TWO-PART TARIFFS, MARGINAL COST PRICING
AND INCREASING RETURNS IN A GENERAL
EQUILIBRIUM MODEL

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1. Introduction

It has long been recognised that there are difficulties in attaining an
efficient allocation of resources in an economy with significant economies of
scale in production. It has generally been argued that the appropriate
response to such a situation is to set prices equal to marginal costs: a variety
of schemes have then been proposed for covering the losses that firms may in
consequence sustain. One of the most widely discussed of such schemes is a
system of two-part tariffs, whereby consumers pay a fixed charge for the
privilege of buying any amount of a good, no matter how small, and then
make an additional per unit payment given by the price, which is set equal
to marginal cost.

Our aim in this paper is to investigate the merits of such a scheme in the
context of a general equilibrium framework recently developed by the
authors [Brown and Heal (1976)], and capable of analysing equilibria in the
presence of nonconvexities. The results of this paper, together with those of
our previous paper [Brown and Heal (1979)] on the welfare properties of
marginal cost pricing equilibria with increasing returns in production, have
rather negative implications for the attractions of a system of marginal cost
pricing and multipart tariffs. There appear to be a number of difficulties in

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implicate them in remaining errors.
such an approach that have not been noted in the previous partial equilibrium analyses.

In two earlier papers [Brown and Heal (1976, 1979)] we have analysed the existence of a certain class of equilibrium under conditions of increasing returns in production, and also discussed its welfare properties. The equilibrium whose existence was established was characterised by the standard pattern of decentralisation on the consumption side, so that each consumer acted to maximise utility subject to a budget constraint determined by prices and his initial endowment, but by complete aggregation on the production side. The analysis of the disaggregated implementation of production decisions under conditions of increasing returns, poses a number of particularly severe problems, and at a formal level these are the main concern of this paper.

The basic point is of course that in the usual competitive equilibrium world one has available the result that the maximum of a linear function over a sum of sets equals the sum of its maxima over the individual sets. Given this result, one can proceed immediately from a characterisation of an equilibrium in terms of the maximisation of value over the aggregate feasible set, to a characterisation in terms of the maximisation of value over each firm's production possibility set. It is this step that cannot be made in a nonconvex framework: the value function that assumes its maximum over the aggregate feasible set at an equilibrium is nonlinear (albeit linear homogeneous), and for such cases no exactly corresponding result exists. It is therefore interesting to see how near to such a result one can come in a world where feasible sets are nonconvex and the maximand is thus nonlinear, and to relate this to marginal cost pricing and two-part tariffs with increasing returns in production. Of course, with a few notable exceptions [for example Guesnerie (1975)] this literature has been partial equilibrium in its approach, but nevertheless it is clear that its intent has been to establish behavioural rules which will guide firms with increasing returns in production to socially optimal production decisions. It must therefore be viewed as a contribution to the analysis of the support and implementation of efficient allocations with nonconvex feasible sets.

We shall show that it is possible to establish results with many of the characteristics of the usual linear-convex case, but with the anonymity of firms no longer preserved. This is perhaps to be expected, given the results of Hurwicz (1973), Calzamiglia (1977) and others on decentralisation in non-classical environments. It should be noted that these writers make the anonymity or informational privacy of firms a defining characteristic of decentralisation, so that in their terminology the equilibria that we shall analyse are not decentralised. However, if increasing returns to scale are important, one is necessarily dealing with an industry composed of a small number of large firms, and it would seem slightly artificial to impose on a
Before proceeding to matters of detail, it is convenient to clarify what will be meant by marginal cost pricing. We shall say that a price vector $p$ constitutes a set of marginal cost prices for firm $i$ at production plan $y_i$ if $p$ is normal to the hyperplane which is a local support to $f_i$'s production possibility frontier at $y_i$. In the differentiable case, this implies that all price ratios are equal to the respective marginal rates of transformation. Note that in an earlier discussion of marginal cost pricing and increasing returns, Arrow and Hurwicz (1961, p. 92) propose a different definition: they define marginal cost prices as prices equal to the first derivatives of cost functions. The obvious shortcoming of such an approach is that a cost function is only defined once a set of prices has been chosen. This makes it much more suitable for partial rather than general equilibrium analysis. Indeed, if one is dealing with a nonconvex economy where a competitive equilibrium price vector does not exist, then this approach is not useful.

In relating our results to the possibility of supporting an efficient allocation by a system of marginal cost pricing with two-part tariffs, we reach some interesting and apparently novel conclusions which seem not to be obtainable by a partial equilibrium approach. In particular, we conclude that under conditions of increasing returns in production, a Pareto-efficient equilibrium, if it exists, may be supported by a system of marginal cost pricing with two-part tariffs, provided that:

(i) the fixed parts of the two-part tariffs depend on individual preferences and therefore in general vary from person to person, and

(ii) the fixed parts are associated, not with purchases from particular firms, but with purchases of particular goods.

This clearly does not conform fully with the usual idea of such a pricing system: normally, such a system is seen as impersonal in that the fixed parts are the same for all, and firm-based, in that these fixed parts are associated with the use of a particular firm.

In fact it is only under extremely special conditions that it will be possible to support an efficient allocation with impersonal two-part tariffs. It should be noted that the fact that the fixed parts of the two-part tariffs depend upon individual preferences, implies that the computation of these fixed parts requires information about preferences. It is clear that any attempt to obtain such information may encounter the general problem of incentive-compatibility in preference revelation: individuals may have an incentive to misrepresent their preferences in such a way as to minimise the fixed charges that they are asked to pay.

The requirement that tariffs should be product-based and not firm-based, is one that appears not to have been noted before. Probably this stems from the fact that most partial equilibrium models consider only a single firm, in
which special case the two are of course identical. This special case is not without interest: it clearly includes for example telephone companies and electric power companies. In fact the idea of relating prices to goods, rather than to firms, is entirely natural and is within the standard economic tradition. If the fixed part of a two-part tariff related to the firm from which the purchase was made, there would in general be several different prices for the same good. We shall see in due course that there are important reasons for avoiding such a situation.

Finally, it should be noted that we said above that a Pareto-efficient equilibrium, if it exists, may be supported in this way. The caveat 'if it exists' is important, for it was shown in Brown and Heal (1979) that for many nonconvex economies there will be no marginal-cost-pricing equilibrium which is Pareto efficient. In particular, it was shown in Brown and Heal (1979) that the existence or otherwise of a Pareto-efficient marginal-cost-pricing equilibrium (an equilibrium where prices equal marginal costs in the sense defined above, and consumers maximise utilities at these prices) depends on the distribution of initial endowments amongst individuals. For some ways of distributing the economy's given total of endowments amongst individuals there is no efficient marginal-cost-pricing equilibrium, whereas for other distributions such an efficient equilibrium exists. Note that what is at issue here is the existence of an efficient equilibrium, not just the existence of an equilibrium: the latter is demonstrated under weak conditions in Brown and Heal (1976).

The fact that with increasing returns in production there may be no marginal-cost-pricing equilibrium which is Pareto efficient, seems to indicate a very major shortcoming in the idea of marginal cost pricing. It has presumably not been noticed before [except by Guesnerie (1975)] because all other studies have been partial equilibrium in nature. The fact that the fixed parts of two-part tariffs need in general to be personalised, must also reduce further the attractions of marginal cost pricing and multipart tariffs.

2. The equilibrium concept

In this section we remind the reader of the nature of the equilibrium whose existence was established in Brown and Heal (1976), and then proceed to prove what seems to us the nearest equivalent to the usual result on the production side. The model with which we work is as in Brown and Heal (1976): \( \mathbb{R}^n \) is the commodity space, and

\( Y_i; \mathbb{C}^R^n \) is firm \( i \)'s production possibility set, \( i = 1, 2, \ldots, f; \)
\( w_j \in \mathbb{R}^n \) is individual \( j \)'s initial endowment, \( j = 1, 2, \ldots, m; \)
\( X_j; \mathbb{C}^R^n \) is individual \( j \)'s consumption possibility set, \( j = 1, 2, \ldots, m; \) and
\( U_j; \mathbb{R}^R^1 \) is individual \( j \)'s utility function.
$w_j$ and $X_j$ are assumed to lie in the non-negative orthant. We remind the reader that a nonlinear value function is a function $V: R^n \rightarrow R^1$ which is linear homogeneous and of class $C^1$. It there has the convenient property that $V(x) = VV(x) \cdot x$, where $VV(x)$ is the gradient of $V(x)$ at $x$, assumed to be non-negative, and we shall refer to $VV(x)$ as the linear part of $V$, or price.

The assumptions made about consumers and producers are:

A1. $Y_j$ is closed, nonempty and contains the negative orthant.

A2. $Y = \sum_{j=1}^{\infty} Y_j + \sum_{j=1}^{n} w_j$ is bounded.

A3. $Y$ is supported by a pointwise bounded and equicontinuous family of nonlinear prices closed in the $C^1$ compact topology.

A4. The $X_j$ are closed, convex and nonempty.

A5. The $U_j$ are continuous, monotonic and quasiconcave.

In addition to assumptions A1 A5 we need an assumption which will ensure that at an equilibrium, each consumer is above the lower boundary of his consumption possibility set. It is difficult to find a single formulation of this that is satisfactory for all of our purposes, so we shall leave the formulation of such an assumption until it is needed. Of these assumptions, the only one that seems to require comment is A3, which requires that any efficient point in $Y$, the aggregate feasible set, is supported by a nonlinear price. Intuitively speaking, A3 requires that the boundary of $Y$ be 'smooth enough'.

We can now describe the equilibrium whose existence was established in Brown and Heal (1976) under the above assumptions.

**Definition.** An equilibrium is a set of consumption vectors $x^*_j \in X_j, j = 1, 2, \ldots, m$, and a nonlinear value system $V$ such that

(i) $x^* = \sum_{j=1}^{m} x^*_j$ maximises $V$ over $Y$, and

(ii) for all $j$, $x^*_j$ maximises $U_j(x_j)$ subject to $VV(x^*) \cdot x_j \leq I_j$.

The first part of the definition requires that aggregate consumption, which must of course equal aggregate net output, maximises value at a nonlinear price over the aggregate feasible set. The second part requires each consumption vector to be utility maximising subject to a budget constraint in which the price vector is the linear part of the nonlinear price evaluated at $x^*$, and $I_j$ is the value of the consumer's endowments plus his share of profits, corrected for any taxes or subsidies levied on these. In Brown and Heal (1976) $I_j$ took the form $\beta_j V(x^*)$, where $\beta_j$ was $j$'s share in the total value of production of the economy. In subsequent sections we shall examine in more detail the components of the right-hand side of the consumer's budget constraint.
We are now in a position to state and prove the following.

**Theorem 1.** Let \(x^*_i, j = 1, 2, \ldots, m\) and \(V\) form an equilibrium. Then there exist production plans \(y^*_i, \ i = 1, 2, \ldots, f\), and vectors \(z_i, \ i = 1, 2, \ldots, f\), such that

\[
\begin{align*}
(i) \quad & \sum_{i=1}^{m} y^*_i = \sum_{j=1}^{m} x^*_j - \sum_{j} w_j, \\
(ii) \quad & V(y^*_i + z_i) \geq V(y^*_i + z_i), \quad \forall y_i \in Y_i, \quad \text{each } i, \\
(iii) \quad & V(V(y^*_i + z_i)) = V(y^*_i + z_i), \quad \forall i, k, \\
(iv) \quad & V(y^*_i + z_i) \geq 0, \quad \forall i.
\end{align*}
\]

The implication of this theorem is that the firm production plans associated with the equilibrium may be supported, not by maximisation of the price system \(V\) that works for the aggregate set \(Y\), but by maximisation of value at price systems derived from this in a very simple fashion, namely by shifting its origin by various constant vectors \(z_i\).

Furthermore, the various nonlinear prices derived by this means and supporting the individual firm production plans, all have the same linear part at an equilibrium configuration, and imply non-negative 'profits' for the firms.

**Proof of theorem 1.**

\[
V\left(\sum_{j=1}^{m} x^*_j\right) = V\left(\sum_{j=1}^{f} y^*_j + \sum_{j} w_j\right) \geq V(y), \quad \forall y \in Y.
\]

i.e.

\[
V\left(\sum_{j=i}^{m} y^*_j + \sum_{j} w_j + y^*_j\right) \geq V\left(\sum_{j=i}^{m} y^*_j + \sum_{j} w_j + y_j\right), \quad \forall y_i \in Y_i.
\]

Let

\[
z_i = \sum_{j=i}^{m} y^*_j + \sum_{j} w_j.
\]

Then

\[
V(y^*_i + z_i) \geq V(Y_i + z_i), \quad \forall y_i \in Y_i, \quad \text{for each } i.
\]

This proves parts (i) and (ii) of the theorem. To prove (iii) note that \(z_i + y^*_i = z_i + y^*_i, \forall i\) and \(k\), so that \(V(V(z_i + y^*_i)) = V(V(z_i + y^*_i))\), as required. (iv) follows trivially from the fact that

\[
V(y^*_i + z_i) = V\left(\sum_{j=1}^{m} x^*_j\right) = VV\left(\sum_{j=1}^{m} x^*_j\right) = V\left(\sum_{j=1}^{m} x^*_j\right) = V\left(\sum_{j=1}^{m} x^*_j\right)
\]
and
\[ \sum_{j=1}^{m} x_{j}^{*} \in R_{+}^{m}, 0. \]

and the price vector \( PV \) is non-negative.

**Remark 1.** At an equilibrium where production plans are supported in this way, it is clear that all of the standard first-order conditions for a welfare optimum are satisfied. An equilibrium is therefore defined by market clearing, utility maximisation by consumers, and by the first-order conditions. For a similar definition of equilibrium in terms of the first-order conditions in the case of non-convexities of preferences, see Chichilnisky (1976).

**Remark 2.** On the production side, an equilibrium may be supported by each firm maximising \( V(z_i + y_i) \), where
\[ z_i = \sum_{k \neq i} y_{k}^{*} + \sum_{j=1}^{m} w_j. \]

\( z_i \) thus contains information about the production plans of all other firms in aggregate, and each firm can be seen to be maximising the value of \( V \) over its production set, taking as given the actions of other firms, which of course affect \( V \). The equilibrium is thus a *Nash equilibrium*. We have therefore constructed a game between firms with the property that any equilibrium set of production plans forms a Nash equilibrium of this game.

### 3. A pricing interpretation

We now turn to the institutional interpretation that can be given to the results just established. It is natural to think of firms producing at \( y_{i}^{*} \), and buying and selling inputs and outputs at prices \( PV(x^{*}) \), where \( x^{*} = \sum_{j} y_{j}^{*} \). On the output side, such behaviour would clearly correspond to marginal cost pricing, for \( PV(x^{*}) \) is a local support to \( Y_i \) at \( y_{i}^{*} \).

Next consider the inequality
\[ V(y_{i}^{*} + z_i) = PV(x^{*}) \cdot y_{i}^{*} + PV(x^{*}) \cdot z_i \geq 0. \]

It is clear from this that the proceeds \( PV(x^{*}) \cdot y_{i}^{*} \) from buying inputs and selling outputs at prices \( PV(x^{*}) \) need not be non-negative — or, more conventionally — that marginal cost pricing may lead to losses if there are increasing returns in production. We can see now that a non-negative value
for the function \( V \) at \( y_{i}^{*} \) is ensured, in effect, by the administration of taxes or subsidies to the value of \( V V(x^{*}) \cdot z_{i} \).

It should be noted that these are not, in the conventional sense, lump sum taxes or subsidies, for

\[
V V(x^{*}) \cdot z_{i} = V (y_{i}^{*} + z_{i}) \cdot z_{i},
\]

showing that the value of this subsidy depends upon the firm's output \( y_{i} \) via the influence that this has on the prices \( V V(x^{*}) \). Although using Euler's theorem does make the nature of the subsidy very clear, the same points can be seen by careful consideration of the fact that a firm's production plan \( y_{i}^{*} \) is supported not by \( V (y_{i}) \) but by \( V (y_{i} + z_{i}) \). The difference between the two amounts to a shift of origin by \( z_{i} \), which can be thought of as an instruction to the firm that in using the function \( V (\cdot) \) to compute the value of a production plan, it should take no account of the first \( z_{i1} \) units of good 1 that it uses, or the first \( z_{i2} \) of good 2, etc. We have clearly left open the issue of the source of any subsidies to be administered to firms, and we now turn to that issue, showing that they can in fact be raised from consumers in a nondistortionary manner.

4. Alternative institutional interpretations

In this section we discuss various levies on consumers for the purpose of covering the losses resulting from marginal cost pricing. These levies are susceptible to several interpretations: they can be regarded as taxes, or as two-part tariffs, or as the consequences of unlimited liability shareholdings or partnerships. We begin by writing the right-hand side of consumer \( j \)'s budget constraint as:

\[
I_{j} = \sum_{i=1}^{f} \beta_{ji} \pi_{i} + V V(x^{*}) \cdot w_{j} - T_{j},
\]

where

\[
\sum_{j=1}^{m} \beta_{ji} = 1, \quad \beta_{ji} \geq 0, \quad \forall i, j,
\]

\( \pi_{i} \) is firm \( i \)'s profits, given by

\[
\pi_{i} = V V(x^{*}) \cdot y_{i} + V V(x^{*}) \cdot z_{i} = V (y_{i} + z_{i}),
\]

and \( T_{j} \) is the levy on consumer \( j \) given by the expression

\[
T_{j} = \sum_{i=1}^{f} \gamma_{ji} V V(x^{*}) \cdot z_{i},
\]
where

\[ \sum_{j=1}^{n} \gamma_{ij} = 1, \quad \gamma_{ij} \geq 0, \quad \forall \ i, j. \]

The consumer is therefore constrained to choose a consumption vector \( x_j \) whose value does not exceed the profits accruing to him by virtue of his ownership of a share \( \beta_{ij} \) of firm \( i \), plus the value of his initial endowment, but net of the levy \( T_j \) imposed on him. This levy is computed as the sum over all firms of a fraction \( \gamma_{ij} \) of subsidy \( VV(x^*) \cdot z_j \) to firm \( i \). An individual is thus liable to receive a fraction \( \beta_{ij} \) of the profits of firm \( i \), and also to meet a fraction \( \gamma_{ij} \) of any subsidy administered to it. Note that

\[
\sum_{j=1}^{n} l_j = \sum_{j=1}^{n} \beta_{ji} \pi_i + \sum_{j=1}^{n} VV \cdot w_j - \sum_{j=1}^{n} T_j \\
= \sum_{i=1}^{n} \pi_i + VV \cdot w - \sum_{i=1}^{n} VV \cdot z_j \\
= \sum_{i=1}^{n} VV \cdot y^*_i + VV \cdot w \\
= V(y^* + w) = V(x^*),
\]

where \( w = \sum_{j=1}^{n} w_j \). This establishes that the budget constraints defined in this way are consistent in the sense that the sum of consumers' incomes is equal to the value of aggregate consumption.

We remarked in section 2 that the proof of the existence of an equilibrium of course requires that each consumer's budget permits him to reach a point interior to his consumption set, and that such a condition is in general difficult to specify. Now that the budget constraint has been analysed in detail, this point should be very clear, and we can do no better than to assume that the levies \( \gamma_{ij} \) are always such that this condition is met. Given the general concern with progressivity in the tax system, this does not seem unreasonable: certainly it is unavoidable.

We have now described a system of subsidies to firms, and levies on individuals, such that firms may price at marginal cost and record non-negative after-tax (or subsidy) profits, all first-order conditions for optimality are met, and the whole is consistent in a general equilibrium framework. The next step, clearly, is to look a little more carefully at the levies on individuals and to investigate the interpretations that may be placed on these. There are, as we have mentioned, several.

1. They may be interpreted as taxes. This is the most obvious, but perhaps least appealing or interesting, interpretation. \( T_j \) may just be viewed as a lump-sum tax on individual \( j \). This is a slight oversimplification since it
has already been remarked that the value of the subsidy received by firm $i$ is not in fact lump sum in the usual sense of being independent of the firm's activities, for it is given by $VV(y_j + z_j)$ and depends explicitly in value terms (though not in physical terms) on these.

(2) If $\gamma_{ji} = \beta_{ji}$ for all $i$ and $j$, the levies may be regarded as a consequence of a form of property ownership that would be termed *unlimited liability shareholding* or *partnership*.

The right-hand side of a budget constraint may be written as:

$$I_j = VV(x^*) \cdot w_j + \sum_{i=1}^{f} \beta_{ji} VV \cdot y_j + \sum_{i=1}^{f} \left( \beta_{ji} - \gamma_{ji} \right) VV \cdot z_i$$

and if $\beta_{ji} = \gamma_{ji}$,

$$I_j = VV \cdot w_j + \sum_{i=1}^{f} \beta_{ji} VV \cdot y_j$$

which is identical in form to the standard general equilibrium budget constraint, with the subtle but important difference that in the usual case profits are non-negative (profit-maximising firms are always seen as having the option of closing down), whereas in the present case the value $VV \cdot y_j$ of a production plan may well be negative in equilibrium. One therefore has a world in which the owners of a firm share both in its profits and its losses. This is by no means an unimaginable situation: most shareholders in the U.K. were in precisely this position until the general legal recognition of limited liability companies in 1856, and members of a partnership are still in this position. Indeed, without excessive use of imagination one can regard all taxpaying members of the general public as shareholders in nationalised industries, and in this capacity their liabilities are very clearly unlimited. General tax revenue provided by them is used to meet losses incurred by these industries. We therefore have an interesting and recognisable institutional interpretation for the levies $T_j$ in the case when $\beta_{ji} = \gamma_{ji}$, an interpretation resting on a slightly different system of property rights from that in the general equilibrium model.

(3) The final interpretation of the budget constraints is a system of firm-based two-part tariffs. Suppose that $\gamma_{ji} > 0$ if and only if individual $j$ consumes the output of firm $i$. Then we can regard the payment by $j$ to $i$ as consisting of two parts, an amount $\gamma_{ji} P_i V \cdot z_i$ which is paid independent of $j$'s consumption of $i$'s outputs, plus payment at the rate given by the appropriate components of $P_i V(x^*)$ for the units actually consumed. This is similar to the usual view of a system of two-part tariffs: a per unit charge designed usually to cover the marginal cost of production, with a fixed charge or connection charge intended to recover some or all of the fixed costs of production. Alternatively, but in the same vein, one could draw on an
analogy with the local public goods literature and regard membership of a 
club as a prerequisite for the consumption of a particular firm's output. 
Payment of a membership fee of $r_{ij}VV(x^*) \cdot z_i$ entitles one to purchase this 
output at marginal cost.

We now have given three alternative interpretations of the levies imposed 
on individuals to cover the subsidies needed to sustain marginal cost pricing. 
As was to be expected, one was the traditional lump-sum tax approach, 
normally viewed as impracticable. But the alternative interpretations are far 
more interesting and suggest that the same outcome can be achieved by 
institutional arrangements that it should in principle be possible to imple-
ment. The unlimited liability shareholding, or partnership, and the two-part 
tariff, are recognisable phenomena. We look in more detail at the use of two- 
part tariffs in the next section.

5. Two-part tariffs: A discussion

It is instructive to look a little more closely at the idea that an equilibrium 
may be sustained by a system of two-part tariffs. Essentially, the results 
established imply that there exist a set of two-part tariffs such that if the 
consumers of an industry's output take their liabilities to the fixed compo-
nents as given and maximise utility at prices given by the variable part, 
subject to a budget constraint net of the fixed part, then, along with suitable 
behaviour on the part of firms, this leads to an equilibrium. The important 
point to note about this is that in their utility maximisation, consumers are 
taking their liability to the fixed component of the tariff as given. If $F_{ji}$ is the 
fixed component of industry $i$'s tariff that is levied on $j$, then his con-
sumption vector is the solution to the problem:

$$\begin{align*}
\text{maximise} & \quad U_j(x_j) \\
\text{subject to} & \quad VV(x^*) \cdot x_j \leq I_j.
\end{align*} \quad (A)$$

This is of course not quite the same as the problem that faces a consumer 
who has the option of not consuming the output of firm $i$, and thereby 
avoiding the charge $F_{ji}$. His problem is:

$$\begin{align*}
\text{maximise} & \quad U_j(x_j) \\
\text{subject to} & \quad VV(x^*) \cdot x_j \leq \begin{cases} 
I_j + F_{ji} & \text{if } x_{ji} = 0, \\
I_j & \text{otherwise.}
\end{cases}
\end{align*} \quad (B)$$

Clearly the budget sets for (A) and (B) are different: fig. 1 illustrates this for 
a case when the consumption of the output of firm 1 has associated with it a 
fixed charge. The set for (A) is the conventional set on or below a
hyperplane, whereas in (B) the budget set consists of the union of this set with a segment of the vertical axis.

Clearly there will be many cases when the inclusion of this extra segment has no effect on the consumer's choice (as with the indifference curve shown in fig. 1): equally clearly, this is not generally true. It is therefore necessary to look very carefully at the precise sense in which we have shown that an equilibrium can be supported by a system of two-part tariffs. This is true only to the extent that the fixed component is something for which the consumer has already been assessed, and which he thus takes as given when choosing his consumption vector. It is in fact possible to strengthen our earlier results, though at the cost of considerable complexity, and we now turn to this extension. First of all, we consider the intuitive explanation of the results we are about to establish.

It is clear from fig. 1 that if the solution to problem (A) involves the consumption of strictly positive amounts of both goods, then there exists \( \epsilon_\mu > 0 \) such that for \( F_\mu \leq \epsilon_\mu \), the solutions to (A) and (B) are the same. In a very loose and heuristic sense, which it will not prove necessary to make precise, \( \epsilon_\mu \) is a measure of individual \( \mu \)'s willingness to contribute to the losses of firm
6. Two-part tariffs: Formal results

We saw in section 4 that the equilibrium discussed in section 2 can be given an interpretation which is at least reminiscent of a system of two-part tariffs and marginal cost pricing. The previous section made it clear that though there are suggestive similarities, the correspondence is by no means exact. In earlier sections the model was not formulated in such a way as to provide for consumers the possibility of avoiding the fixed parts by appropriate choices of consumption bundles.

The intuitive arguments presented above concerning consumer surplus suggest that a suitable reformulation may be possible. In this section we provide such a reformulation though it should be noted that it is in no sense a formalisation of the intuitive arguments about consumer surplus. Being partial equilibrium in nature, these do not carry through to the general equilibrium context. Indeed, as mentioned in the introduction, we do not prove that an efficient equilibrium can be supported by the standard two-part tariff system where the fixed parts are impersonal and associated with purchases from particular firms. It is in general necessary that the fixed parts vary from person to person, and are associated with purchases of particular goods, rather than from particular firms.

This raises some questions about the precise financial arrangements implicit in our results: tariffs are associated with goods, not firms, but losses are made by firms, not goods. In the models implicit in much previous theorising on this issue, there is a single producer whose sole output is the increasing-returns good and who is the only producer of this. Obviously in this context there is no problem: the fixed parts of consumer payments automatically go where they are needed. Otherwise, one has to envisage a centralised system for collecting these and distributing them as appropriate.

The first theorem in this section, theorem 2, is an essentially technical result used in the proof of the main theorem. This result could be regarded as an aggregative form of the intuition discussed in the last section (see fig.
2. The aggregate preferred-or-indifferent set at \( x^* \), \( PI(x^*) \), is shaded, and the budget set given by the theorem is the set of points on or below the line through \( x^* \) supporting both sets at \( x^* \), together with the segment of the subspace \( L \) (generated by the origin and the maximiser of \( P V(x^*) \cdot x \) over \( \hat{Y} \)) that lies inside \( \hat{Y} \). This is a budget set that corresponds to marginal cost pricing at the prices that are local supports at \( x^* \), with fixed charges associated with consumption of any vector outside the subspace \( L \). The theorem asserts that it is always possible to construct an aggregate budget set of this sort whose unique intersection with the aggregate preferred-or-indifferent set at \( x^* \), is \( x^* \).

Having established this, we then show in theorem 3 that this aggregate budget set can be viewed as the sum of individual budget sets of the type discussed in the last section, and hence go on to establish the possibility of decentralising consumer decisions at an efficient allocation via a two-part tariff system.

The subspace \( L \), whose structure is discussed at length below, can be thought of as containing goods with which no production losses are associated. At the aggregate level the fixed cost associated with consuming a vector not in \( L \) is

\[
\max_{x \in \hat{Y}} PV \cdot x - PV \cdot x^*; \quad \hat{Y} = Y \cap R^n. 
\]

Fig. 2. \( \hat{Y} = Y \cap R^n \).
As the origin is in $\hat{Y}$, this difference must be greater than or equal to the sum of the losses incurred at $x^*$. What is therefore being shown is that, at the aggregate level, a spike such as that on the vertical axis of fig. 1, big enough to cover losses, may be appended to the budget set without disturbing the equilibrium.

**Theorem 2.** Let $L$ be a subspace of $R^n$. Let $A(L)$ be:

$$A(L) = \sum_j PI_j(x^*_j) \cap \begin{cases} \{ x : \max V \cdot x = \frac{\max V \cdot x^*_j}{V \cdot x^*_j} \leq \frac{\max V \cdot x^*}{V \cdot x^*} \} & \text{if } x \in L, \\ \{ x : \max V \cdot x = \frac{\max V \cdot x^*_j}{V \cdot x^*_j} \leq \frac{\max V \cdot x^*}{V \cdot x^*} \} & \text{if } x \notin L, \end{cases}$$

where the $x^*_j, j = 1, \ldots, m$, form a Pareto-optimal equilibrium, and $VV$ is the associated linear price system. If

$$\sum_j PI_j(x^*_j) \cap \hat{Y} = \{ x^* \}, \quad (1)$$

then there exists a subspace $L$ such that $A(L) = \{ x^* \}$.

**Proof.** Let $\hat{x}$ satisfy $\hat{x} \in \hat{Y}$ and

$$VV \cdot \hat{x} = \max_{x \in \hat{Y}} VV \cdot x$$

and let $L$ be the subspace spanned by $(0, \hat{x})$. Clearly $x^* \in A(L)$. It is also clear from the fact that $VV \cdot x = VV \cdot x^*$ supports $\sum_j PI_j(x^*_j)$ at $x^*$, and from (1) that $x^*$ is the only point in $A(L)$ satisfying $VV \cdot x \leq VV \cdot x^*$. Hence any other point in $A(L)$ must be in $L$. Suppose then that there exists $x' \in A(L), x' \neq x^*$. Then

$$x' \in L, VV \cdot x' > VV \cdot x^*.$$

As

$$VV \cdot x' \leq \max_{x \in \hat{Y}} VV \cdot x, \quad x' \in \hat{Y};$$

but as $x' \in A(L), x' \in \sum_j PI_j(x^*_j)$. This violates the assumption that

$$\sum_j PI_j(x^*_j) \cap Y = \{ x^* \}.$$

Hence there is no such $x'$, and the chosen $L$ meets the conditions of the theorem.
Remark 1. If

$$PV \cdot x^* = \max_{x \in Y} PV \cdot x,$$

then there are no losses in total.

If, in addition,

$$x^* \max PV \cdot x \quad \text{over} \quad x \in \left( \sum_{i} Y_i + w \right)$$

then clearly each firm maximises profits at the prices $PV$ at the equilibrium production plan $y_i^*$.

In general, if

$$PV \cdot x^* = \max_{x \in Y} PV \cdot x,$$

then we can pick the $z_i$ so that

$$PV \cdot y_i^* + PV \cdot z_i \geq 0, \quad \forall i, \quad \text{and} \quad \sum_i PV \cdot z_i = 0.$$

Thus, there is in this case no need to impose levies on consumers.

Remark 2. The subspace $L$ will be used to define the commodities with whose purchase no fixed charge is associated. As defined above, $L$ is an arbitrary subspace of $R^n$, and this poses problems of interpretation. The fixed charge might for example be avoidable only by buying positive amounts of all commodities in fixed proportions – an unappealing situation. In fact we can make a more felicitous choice of $L$.

Consider

$$\sum_j P_i(x_j^*) \cap \left\{ x / PV \cdot x = \max_{x \in Y} PV \cdot x \right\}.$$

This is the intersection of two convex sets. Suppose that this were to contain $n$ vectors of the form $(x_1, 0, \ldots, 0)$, $(0, x_2, 0, \ldots, 0)$, $(0, 0, x_3, \ldots, 0)$, \ldots, $(0, \ldots, 0, x_n)$.
for \( x_i > 0, \ i = 1, \ldots, n \). Then it would contain

\[
\left\{ x / \forall V \cdot x = \max_{x \in Y} \forall V \cdot x \right\} \cap R^*_+, 
\]

so that

\[
\sum_j P_{j}(x^*_j) C R^*_+, \ C \left( \sum_j P_{j}(x^*_j) \right) \cap \left\{ x / \forall V \cdot x = \max_{x \in Y} \forall V \cdot x \right\} = \emptyset, 
\]

where \( C(X) \) is the complement of \( X \). But this is clearly false. Hence

\[
C \left( \sum_j P_{j}(x^*_j) \right) \cap \left\{ x / \forall V \cdot x = \max_{x \in Y} \forall V \cdot x \right\} 
\]

contains at least one vector of the form \((0, 0, \ldots, x_i, \ldots, 0)\) for some \( i, x_i > 0 \). Let \( x^0 \) be such a vector, and pick as \( L \) the subspace generated by \((0, x^0)\). Then theorem 2 is still true: the proof only needs to be amended as follows.

Suppose there exists \( x' \in A(L) \), \( x' \neq x^* \). Then \( x' \in L, \forall V \cdot x' \leq \max_{x \in Y} \forall V \cdot x \). But \( x' \in \sum_j P_{j}(x^*_j) \) by construction of \( A(L) \), contradicting the fact that \( x^0 \in C \left( \sum_j P_{j}(x^*_j) \right) \).

We can thus choose the subspace \( L \) so that it contains at least one axis of the commodity space, which gives the possibility of a more natural interpretation.

**Theorem 3.** Let \( x^*_j, j = 1, 2, \ldots, m \) form a Pareto-optimal equilibrium satisfying

\[
\sum_j P_{j}(x^*_j) \cap Y = \{ x^* \}. 
\]

The associated prices are \( \forall V \cdot x \), and individual \( j \)'s budget constraint is:

\[
\forall V \cdot x_j \leq I_j, 
\]

where \( \sum_j I_j = \forall V \cdot x^* \). Firm \( i \) produces \( x^*_j \) making a profit or loss of \( v^*_i \cdot \forall V \). Then there exist fixed tariffs \( T_j, j = 1, \ldots, m \), and a subspace \( I \) of \( R^* \) such that

(i) for each \( j \), \( x^*_j \) is the solution of

\[
\max U_j(x_j) 
\]

subject to

\[
\forall V \cdot x_j \leq \begin{cases} 
I_j + T_j & \text{if } x_j \in I, \\
I_j & \text{if } x_j \notin I. 
\end{cases}
\]
(ii) if
\[ \max_{x \in \bar{V}} \mathcal{V} \cdot x = \max_{x \in \mathcal{Y}} \mathcal{V} \cdot x, \]
then
\[ \sum_{j} T_j \geq \sum_{i \in I^-} (-w \cdot y_i^*), \]
where \( I^- = \{ i : \mathcal{V} \cdot y_i^* < 0 \} \); otherwise, \( \sum_{j} T_j \geq \sum_{i} (\mathcal{V} \cdot y_i^*) \).

**Remark.** We are here identifying a subspace \( L \) with the property that as soon as a consumer chooses a consumption vector in the complementary subspace, he is required to pay a fixed charge. So \( L \) represents consumption vectors on which no fixed charge is levied, and \( C(L) \), its complementary subspace, represents those with an associated fixed charge. The theorem, then, asserts that it is possible to pick \( L \) and the fixed charges in such a way that the original consumer’s choices at the efficient equilibrium are not disturbed, so that the equilibrium is fully sustainable on the consumption side by these two-part tariffs. Furthermore, the sum of the fixed charges paid by consumers at equilibrium is at least sufficient to cover the total losses of all loss-making firms.

An unattractive feature of this result is clearly the rather general and aggregative nature of the subspace \( L \). Theorem 2 and the following remark assures us that \( L \) contains at least one axis of the commodity space – it does not consist entirely of consumption vectors with a number of commodities in fixed proportions. However, one would clearly like stronger results than this. It would be more satisfactory to be able to associate fixed charges with the consumption of the outputs of particular firms. Unfortunately, it seems impossible to be more precise about the subspace \( L \) and to particularise the fixed charges and associate them with individual firms without putting a great deal more structure on the underlying model of firms’ production possibilities. In our present framework there is no way of identifying which firms have increasing returns, which firms operate at a loss or which goods are produced at a loss. Only if one specified a more detailed model with these features built in would it be possible to be more precise about the tariff structure.

A further comment that should probably be made by way of interpretation is that it is natural to assume that whether or not a consumer is liable to the fixed part of a two-part tariff should depend on his excess demand, rather than on his demand. We would presumably not expect him to pay a fixed charge to consume his own initial endowments. Again this is a shortcoming that can really only be fully overcome in a more detailed model than the
present: here we can only cover it by assuming that consumer's have zero
endowments of goods which are produced under increasing returns and on
which a fixed charge is levied. This is not unreasonable: one does not enter
the world with a stock of telephone services, electricity, etc.

The stronger version of (ii) of the theorem – that the \( T_j \) cover the total
losses of loss-making firms, as opposed to merely covering the net loss of all
firms – depends on the maximum of \( \mathcal{P} \mathcal{V} \) over \( \hat{Y} \) (which is \( Y \) restricted to \( R^* \))
equalling its maximum over \( Y \). This is clearly a possible case, maybe even a
likely one, but will not always be true. The same stronger version of (ii) can
be established if theorem 2 can be strengthened so that in defining \( A(L) \) the
maximum is over \( Y \) rather than \( \hat{Y} \). For simplicity, details are omitted. And
again for simplicity the following proof deals only with the stronger version
of (ii) when the maximum of \( \mathcal{P} \mathcal{V} \) over \( \hat{Y} \) equals its maximum over \( Y \). The
interested reader can no doubt supply the missing arguments.

**Proof**

\[
\begin{cases}
    x_j \mathcal{P} \mathcal{V} \cdot x_j \leq I_j + T_j & \text{if } x \in L \\
    I_j & \text{if } x \notin L
\end{cases}
\]

\[
= \{ x_j \in L | \mathcal{P} \mathcal{V} \cdot x_j \leq I_j + T_j \} \cup \{ x_j \notin L | \mathcal{P} \mathcal{V} \cdot x_j \leq I_j \}.
\]

From theorem 2 we can pick \( L \) so that

\[
\sum_j PI_j(x^*) \cap \{ x | \mathcal{P} \mathcal{V} \cdot x \leq \begin{cases}
    \max_{x \in \hat{Y}} \mathcal{P} \mathcal{V} \cdot x, & \text{if } x \in L \\
    \mathcal{P} \mathcal{V} \cdot x^*, & \text{if } x \notin L
\end{cases} \}= \{ x^* \}.
\]

Choose \( I_j \) and \( T_j \) so that

\[
\sum_j I_j = \mathcal{P} \mathcal{V} \cdot x^* \quad \text{and} \quad \sum_j (I_j + T_j) = \max_{x \in \hat{Y}} \mathcal{P} \mathcal{V} \cdot x.
\]

Then we can pick \( L \) so that

\[
\sum_j PI_j(x^*) \cap \{ x | \mathcal{P} \mathcal{V} \cdot x \leq \sum_j (I_j + T_j) \} = \{ x^* \},
\]

i.e.

\[
\sum_j PI_j(x^*) \cap \left\{ x \in L | \mathcal{P} \mathcal{V} \cdot x \leq \sum_j (I_j + T_j) \right\} \cup \left\{ x \notin L | \mathcal{P} \mathcal{V} \cdot x \leq \sum_j I_j \right\} = \{ x^* \},
\]
\[
\left[ \sum_{j} PI_{j}(x_{+}^{\ast}) \cap \left\{ x \in L \mid \forall V \cdot x \leq \sum_{j} (I_{j} + T_{j}) \right\} \right] \\
\cup \left[ \sum_{j} PI_{j}(x_{+}^{\ast}) \cap \left\{ x \notin L \mid \forall V \cdot x \leq \sum_{j} I_{j} \right\} \right] = \{ x^{\ast} \},
\]

i.e.
\[
\left[ \sum_{j} PI_{j}(x_{+}^{\ast}) \cap \sum_{j} \left\{ x_{j} \in L \mid \forall V \cdot x_{j} \leq I_{j} + T_{j} \right\} \right] \\
\cup \left[ \sum_{j} PI_{j}(x_{+}^{\ast}) \cap \left\{ x \notin L \mid \forall V \cdot x \leq \sum_{j} I_{j} \right\} \right] = \{ x^{\ast} \}.
\]

Here we have two sets whose union is \( \{ x^{\ast} \} \). Hence either both equal \( \{ x^{\ast} \} \), or one equals \( \{ x^{\ast} \} \) and one is empty. But they have no point in common: hence one equals \( \{ x^{\ast} \} \) and the other is empty. We can certainly choose \( L \) so that \( x^{\ast} \notin L \). Hence

\[
\sum_{j} PI_{j}(x_{+}^{\ast}) \cap \sum_{j} \left\{ x_{j} \in L \mid \forall V \cdot x_{j} \leq I_{j} + T_{j} \right\} = \emptyset,
\]

i.e.
\[
\sum_{j} \left[ PI_{j}(x_{+}^{\ast}) \cap \left\{ x_{j} \in L \mid \forall V \cdot x_{j} \leq I_{j} + T_{j} \right\} \right] = \emptyset,
\]

i.e.
\[
PI_{j}(x_{+}^{\ast}) \cap \left\{ x_{j} \in L \mid \forall V \cdot x_{j} \leq I_{j} + T_{j} \right\} = \emptyset, \quad \text{for all} \; j.
\]

This establishes that augmenting the budget set by \( T_{j} \) on the subspace \( I_{j} \) does not change the consumer's choice, proving the first assertion of the theorem.

Now note that by construction

\[
\sum_{j} I_{j} = \forall V \cdot x^{\ast} \quad \text{and} \quad \sum_{j} (I_{j} + T_{j}) = \max_{x \in \mathcal{Y}} \forall V \cdot x.
\]

Hence

\[
\sum_{j} T_{j} = \max_{x \in \mathcal{Y}} \forall V \cdot x - \forall V \cdot x^{\ast}.
\]

But

\[
\forall V \cdot x^{\ast} = \forall V \cdot x^{\ast} - \forall V \cdot w.
\]
So
\[ \sum_{j} T_j = \max_{x \in \mathcal{I}} PV \cdot x - PV \cdot (y^* + w) \]
\[ = \max_{x \in \mathcal{I}, y \in \mathcal{Y}} PV \cdot x - PV \cdot (y^* + w). \]

Subtracting $PV \cdot w$ from both terms on the right-hand side
\[ \sum_{j} T_j = \max_{y \in \mathcal{Y}} PV \cdot y - PV \cdot y^*. \]

So
\[ \sum_{j} T_j + PV \cdot y^* = \max_{y \in \mathcal{Y}} PV \cdot y \geq 0. \]

Now
\[ \max PV \cdot y = \sum_{i} \max_{y \in \mathcal{Y}_i} PV \cdot y_i. \]

So
\[ \sum_{j} T_j = \sum_{i} \max_{y \in \mathcal{Y}_i} PV \cdot (y_i - y_i^*). \]

For all $i$,
\[ \max_{y \in \mathcal{Y}_i} PV \cdot (y_i - y_i^*) \geq 0. \]

Partition firms into $I^-$ and $I^+$ given by
\[ I^- = \{i | PV \cdot y_i^* < 0\}; \quad I^+ = \{i | PV \cdot y_i^* \geq 0\}. \]

Then
\[ \sum_{j} T_j = \sum_{i \in I^- \cap \mathcal{Y}_i} \max PV \cdot (y_i - y_i^*) \]
\[ + \sum_{i \in I^+ \cap \mathcal{Y}_i} \max PV \cdot (y_i - y_i^*). \]

Hence
\[ \sum_{j} T_j = \sum_{i \in I^- \cap \mathcal{Y}_i} \max PV \cdot (y_i - y_i^*) \]
\[ = \sum_{i \in I^- \cap \mathcal{Y}_i} \left( \max PV \cdot y_i - PV \cdot y_i^* \right). \]

Now
\[ \max_{y \in \mathcal{Y}_i} PV \cdot y_i \geq 0, \quad \text{as } 0 \in \mathcal{Y}_i, \]
\[ \sum_j T_j = \sum_{i \in T^-} (-VV_i)^\gamma. \]

This proves (ii) of the theorem.

Now we have established in a fully satisfactory way that an efficient equilibrium may be sustained by marginal cost pricing and two-part tariffs. As noted, it is impossible to characterise the structure of these tariffs, or of the subspace \( L \), more fully in a model as general as the present one. There are two points about the analysis that seem worth drawing attention to.

(1) Nothing in the results established indicates how the \( T_j \) can be computed. They clearly vary from person to person, depending on preferences. Sufficient conditions for them to be equal would be that all individuals have identical preferences and either that these give rise to no income effects or that all have identical incomes. We might note in passing that many of the partial equilibrium studies in this field make such assumptions, and work with two-part tariffs where all individuals face the same fixed part. When tariffs must vary from person to person, it seems that any method of eliciting information on which they could be based would encounter the standard preference revelation problem.

(2) In defining fixed tariffs so that

\[ \sum_j (I_j + T_j) = \max_{x \in \mathbb{R}} P \cdot V \cdot x, \]

we have actually proved more than is strictly necessary. It will typically be possible to meet all losses with a smaller \( \sum_j T_j \) than implied by this approach. In situations where this is indeed the case, there will be considerably more latitude in choosing the subspace \( L \).

The fixed charge paid by individual \( j \) for consuming outside the subspace \( L \) is denoted by \( T_j \), and in general varies from person to person. The intuitive discussion associated with fig. 1 should make it clear why this is so. It is natural to enquire whether there are any circumstances in which these fixed parts are uniform. One such case is clearly that in which individual difference curves never intersect the axes of the commodity space, as in fig. 3.

Formally this is characterised by the condition that for all \( j \) and all \( x \) in the positive orthant, \( P_if_j(x) \) is contained in the positive orthant. Although an unattractive assumption, utility functions which satisfy this condition are widely used – for example, any CES function with the elasticity of substitution not less than unity. In this case the relevant consumer surplus measure is unbounded, and any individual may be charged any fixed part. In particular, all may be charged the same.

If indifference curves are allowed to intersect the axes it becomes more
difficult to find interesting conditions which will ensure the uniformity of the fixed parts of the tariffs. A sufficient condition is that preferences and income levels are identical for all consumers; significantly weaker conditions are not obvious. In the case of identical preferences and incomes either every consumer pays the fixed tariff, or none pays it – in which case it is of no importance.

There are several interpretative comments that should be made on the equilibrium with two-part tariffs. The first is a standard one, but one which bears repeating: two-part tariffs can in general only be used on goods whose purchasers cannot resell them, for otherwise all consumers of a particular good would nominate one individual to purchase their total requirements and then buy from him, thus incurring only one fixed cost in total. This is of course exactly the restriction that applies to any system of price discrimination, of which a two-part tariff system can be regarded as a particular case. It would then appear that the problem of achieving an efficient pricing system for a good produced under increasing returns is a great deal more acute if its resale cannot be prohibited, so that no discriminatory pricing system is sustainable.

The second point is more subtle. In our model, liability to the fixed part of a two-part tariff is associated with the purchase of a particular good, not with the purchase of the output of a particular firm. To see that this difference is considerably more than semantic, consider the following situation. There are two producers of a good, both of whom produce positive quantities and make a loss at marginal cost at an efficient equilibrium. There is a single consumer of this good, who must thus buy the entire output of both firms at equilibrium. If a two-part tariff is associated with each firm, the
consumer will naturally try to avoid buying from more than one firm, so as to avoid more than one fixed cost. In our hypothetical example he will therefore have to be quantitatively rationed by each firm. However, if, as in our model, a two-part tariff is associated with the purchase of a particular good, no such problem arises: a fixed payment is associated with entry to the market for this good, and after entering the market the consumer is indifferent about the firm from which he buys. So it is clear that the association of fixed charges with goods rather than firms is a point of some substance, but one that seems not to have been appreciated before. No doubt this is in part because with the type of public utility to which this kind of pricing system is usually applied, there is normally only a single producer, so that the distinction does not arise.

7. Conclusions

We have taken the equilibrium concept presented in Brown and Heal (1976), which was aggregated on the production side, and explored the possibility of decentralising this. We have shown that decentralisation on the production side is possible, and that the result may be interpreted either as an equilibrium at which each firm maximises profits at a nonlinear price system whose origin is specific to that firm, or as one in which all firms sell at marginal cost, and the resulting losses are covered by levies on consumers. We have discussed ways of raising these levies, and noted that this may be achieved either by a redefinition of the property rights involved in shareholding, or by a modified system of two-part tariffs. In particular we have shown that any efficient equilibrium can be supported by marginal cost pricing and two-part tariffs, though we emphasised as in Brown and Heal (1979) that efficient marginal cost pricing equilibria may not exist. With the modified two-part tariff system analysed, the resulting equilibrium appears vulnerable to problems of preference revelation and free-riding. There is clearly scope for far more work on this aspect of the problem.

References

