and multiproduct firms may exhibit increasing returns.

Here, our intended model is an economy with a competitive sector consisting of households and firms with convex technologies, and a public sector consisting of firms with nonconvex technologies. A special case is a single multiproduct firm which produces products for regulated markets (with a nonconvex technology) and produces products for unregulated markets (with a convex technology), for example, AT&T. We consider for such an economy two of the general equilibrium concepts that have been investigated in this literature. One is Harold Hotelling's notion of a marginal cost-pricing (*MCP*) equilibrium, and the other is M. Boiteux's notion of an average cost-pricing (*ACP*) equilibrium.

A marginal cost-pricing equilibrium is a family of consumption plans, production plans, prices, and lump sum taxes such that households are maximizing utility subject to after-tax income; firms with constant or decreasing returns are maximizing profits; the public monopoly is pricing at marginal cost, where potential losses are covered by the lump sum taxes; and all markets clear.

An average cost-pricing equilibrium is a family of consumption plans, production plans and prices such that households are maximizing utility subject to their budget constraint; firms with constant or decreasing returns are maximizing profits; the public monopoly is pricing at average cost, that is, breaking even or making zero profits; and all markets clear.

Unfortunately, all of the extant proofs of existence of a *MCP* or an *ACP* equilibrium are somewhat technical in nature and lack the transparency of counting equations and unknowns which many economists accept as an intuitive, if not formally correct, proof of existence. In view of this, one of the purposes of this paper is to demonstrate the existence of a *MCP* and an *ACP* equilibrium in a simple economy with increasing returns, where the equilibrium notions are characterized by systems of behavioral equations and market-clearing conditions. We give both an intuitive proof of existence by counting equations and unknowns, and a formal argument that these systems of equations have a solution by use of a simple fixed-point argument.

In addition, we review several of the standard partial equilibrium prescriptions for the regulation of a public monopoly and show that in a general equilibrium model they can be interpreted as *MCP* or *ACP* equilibria.

I. The Model

Our model will be the neoclassical two-input, two-output, two-household, two-firm

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economy where inputs are inelastically supplied. The inputs are capital (K) and labor (L). The outputs are grain (G) and electricity (E). Each household has a utility function denoted U_x and U_y. Endowments and shareholdings in firms are given by (K_x, L_x), (K_y, L_y); $(\theta_{xG}, \theta_{xE})$, $(\theta_{yG}, \theta_{yE})$. Each firm has a production function, F_G and F_E, or equivalently, cost functions c_G and c_E. Let $K = K_x + K_y$ and $L = L_x + L_y$.

We make the same assumptions regarding firms and households as does Francis Bator in his classic 1957 expository piece on welfare economics, with one exception: we do not assume constant returns to scale, although firms are assumed to exhibit diminishing marginal rate of substitution along any isoquant, that is, the markets for inputs are competitive.

Under these assumptions, we construct the Edgeworth-Bowley box for production and the social production possibility frontier, *PPF*. In general, the social production possibility set is nonconvex.

Let P_G and P_E denote the prices of grain and electricity, and w and r denote the prices of labor and capital. The marginal rate of transformation (*MRT*) at a point $(\tilde{G}, \tilde{E})$ on the social production possibility frontier is simply the absolute value of the slope of the frontier at that point and will be denoted $P_{\tilde{E}}/P_{\tilde{G}}$. A point $(\tilde{G}, \tilde{E})$ is said to be production efficient if it lies on this frontier.

Each point $(\tilde{G}, \tilde{E})$ on the frontier also determines a unique point in the Edgeworth-Bowley box for production, that is, the point on the efficiency locus corresponding to the tangency of the isoquants defined by $F_E(L_E, K_E) = \tilde{E}$ and $F_G(L_G, K_G) = \tilde{G}$. The slope of their common tangent line at this point will be denoted as w/r and is the marginal rate of technical substitution (*MRTS*) at this point.

We shall use repeatedly that the *MRT* at a point $(\tilde{G}, \tilde{E})$ is the ratio of the marginal costs; that is, $P_{\tilde{E}}/P_{\tilde{G}} = \partial c_E(w/r, \tilde{E})/\partial E \big/ \partial c_G(w/r, \tilde{G})/\partial G$.

II. Marginal Cost Pricing

As is common in this literature, we assume that each household's income at the prevailing prices is a fixed proportion of *GNP*. This assumption, which is called a fixed structure of revenues, guarantees that households have positive after-tax income.

Formally, a fixed structure of revenues is defined as follows: there are fixed α_x and α_y where $\alpha_x, \alpha_y > 0$ and $\alpha_x + \alpha_y = 1$ such that $(K_x, L_x) = \alpha_x(K, L)$, $\alpha_x = \theta_{xG} = \theta_{xE}$; $(K_y, L_y) = \alpha_y(K, L)$, $\alpha_y = \theta_{yG} = \theta_{yE}$. Hence, the income of household x,

$$\begin{aligned} I_x &= wL_x + rK_x + \theta_{xG}(p_G G - wL_G - rK_G) \\ &\quad + \theta_{xE}(p_E E - wL_E - rK_E) \\ &= \alpha_x(wL + rK) \\ &\quad + \alpha_x(p_G G + p_E E - w(L_G + L_E) \\ &\quad - r(K_G + K_E)) \\ &= \alpha_x(wL + rK) \\ &\quad + \alpha_x(p_G G + p_E E - wL - rK) \\ &= \alpha_x(P_G G + p_E E), \end{aligned}$$

if production is efficient. Similarly, the income of household y, $I_y = \alpha_y(p_G G + p_E E)$. Consequently, a household's income depends only on the relative product prices and outputs of firms.

Note that each agent's income is positive and is net of the lump sum taxes necessary to cover the losses of firms producing with increasing returns. Hence, if electricity is produced with increasing returns, then $\theta_{xE}(P_E E - wL_E - rk_E)$ is the lump sum tax imposed on household x. Another interpretation of the lump sum taxes is that the shareholdings carry unlimited liability.

A household's demand for products derive from utility maximization subject to its budget constraint:[1]

$$(1) \qquad \partial U_i/\partial E_i \Big/ \partial U_i/\partial G_i = P_E/P_G,$$

$$(2) \quad p_E E_i + p_G G_i = \alpha_i(p_E E + p_G G) \; i = x, y.$$

[1]Throughout this discussion we ignore the possibility of corner solutions. These will never occur if household's indifference curves do not cut the coordinate axes and both marginal and average costs are well behaved at zero output.

A firm's demand for factors derive from cost minimization subject to its output constraint:

$$\partial F_j/\partial L_j \Big/ \partial F_j/\partial K_j = w/r, \tag{3}$$

$$F_j(L_j, K_j) = j \;\; j = E, G. \tag{4}$$

Equilibrium is defined as a set of relative prices P_E/P_G and w/r; product demands E_x, G_x and E_y, G_y; factor demands L_E, K_E and L_G, K_G; and output levels E and G, such that all markets clear. That is,

Product Markets:

$$E_x + E_y = E, \quad (5) \qquad G_x + G_y = G, \quad (6)$$

Factor Markets:

$$L_E + L_G = L, \quad (7) \qquad K_E + K_G = K. \quad (8)$$

Because of Walras' law, equation (6) is redundant. Our final equation is the market pricing equation which gives the relationship between relative prices in the factor and product markets:

(9)

$$\frac{\partial c_E(w/r, E)}{\partial E} \Big/ \frac{\partial c_G(w/r, G)}{\partial G} = P_E/P_G$$

We only need two rather than three relative prices because of the fixed income distribution assumption.

Hence, a *MCP* equilibrium in this economy is characterized by a system of twelve independent equations in twelve unknowns. We now use a fixed-point argument to demonstrate the existence of a solution to this system of equations.

LEMMA: *A continuous map, $f(x)$, of a compact interval of the real line into itself has a fixed point.*

PROOF:

Let the interval be $[-1,1]$ and f: $[-1,1] \to [-1,1]$. Let $g(x) = f(x) - x$, then $g(-1) \geqslant 0$ and $g(1) \leqslant 0$. Hence, there is some $x \in [-1,1]$ such that $g(\bar{x}) = 0$, i.e., $f(\bar{x}) = \bar{x}$.

The social production possibility frontier, *PPF*, can be "stretched" onto a compact interval of the real line without tearing it. More formally, *PPF* is homeomorphic to a compact interval of the real line. Therefore, by the lemma, any continuous map Φ of *PPF* into itself will have a fixed point.

Consider the following continuous map, Φ, of *PPF* into *PPF* $(E_1, G_1) \to P_{E_1}/P_{G_1} \to (E_2, G_2) \to (E_3, G_3)$, where

(i) (E_1, G_1) is an arbitrary point on *PPF*

(ii) P_{E_1}/P_{G_1} is the *MRT* at (E_1, G_1)

(iii) (E_2, G_2) is the aggregate demand at relative product prices P_{E_1}/P_{G_1}, given production outputs E_1 and G_1.

(iv) (E_3, G_3) is the intersection of the ray from the origin through (E_2, G_2) and *PPF*—under our assumptions on the technology, this intersection is unique.

Note that (E_2, G_2) lies on the line through the point (E_1, G_1) with slope $-P_{E_1}/P_{G_1}$ by Walras' law. We can now prove the following theorem.

THEOREM 1: *A production efficient MCP equilibrium exists, i.e., equations* (1) *through* (9) *have a solution, which is production efficient.*[2]

PROOF:

Let (E^*, G^*) be a fixed-point of the map Φ. At such a point $(E_1, G_1) = (E_2, G_2) = (E_3, G_3) = (E^*, G^*)$. Hence, demand, $(E_2, G_2) = (E_1, G_1)$, supply, at the relative product prices P_E^*/P_G^*, the *MRT* at (E^*, G^*). The equilibrium relative prices in the factor markets, w^*/r^*, is the *MRTS* at the tangency of $F_E(L_E^*, K_E^*) = E^*$ and $F_G(L_G^*, K_G^*) = G$ in the Edgeworth Bowley box for production. This point of tangency gives us the equilibrium values of L_E^*, K_E^* and L_G^*, K_G^*; where $L_E^* + L_G^* = L$ and $K_E^* + K_G^* = K$. Finally, the household demands G_x^*, E_x^* and G_y^*, E_y^* total to the aggregate demand G^* and E^*. This completes the proof.

[2]As described, the equilibrium involves a decision by the planner, not only about how much electricity should be produced, but also about the production of grain. Each output price is then set equal to marginal cost. (Actually, all we require is that the price ratio be equal to the ratio of marginal costs.) However, profit maximization by a price-taking grain producer would also result in an output choice equating price and marginal cost so the equilibrium of this theorem remains viable with intervention only in the increasing returns to scale sector.

Note that in this model the existence of a socially inefficient *MCP* equilibrium is precluded by the assumption that inputs are fully employed. Note, also, that a *MCP* equilibrium need not be Pareto efficient despite the fact that the first-order conditions for Pareto efficiency hold at a *MCP* equilibrium. The reason is that for nonconvex production possibility sets, the first-order conditions are not sufficient to insure optimality. In a recent paper, we constructed an example of an economy, with increasing returns to scale, where all of the *MCP* equilibria are Pareto inefficient.

III. Average Cost Pricing

Initially, we assume that only electricity is produced with increasing returns and is priced at average costs; while grain is produced with constant or decreasing returns and is priced at marginal cost. Consequently, the income of household x, $I_x = wL_x + rK_x + \theta_{xG}(p_G G - wL_G - rK)$. Similarly, the income of household y, $I_y = wL_y + rK_y + \theta_{yG}(p_G G - wL_G - rK_G)$.

Note that we do not assume a fixed schedule of revenues in this section of the paper.

The system of equations describing an *ACP* equilibrium differ from the *MCP* equilibrium equations in the following manner: Equations (1), (3), (4), (5), (6), (7) and (8) remain the same. We shall denote the new equations with primes.

$$(2')\qquad P_E E_i + P_G G_i = wL_i + rK_i + \theta_{xG}(P_G G - wL_G - rK_G)\quad i = x, y$$

$$(9')\qquad P_G/r = \frac{\partial c_G(w/r, G)}{\partial G}$$

$$\text{and}\qquad P_E/r = c_E(w/r, E)/E.$$

Equilibrium is defined as a set of relative prices P_E/r, P_G/r, and w/r; product demands E_x, G_x and E_y, G_y; factor demands L_E, K_E and L_G, K_G; and output levels E and G, such that all markets clear. Hence, an *ACP* equilibrium in this model, is characterized by a system of thirteen independent equations in thirteen unknowns.

Consider the following continuous map, Ψ, of *PPF* into *PPF*:

$$(E_1, G_1) \to (w/r) \to (P_{E_1}/r, P_{G_1}/r) \to (E_2, G_2) \to (E_3, G_3),$$

where

(i) (E_1, G_1) is an arbitrary point on *PPF*,

(ii) w/r is the *MRTS* at the tangency of $F_E(L_{E_1}, K_{E_1}) = E_1$ and $F_G(L_{G_1}, K_{G_1}) = G_1$ in the Edgeworth-Bowley box for production,

(iii) $P_E/r = c_E(w/r, E_1)/E_1$

(iv) $P_G/r = \partial c_G(w/r, G_1)/\partial G$

(v) (E_2, G_2) is the aggregate demand at the relative prices w/r, P_{E_1}/r, P_{G_1}/r given production outputs E_1 and G_1,

(vi) (E_3, G_3) is the intersection of the ray from the origin through (E_2, G_2) and *PPF*.

THEOREM 2: *A production efficient ACP equilibrium exists.*

PROOF:

Let (E^*, G^*) be a fixed point of the map Ψ, then use the argument in the proof of Theorem (1) to complete the proof.

Finally, if both firms produce with increasing returns to scale, we require that grain is also sold at average cost: $P_G/r = c_G(w/r, G)/G$; and making an obvious change in the definition of Ψ, that is, (iv) is now $P_{G_1}/r = c_G(w/r, G_1)/G$, we can prove the following theorem.

THEOREM 3: *A production efficient ACP equilibrium exists, where each firm breaks even.*

IV. Regulation

We now review several of the standard partial equilibrium policy prescriptions for regulating a public monopoly. In the present general equilibrium model, they are simply decentralized interpretations of a *MCP* or an

ACP equilibrium, and their consistency and production efficiency are therefore assured by Theorems 1 and 2.

We consider first the policies of Oskar Lange and Abba Lerner. Lange proposed that the public monopoly, electricity (E), be given the desired output E^* which it should produce at minimum cost and sell at average cost, subject to the prevailing relative factor prices w^*/r^*. Although this policy was put forward by Lange as an application of the marginal cost pricing principle, it is clearly only consistent with the notion of an *ACP* equilibrium, if electricity is produced with increasing returns to scale.

Later, Lerner modified Lange's proposal by suggesting that the desired output E^* must be sold at marginal cost if the pricing rule is to satisfy the necessary conditions for Pareto optimality. The appropriate notion of equilibrium in this case is that of a *MCP* equilibrium.

Turning to more current policy prescriptions, we note that the primary activity of a public utility's regulatory commission is the setting of rates, that is, prices. A common prescription is to fix the rate of return for the public monopoly; require it to meet all demand; and have it produce efficiently.

In our model, this corresponds to an *ACP* equilibrium where the regulator sets the price P_{E^*}/r; requires the public monopoly to meet all demand; and to produce the demand E^* at minimum cost. In this case, the public monopoly makes normal economic profits, that is, breaks even.

Another regulatory policy, which one sees in real world markets, for example, British Rail, is that the public monopoly, electricity (E), is first given a subsidy S, and then required to price at marginal cost subject to a break-even constraint. That is, given the prevailing factor price ratio w/r find an output E which satisfies the following equation:

$$(10) \quad c_E(w/r, E) - E\frac{\partial c_E(w/r, E)}{\partial E} = S;$$

and sell E at marginal cost.

The efficacy of this regulatory policy in a general equilibrium model reduces to a question of the existence of a subsidy S, such that the output and price of the public monopoly are consistent with the utility-maximizing behavior of households; the profit-maximizing behavior of competitive firms; and the clearing of product and factor markets.

If E is produced with decreasing marginal costs, then (10) has at most one solution. Note that if $F_E(L_E, K_E)$, the production function for electricity, is homogeneous of degree r, where $r > 1$, then $c_E(w/r, E)$ is concave, for example, F_E is Cobb-Douglas with increasing returns to scale.

Clearly, any *MCP* equilibrium implicitly defines a subsidy S^* with the desired properties. Simply let $S^* = c_E(w^*/r^*, E^*) - E^*\partial c_E(w^*/r^*, E^*)/\partial E$, where w^*/r^* and E^* are the *MCP* equilibrium values of w/r and E. Note that S^* is raised through lump sum taxation.

It should be noted that all these regulatory policies lack behavioral incentives, both in the *MCP* or *ACP* interpretations, and that the development of incentive compatible variants is an important and difficult task.

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