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OPINIONS ON HOW ONE SHOULD PLAY A THREE-PERSON NONCONSTANT SUM GAME

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In 1973 at several Australian universities several different sets of individuals were asked to give their opinions on how a certain three-person nonconstant sum game should be played. The individuals were presented with a diagram as shown in Figure 1.

This figure shows various solutions to a three-person nonconstant sum game with the following characteristic function:

$$v(1) = v(2) = v(3) = 0$$
$$v(12) = 1,\ v(13) = 2,\ v(23) = 3$$
$$v(123) = 4.$$

On the figure various suggested solutions have been noted.

The core is the area bounded by ACDE. No individual or pair of individuals can obtain more by independent action than they can obtain when offered any imputation in the core. An imputation $a = (a_1, a_2, a_3)$ in the core satisfied the inequalities:

$$a_1 + a_2 \geqslant 1$$
$$a_1 + a_3 \geqslant 2$$
$$a_2 + a_3 \geqslant 3$$

For this game the *bargaining set* and the core coincide.

The *Shapely Value* V is given by (5/6, 8/6, 11/6). We use the Harsanyi technique of giving (positive or negative) dividends to

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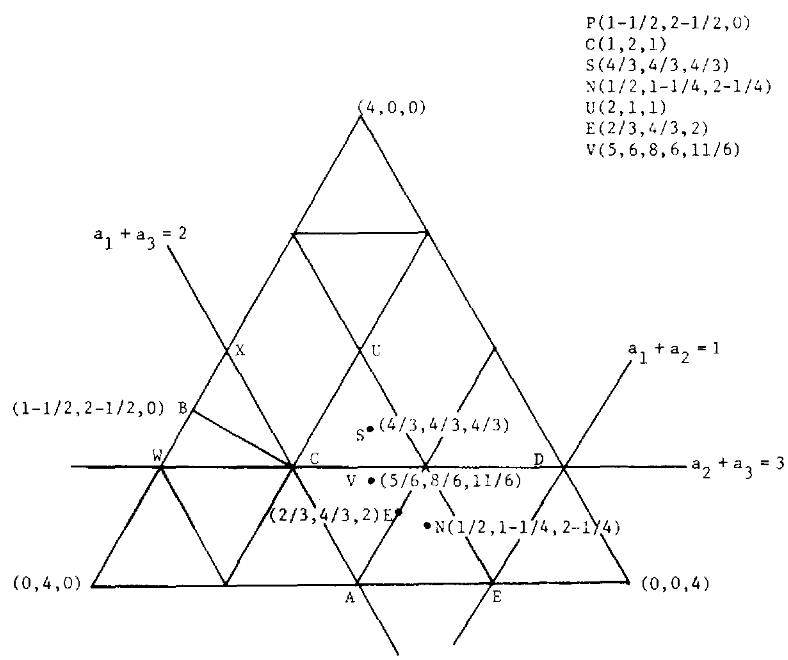

Figure 1

all coalitions based upon their "earnings" minus the earnings of all subcoalitions. The surplus is split evenly among all members of a coalition.

	1	2	3	Player
Coalition dividend from 1	0			
2		0		
3			0	
12	1/2	1/2		
13	1		1	
23		3/2	3/2	
123	-2/3	-2/3	-2/3	
	5/6	8/6	11/6	

The *nucleolus* (½, 1¼, 2¼) is the center of gravity of the core.

It is possible to regard the game as having arisen from a production technology with each individual possessing one unit of a production good which can be combined in different ways to produce different final goods of values 1, 2, 3, and 4, respectively. Efficiency would call for producing the most valuable consumer good. Raw material prices of:

$$p_1 = 2/3,\ p_2 = 4/3,\ p_3 = 2$$

and a final product price of $p_7 = 4$ clear the markets. Thus (2/3, 4/3, 2) noted at E gives a *competitive price system.*

The point S of (4/3, 4/3, 4/3) is the *equal division point* which is arrived at by crude symmetry which ignores differences in individual endowments or talents and concentrates on equity regardless of original contribution.

EXPERIMENTS AND RESULTS

The game was used six times in Australia; four times with an abstract briefing. In the first three instances the audience was comprised of social scientists. They were asked what the outcome "should" be. In the fourth instance the group involved were mathematicians. The fifth and sixth runs were with economists provided with an economic briefing. The seventh use of the game was with a class in game theory at Yale University in 1977 who had been provided with an economic briefing.

Table 1 shows the outcomes of the seven runs of the game. In the last instance students were asked to make more than one choice if they were so inclined and to offer an explanation of their choice. Their first two choices are noted.

COMMENTS AND INTERPRETATION

The results at first glance appear to be highly diverse. Yet upon further investigation several features emerge.

Except for S which has a strong symmetry property all other points chosen, but one, were in the core.

TABLE 1

	Abstract Game						Econ Briefing Australia			U.S. Students			Total
	1	2	3	%	4	%	5	6	%	7a	7b	7a%	1-7
S (4/3,4/3,4/3)	8	3	7	31.6	3	15	5	4	29				30
E (2/3,4/3,2)	2	1	2	8.8	4	20	6	2	25.8	2	1	11.1	19
V (5/6,8/6,11/6)	1	2	1	7.0	2	10	2	3	16.1	4		22.2	15
N (1/2,1-1/4,2-1/4)	2	3	1	10.5	3	15		1		6	4	33.3	16
Ec (1,1-1/2,1-1/2)	6		2	14.0						3		16.7	11
(1,4/3,5/3)		1	1		2	10	1				1		5
(0,2,2)	4			7.0									4
(7/6,8/6,9/6)	1				1		1	1	6.5				4
(1,1,2)		1	1				1						3
(0,3/2,3/2)			2										2
(1,2,1)		1			1								2
other points chosen once	2	1	1	7.0			1						5
core										3			3
no reply or errors					4	20		3	9.7		12		7
	26	13	18		20		17	14		18	18		126

The importance of the role of the briefing, the training and background of participants and the importance of interpersonal comparisons appear to be reflected in these experiments and in allied experiments by Roth (1975). In particular the U.S. students were enrolled in a class in game theory and had listened to lectures on the subject.

Table 2 presents some aggregated features of the games. Perhaps the most striking difference is the absence of selection of the equal shares point by the U.S. students. Referring to Table 3 it is of interest to see the change in selection of this solution.

Leaving aside S the attractiveness of the core was clear. Furthermore the value, nucleolus, price system, and equal shares core accounted for around 60% of the points selected in the core.

TABLE 2

	Australian	U.S.	Total
S	30	--	30
E,V,N,Ec	43	15	58
in core	77	18	95
Total	108	18	126

Roth's experiments subjected the characteristic function to linear transformations. He found that the results were extremely sensitive to these transformations, suggesting that the zero points and scale do matter.

TABLE 3

Australian	Abstract game:	social scientists	31.6%
Australian	Abstract game:	mathematicians	15%
Australian	Economic briefing:	social scientists	29%
	Economic briefing:	U.S. game theory students	0%

TEACHING, OPINIONS, AND EXPERIMENTS

The simple experiments with normative concern for how a three-person game should be played were all run in conjunction with lectures or classes as material used to introduce the various solution concepts of cooperative game theory. They were run at essentially zero cost with the additional feature that they also served a teaching purpose. Furthermore the characteristic function form of a game poses peculiar difficulties when one tries to use it experimentally. Von Neumann and Morgenstern make it transparently clear that they leave out costs of bargaining, communication, and negotiation in their description of the characteristic function as a set of costlessly createable coalitions. Yet any experimental procedure immediately violates this if the game is actually played.

The normative or opinion format appears to be more in keeping with the characteristic function than is actual play.

Several questions are raised by the results noted here. Is the choice of the equal division point by the Australians but not the U.S. students a culture or a training bias, or both? The behavior of Australian mathematicians gives some credence to the possibility that the mathematical training biases away from choosing the equal division, but there may be a cultural bias toward selecting it.

It would be desirable to see how the results are influenced by selecting different characteristic functions.

The Appendix presents some insights into the reasoning of the U.S. students who played in game 7.

REFERENCES

ROTH, B. M. (1975) "The game theoretic analysis of coalition formation in three-person games." Ph.D. dissertation, New School for Social Research.

Appendix

COMMENTS BY U.S. STUDENTS

S1: I would choose (1, 4/3, 5/3). It seems to best allocate the goods according to how much each contributed. Three is the best worker so should receive most, and 1 is the worst worker so should receive least. I took the amount contributed as the average of the two cooperative values. (1 contributes 1.5, 2 contributes 2, 3 contributes 2.5.) This totals 6. Translating this into a total of 4, we get the values given.

S2: Three seems to be the most productive so I eliminated all cases where 3 does not get the most. Similarly, 2 should get more than 1. So E, V are the ones I should discuss: E isn't stable because 2 and 3 can get more on their own. V and N are the only solutions which offer more to each pair than the available 2-person pair-offs and obey the ordering conditions.

S3: If all pull out of a common coalition consisting of 3, then all lose. But any one might threaten to do so nevertheless. Threatening power is greatest for player 3, least for player 1. Thus N, E, or V are feasible. Which one? I chose one with greatest equity. God knows why.

S4: Accept any point in CAED since all other imputations are not stable in the sense that some coalition would withdraw and by playing among themselves they could do better.

S5: Player #3 is obviously the most productive when combined with another person. The key is that he would have to offer #2 more to to work with him than he could get from working with #1 (say 1 1/8). Once he has #2 on board it would be easy to get #1 to join since he is out in the cold without and could gain to the extent of getting a 1/2. This is all fine and good as long as everyone maximizes.

S6: "N"—by sidepayments and blocking by coalitions.

S7: "(1, 3/2, 3/2)." This assures at least 50% of the value of any coalition between any two players to each member of the coalition.

S8: As always, not enough structure, so I impose some. Assume that greater "productivity" is a result of past (costly) investments by the individual and so we will reward that investment. (Alternatively assume greater productivity is due to "working harder.") Therefore reward 3 the most, 2 next, 1 least (compare the effect of their joining various of the two-member coalitions). Therefore, eliminate C, S, U. Within this constraint, and that no two members do worse when they could be going alone in a 2-person coalition, we are in the area ACDE??

S9: The relative producing ability of each player is 1, 2, and 3, respectively. Therefore the players should all form an agreement and distribute the "loot" according to relative portions 1/6, 1/3, 1/2.

S10: It would be fair to rank the payoffs in this order: player 3 is most productive, 2 is next, player 1 is least. The best that player #1 can do in dual combination is 1; player #2 is 1.5; player #3 is 1.5. In this sense it looks like the payoff should be: (1, 3/2, 3/2). But when we add the first solution we need to give player #3 a little of player #2's spoils such as at point E.

S11: In this case I like the Shapley Value, namely to give each player his average marginal contribution to the grand coalition which is assumed to form in random order. On this diagram, that is choice E, I believe.

S12: Clearly a solution will have to be "above" all three constraint lines. This leaves points C, V, E, N. From structure of problem it would seem that player #3 has best possibility against #2 and #1 while #1 has worst, so I choose N. #1 and #2 could not get together and improve themselves.

S13: First, the solution point should be within the shaded area, since otherwise, one of three 2-person coalitions could have improved their collective payoffs. Second, 3 points in the shaded area, N is located more nearly in the center than the other two, E and V.

S14: I like the solutions E and N, because they seem to best reflect the "ability" of the players to earn their share of the payoffs; their worth in coalition with the others. In some sense, #3 "deserves" more than #2 or #1, and #2 more than #1. (I am taking Professor Dubey's course, so I know that N is the nucleolis.)

S15: I like N, in which #1, #2, and #3 can get greater than or equal to what they could possibly get otherwise combined (in 2-person coalition).

S16: (1, 1½, 1½). Balance between keeping productive members in coalition and not making unproductive player too badly off.

S17: Select point N: gives more payoff to #2 and #3 over #1; can't be outside of ACDE polygon, within ACDE solution seems fair.

S18: The reward to #3 should be greater than the reward to #2 and the reward to #2 should be greater than the reward to #1. This is because #3 has the power to force either #1 or #2 into taking zero by dealing with the other. Likewide, #2 can force #1 into taking zero by dealing with #3. Furthermore, the relation of the rewards should be ordered by taking the maximum reward that each player would get if only two-player coalitions were formed. By using some type of proportional system, a "fair" distribution will be achieved. Point N seems to achieve this. Note: what really seems fair is point S but there doesn't seem to be any way to achieve S because both #3 and #2 would do better by forming a coalition of their own. Maybe the government should force #3 and #2 to accept point S.