

**A NOTE ON OVEREMPLOYMENT/UNDEREMPLOYMENT IN
LABOR CONTRACTS UNDER ASYMMETRIC INFORMATION ***

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Models of labor contracts under asymmetric information may predict either overemployment or underemployment. This paper shows that this result depends crucially on whether or not leisure is a normal good if firms are risk neutral.

1. Purpose

Recent papers on labor contracts under asymmetric information have yielded both underemployment [e.g., Azariadis (1980), and Grossman–Hart (1981)] and overemployment [Cooper (1982), Green–Kahn (1981)]. The key issue in these results appears to be the specification of workers' preferences – in particular, the normality of leisure. The purpose of this note is to make this relationship explicit and to propose a methodology for solving similar problems.¹

* This note was inspired by a comment made by Eric Maskin at the NBER Conference on Implicit Contracts and Fixed Price Equilibria, Princeton University, December 12 and 13, 1980. I am also grateful to Costas Azariadis, Roger Farmer, Jerry Green, Beth Hayes, Charles Kahn and Michael Riordan for discussions on this topic. All remaining errors are my own.

¹ Green–Kahn also show that if leisure is normal, overemployment occurs in a model with a continuum of states. This paper shows this same result with a finite number of states and also shows that if leisure is inferior, underemployment results. One can obtain this result in the Green–Kahn model as well. Many of these results are shown independently by Chari using a different methodology.

2. Preferences of workers (all workers identical)

Workers have preferences over their consumption, c , and their work, n , represented by $U(c, n)$. We assume $U_c > 0$, $U_n < 0$, $U_{cc} < 0$, and $U_{nn} < 0$ as is customary. Note that workers are strictly risk averse over consumption. At this stage we make no explicit assumptions on the sign of U_{cn} . Note, however, that leisure is a normal good if $U_{cc}U_n/U_c - U_{cn} > 0$. Hence $U_{cn} \leq 0$ is sufficient for the normality of leisure given our other assumptions. This condition of normality will be critical in the analysis.

3. Preferences of the firm

The firm's profits (π) are given by $\bar{s}f(n) - w$. Here \bar{s} represents a shock to the firm's technology, n is employment and w is compensation. With identical workers, we can assume there is only a single worker whose consumption equals the compensation paid by the firm. The firm is strictly risk neutral. At this stage, we allow s to be a continuous random variable taking values in $[\underline{s}, \bar{s}]$ where $\underline{s} > 0$. The production function, $f(\cdot)$, is increasing and concave.

4. Full-information solution

When realizations of s are public information, we characterize the full-information solution as the schedules $n^*(s)$, $w^*(s)$ which

$$\max E_s\{\lambda U + (1 - \lambda)\pi\}. \quad (1)$$

Here $0 \leq \lambda \leq 1$ is an arbitrary weight used to parameterize the expected utility possibility frontier. The first-order conditions for (1) are

$$U_c(w^*, n^*) = (1 - \lambda)/\lambda = k \quad \text{for all } s, \quad \text{and} \quad (2)$$

$$U_n(w^*, n^*) = -ksf'(n^*(s)) \quad \text{for all } s. \quad (3)$$

Condition (2) represents optimal risk sharing while (3) ensures productive efficiency ex-post.

Differentiating (2) and (3) with respect to s yields the following information on the employment and compensation schedules:

- (i) $n^*(s)$ increases with s for all $U(c, n)$,
- (ii) $w^*(s)$ increases with s if $U_{cn} > 0$,
 $w^*(s)$ is independent of s if $U_{cn} = 0$,
 $w^*(s)$ decreases with s if $U_{cn} < 0$.

For productive efficiency, employment always increases with s . To maintain the constancy of U_c , w must adjust as n changes with s unless preferences are separable. This adjustment depends on the sign of U_{cn} .

5. Implementability of full-information solution under asymmetric information

If realizations of \hat{s} are observed only by the firm, we have a situation of asymmetric information. Our ultimate interest is to characterize the distortion in the full-information contract due to asymmetric information.

One hint on the form of the distortion can be obtained at this point. We can always ask whether the full-information solution is implementable under asymmetric information. If it is not, the way in which a firm lies will be helpful information later on.

As is now customary in the literature on asymmetric information, we will confine our attention to the implementation of contracts via direct revelation mechanisms (see Myerson and Harris–Townsend). For any contract $\{n(s), w(s)\}$ we define $\pi(s/\hat{s})$ as the firm's profit when the true state is \hat{s} and the firm announces that s has occurred. That is

$$\pi(s/\hat{s}) = \hat{s}f(n(s)) - w(s). \quad (4)$$

For a given contract, we can define an *announcement function*, $m(\hat{s})$, which is the firm's announced state when \hat{s} occurs. Formally, $m(\hat{s})$ is the

$$\operatorname{argmax}_{s \leq \hat{s} \leq \bar{s}} \pi(s/\hat{s}). \quad (5)$$

Using (4) and (5) we can now ask whether the full-information solution characterized by (2) and (3) is implementable under asymmetric information. Implementability of $\{n^*(s), w^*(s)\}$ will require that under this contract $m(s) = s$ for all s . Since $m(s)$ is given by (5), implementability requires ²

² These are necessary conditions.

$$sf'(n^*(s)) \frac{dn^*(s)}{ds} - \frac{dw^*(s)}{ds} = 0 \quad \text{for all } s. \quad (6)$$

We can obtain the derivatives of $n^*(s)$ and $w^*(s)$ from (2) and (3). Using these as well as (2) and (3), (6) becomes

$$(U_n/U_c)U_{cc} - U_{cn} = 0. \quad (7)$$

Of course, all the derivatives in (7) are evaluated at $\{n^*(s), w^*(s)\}$.

Proposition 1. *The full-information solution is implementable under asymmetric information iff the demand for leisure is independent of income.*

Proof. It is straightforward to show that (7) is equivalent to a zero income effect for leisure. Since (7) is equivalent to (6), the proposition holds. Q.E.D.

Utility functions satisfying (7) are common in the labor contracts literature. The papers by Hall–Lilien, Azariadis and Grossman–Hart all assume that $U(c, n) = V(c - h(n))$ which satisfies (7).

When leisure demand is not independent of income, (7) will not hold. In this case, we need to determine whether $m(s)$ is greater than or less than s .

Proposition 2. *If leisure is a normal good, then $m(s) > s$ for all $s < \bar{s}$ and $m(\bar{s}) = \bar{s}$. If leisure is an inferior good, $m(s) < s$ for all $s > \underline{s}$ and $m(\underline{s}) = \underline{s}$.*

Proof. First assume that leisure is a normal good. The announcement function $m(s)$ satisfies

$$sf'(n^*(m(s))) \frac{dn^*(m(s))}{ds} - \frac{dw^*(m(s))}{ds} = 0. \quad (8)$$

Suppose $m(s) \leq s < \bar{s}$. We know that $m(s) = s$ leads to a contradiction since the normality of leisure violates (7). If $m(s) < s$, then we can use (2) and (3) as before to rewrite (8) as

$$\frac{s}{m(s)} \frac{U_n}{U_c} U_{cc} - U_{cn} = 0. \quad (9)$$

Here the derivatives are evaluated at $\{n^*(m(s)), w^*(m(s))\}$. With leisure normal, $U_{cc} < 0$ and $U_n < 0$, (9) cannot hold for $s > m(s)$. When $s = \bar{s}$, $m(\bar{s}) = \bar{s}$ and (8) is met with an inequality since the solution is not inferior. In a similar manner, one can show that if leisure is inferior, $m(s) > s$ for $s \neq \underline{s}$.

This proposition tells us exactly which way the firm will lie under the full-information solution. When $U_{cn} \leq 0$ (a sufficient condition for the normality of leisure) we noted earlier that $w^*(s)$ will be a non-increasing function of s . Since $n^*(s)$ increases with s , it is obvious that if $U_{cn} \leq 0$, $m(s) = \bar{s}$ for all s .

6. Optimal contract under asymmetric information

The easiest way to demonstrate the relationship between the normality of leisure and the overemployment/underemployment issue is via a two-state example. Assume s takes the value s_H with probability p_H and the value $s_L < s_H$ with probability $1 - p_H$. Using a direct revelation mechanism, the optimal contract under asymmetric information, $\{\bar{n}(s), \bar{w}(s)\}$, solves

$$\max E_s\{\lambda U + (1 - \lambda)\pi\}, \quad \text{subject to} \quad (10)$$

$$\pi(s_H|s_H) \geq \pi(s_L|s_H), \quad (10')$$

$$\pi(s_L|s_L) \geq \pi(s_H|s_L). \quad (10'')$$

Using δ and ϕ as the Lagrange multipliers for (10') and (10'') respectively, the first-order conditions imply

$$s_L f'(n_L)\{\geq\} - U_n(w_L, n_L)/U_c(w_L, n_L) \quad \text{when} \quad \delta\{\geq\} 0, \quad (11)$$

$$s_H f'(n_H)\{\leq\} - U_n(w_H, n_H)/U_c(w_H, n_H) \quad \text{when} \quad \phi\{\geq\} 0. \quad (12)$$

From (11) $\delta > 0$ implies underemployment in the low state of nature. From (12), $\phi > 0$ implies overemployment in the high state of nature. As is now well-known in these types of problems both δ and ϕ cannot be positive simultaneously. Hence which one of them is positive determines whether we have overemployment or underemployment.

This is where Propositions 1 and 2 come into the picture. If leisure is a

normal good, then from Proposition 2 the firm had an incentive, under the full-information solution, to announce $m(s) > s$. Suppose that, with leisure a normal good, the solution to (10) had (10') binding and therefore (10'') not binding. It is obvious that the solution to (10) with (10') the only constraint must be the full-information solution since that solution did not violate (10'). Yet, from Proposition 1 we know that this solution violates (10''). Hence in the solution to (10), if leisure is normal (10'') must be binding. A similar argument shows that if leisure is inferior, (10') must be binding.

Proposition 3. In the solution to (10), we have:

- (i) overemployment in state s_H if leisure is a normal good, and
- (ii) underemployment in state s_L if leisure is an inferior good.

Proof. As discussed above, Propositions 1 and 2 indicate which incentive compatibility constraint must be binding in the solution to (10). Using this, (11) and (12) yield the results. Q.E.D.

7. Summary

Using this two-state example, we see the relationship between leisure as a normal (inferior) good and overemployment (underemployment) in the optimal contract under asymmetric information. This result should easily generalize to the multi-state case by extending the above arguments.³

Another extension concerns the preference of the firm. Azariadis and Grossman–Hart assumed the firm was risk averse as a means of generating underemployment when consumption and leisure are perfect substitutes. The relationship between the risk preferences of the firm and the normality of leisure for consumers remains an open issue.⁴

Since many problems in the economics of asymmetric information take the same form as the contracting problem, this paper suggests a methodology for their solution. First determine why the full information

³ Showing the N -state case is a messy extension of Proposition 3. If the random variable is continuous, Green–Kahn show that overemployment occurs when leisure is a normal good. Using their model, it is straightforward to show that underemployment occurs when leisure is inferior.

⁴ I am grateful to Roger Farmer for discussions on this point.

solution is not incentive compatible. Then use this information in solving the programming problem to determine the distortions created by the asymmetric information.

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