THE LONG RUN CONSEQUENCES OF MONETARY AND FISCAL POLICIES WHEN THE GOVERNMENT'S BUDGET IS NOT BALANCED*

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1. Introduction

The familiar IS–LM apparatus describes the short run impact effects on the economy of various shocks, usually changes in policy instruments such as federal expenditures or open market operations. As time passes the stocks of money, bonds, and capital will change and a sequence of new equilibria will evolve. Although the short run nature of IS–LM analysis has seemingly long been recognized, there was surprisingly little work done on the dynamic evolution of an economy after the initial impact of a policy change. This neglect was apparently terminated by the suspicions that were voiced by Milton Friedman (1972) and others that short run analyses are misleading since the cumulative effects of policies may differ qualitatively from the impact effects. Specifically, Friedman conjectured that the effects of fiscal policies on spending are ‘certain to be temporary and likely to be minor.’

Since the dynamic evolution of an economy is quite complicated and time dependent, a natural target of longer run analyses is that hypothetical distant time when the economy has converged to a steady state with replicative temporary equilibria. The seminal analyses by Blinder and Solow (1973, 1974) and by Tobin and Bitter (1976) are concerned solely with stationary states in which prices and the nominal stocks of money and bonds are constant. This imposes the analytically powerful constraint that the government’s budget must in the long run be balanced. If the real after tax interest payments on federal debt are considered part of government spending, then a higher level of government spending can only be sustained by a higher tax base, and hence higher national income if this is the only source of tax revenue. Monetary policies which do not change government

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spending can have no long run effect on national income since any change in tax revenue would imply an unbalanced budget.

Somewhat more freedom is provided if interest payments are separated from other expenditures since this latter category could be sustained by lower real interest payments (higher prices, lower interest rates, or a smaller level of federal interest bearing debt) rather than higher tax revenue. However, the relevance of these limited possibilities is weakened in practice by the small size of federal interest payments. Unless the government were to become a creditor, an increase in government spending by more than the size of these interest payments must in a stationary state be financed by higher taxes.

This concern with stationary states and a concomitant balanced federal budget necessarily follows from the assumption that the price level or the nominal stock of money or bonds is fixed. In the present paper I analyze inflationary steady states in which the government runs a perpetual budget deficit. Long run equilibrium in an inflationary environment requires nominal asset stocks to grow at the same rate as prices. There are a variety of deficit financing rules which are consistent with this outcome, and I consider three different rules here: (1) the real money supply is pegged while the real supply of bonds fluctuates; (2) the real bond supply is pegged and money is the residual financing instrument; and (3) a mixed policy is employed with deficits financed by a fixed proportion of money to bonds. The details of the analysis for the third rule are presented in this paper. Only the results for the other two rules are given, since the analytics are analogous.

In the unemployment version of the model the rate of inflation rather than the price level (as in Tobin-Buiter) is a predetermined constant. This turns out to not qualitatively affect the long run consequences of an increase in government spending. Output will converge to a permanently higher level. With mixed or money financing the interest rate will eventually decline and the capital stock increase. With pure bond financing, the interest rate will remain higher than initially and the capital stock may decline. There is an enlarged role for monetary policy in the inflationary model, in that an expansionary monetary policy not only reduces $r$ and increases $K$, but also increases $Y$ in the long run.

In the full employment model, the commodity market is continuously cleared by an endogenous price level. With the deficit financing policies considered here, the steady state rate of inflation need not be zero. Again it turns out that the qualitative long run comparative static effects of an increase in government spending are remarkably consistent. Regardless of the financing rule, the new long run equilibrium is characterized by a lower interest rate, larger capital stock, and increased output. Expansionary monetary policies shed their neutrality, since they also reduce the interest rate, enlarge the capital stock and increase output. The primary qualitative
distinction between expansionary monetary and fiscal policies is that the
former reduces while the latter increases the equilibrium rate of inflation. As
Tobin-Buitier also found, the stability of the full employment long run
equilibria is suspect. One interesting finding is that, with static expectations,
an increased monetization of deficits tends to stabilize the model.

More generally, while the composition of the financing of government
budget imbalances is irrelevant in the short run it is very important in the
long run. However, no support is found for Friedman’s conjecture that the
consequences of expansionary fiscal policies are ‘certain to be temporary and
likely to be minor.’ Instead, such policies are either permanently expansionary
or catastrophically contractionary. Not surprisingly, unemployment
scenarios are more receptive to stimulative policies. In addition, persistent
monetary policies, unlike once-and-for-all actions, are seldom neutral.

The notation used in this paper is described here:

\[ Y = \text{real net national product}, \]
\[ C = \text{real private consumption}, \]
\[ K = \text{capital stock}, \]
\[ P = \text{price level}, \]
\[ \pi = \text{rate of inflation}, \frac{\dot{P}}{P}, \]
\[ \pi^* = \text{anticipated rate of inflation}, \]
\[ B = \text{nominal government interest bearing debt}, \]
\[ M = \text{nominal government monetary debt}, \]
\[ b = \text{real government interest bearing debt}, \frac{B}{P}, \]
\[ m = \text{real government monetary debt}, \frac{M}{P}, \]
\[ L = \text{real private demand for } m, \]
\[ r = \text{anticipated real rate of return on debt}, \]
\[ \tau = \text{income tax rate}, \]
\[ \dot{x} = \text{time derivative of } x, \]
\[ G = \text{real government purchases of national product}, \]
\[ G' = G + \text{plus real net debt interest}, \]
\[ G'' = G' - \text{minus real capital gains on government debt}, \]

2. Steady states and the government budget constraint

Consider the following general IS–LM model:

\[ Y + (1 - \tau) (r + \pi^*) b = C[r, \pi^*, \tau, Y, K, m, b] + K + G', \]
\[ m = L[r, \pi^*, \tau, Y, K, m, b]. \] (1)

The important assumption here is that the price level does not appear as a
separate argument, owing to the conventional dismissal of money illusion.
Differentiating (1) with respect to time and imposing the steady state...
conditions \( K = \dot{Y} = \dot{r} = \pi^* = 0 \) gives

\[
\frac{\partial C}{\partial m} \dot{m} + \left( \frac{\partial C}{\partial b} - (1 - \tau) (r + \pi^*) \right) \dot{b} = -\dot{G},
\]

\[
\left( \frac{\partial L}{\partial m} - 1 \right) \dot{m} + \frac{\partial L}{\partial b} \dot{b} = 0.
\]

(2)

If \( \dot{G} = 0 \), then the solution of (2) is

\[
\dot{m} = \dot{b} = 0,
\]

except in the special case

\[
\frac{\partial C}{\partial m} \frac{\partial L}{\partial b} + \left( 1 - \frac{\partial L}{\partial m} \right) \left( \frac{\partial C}{\partial b} - (1 - \tau) (r + \pi^*) \right) = 0,
\]

which is ruled out by the natural assumptions

\[
\frac{\partial C}{\partial m} > 0; \quad \frac{\partial L}{\partial b} > 0; \quad \frac{\partial L}{\partial m} < 1; \quad \frac{\partial C}{\partial b} > (1 - \tau) (r + \pi^*).
\]

This steady state condition (3) and the government's budget constraint

\[
G' - \tau Y = (\dot{M} + \dot{B})/P = \dot{m} + \dot{b} + (m + b)\pi
\]

(4)

give the steady state government deficit

\[
G' - \tau Y = (m + b)\pi.
\]

(5)

Thus, a steady state requires constant real stocks of money and bonds. The ratio of the government deficit to the stock of debt (i.e. the rate of change of government debt) will equal the rate of inflation. When the nominal stock of one asset is held constant the steady state is with a zero rate of inflation and a balanced government budget. There are many asset

\[1\] If the government instead follows a \( G' = 0 \) policy, then \( \dot{m} = \dot{b} = 0 \) unless

\[
\frac{\partial C}{\partial m} \frac{\partial L}{\partial b} + \left( 1 - \frac{\partial L}{\partial m} \right) \frac{\partial C}{\partial b} = 0,
\]

which is ruled out by

\[
\frac{\partial C}{\partial m} > 0; \quad \frac{\partial L}{\partial b} < 0; \quad \frac{\partial C}{\partial b} > 0; \quad \frac{\partial L}{\partial m} < 0.
\]
supply policies compatible with a steady state inflation and a persistent government deficit. The deficit financing rule that I will analyze in this paper is

\[ \dot{M} = \gamma(G' - \tau Y)P, \]
\[ \dot{B} = (1 - \gamma)(G' - \tau Y)P \]

(6)

or, in real terms,

\[ \dot{m} = \gamma(G' - \tau Y) - m\pi, \]
\[ \dot{b} = (1 - \gamma)(G' - \tau Y) - b\pi. \]

(7)

I will also display the results but not the underlying mathematics for the two alternative financing policies \( \dot{m} = 0 \) and \( \dot{b} = 0 \).

The policy (7) and the steady state conditions (3) and (5) imply

\[ 0 = \dot{m} = \gamma(G' - \tau Y) - m\pi = \pi(\gamma(m + b) - m)). \]

And hence that

\[ m = \gamma(m + b) \]
\[ b = (1 - \gamma)(m + b) \]

(8)

for any nonzero rate of inflation. That is to say, in a steady state the stock ratio of money to bonds will be identical to the flow ratio set by the government’s debt financing policy unless \( \pi = 0 \).

Tobin–Buiter support the analysis of \( G' \) rather than \( G \) policies by appeals to simplicity and the interpretation of federal interest payments as a continuing demand stimulus. In an inflationary context these arguments suggest the consideration of \( G'' = G' - \pi(m + b) \) policies which incorporate capital gains on federal debt. Results are presented in this paper for both \( G' \) and \( G'' \) policies.

Following Tobin–Buiter, I will use the specific private sector behavioral equations

\[ C = (1 - \tau)(Y + (r + \pi^e)b) - (m + b)\pi^e - s(\hat{\mu}Y - m - b - K) \]

\[ (s > \pi^e), \]
\[ \dot{K} = i \left( \frac{\alpha Y}{r} - K \right) \]
\[ (i > 0, i\pi/r < i + s\hat{\mu}), \]
\[ L = L \left[ r + \pi^e, \frac{Y}{m + b + K} \right](m + b + K) \]
\[ (i_1 < 0, 0 < i_2 < m/Y), \]
\[ \pi^e = \beta(\pi - \pi^e) \]
\[ (\beta > 0), \]

(9)
where \( s(\mu Y - m - b - K) \) is the planned time derivative of wealth. Anticipated revaluations of the capital stock do not affect consumption or the demand for money.\(^2\) The restriction \( t + s\mu - iz/r > 0 \) gives the usual presumption that an increase in income raises saving relative to investment. The assumption that the demand for money has an income elasticity less than one \( (\frac{\mu}{\gamma} < \frac{m}{r}) \) is widely supported and is here needed to give money a positive wealth elasticity. The assumption \( s > \pi^\psi \) is needed to impose the conventional assumption \( \partial C/\partial (m + b) > 0 \). The unusual parameters involved here arise from the separation of the conflicting effects of an increase in government debt on consumption. An increase in \( m + b \) raises actual wealth relative to target wealth and hence reduces saving. At the same time a higher level of \( m + b \) requires a permanently higher level of saving to offset the erosion of wealth from inflation. In a more complex model, the assumption \( \partial C/\partial (m + b) > 0 \) might be further motivated by the observation that an increase in the anticipated rate of inflation probably reduces desired wealth. This is because, even with a constant real rate of interest, inflation is not neutral since money holdings do not earn interest and taxes are paid on nominal rather than real returns.

An attractive price determination equation is a Phillips curve relation

\[
\pi = g(Y - F[K]) + \Psi \pi^\psi \quad (g \geq 0, 0 \leq \Psi \leq 1).
\]  

(10)

This relation (10) together with (1), (7), and (9) gives the dynamic IS–LM model

\[
\begin{align*}
\dot{K} & = s(\mu Y - m - b - K) + \tau Y - G' + \pi^*(m + b), \\
\dot{l} & \left[ r + \pi^*, \frac{Y}{m + b + K} \right](m + b + K) = m, \\
\dot{K} & = i \left( \frac{2Y}{r} - K \right), \\
\dot{m} & = \gamma (G' - \tau Y) - \pi m, \\
\dot{b} & = (1 - \gamma) (G' - \tau Y) - \pi b, \\
\pi & = g(Y - F[K]) + \Psi \pi^\psi, \\
\dot{\pi} & = \beta (\pi - \pi^\psi).
\end{align*}
\]  

(11)

Unfortunately, as is shown in the appendix to this paper, the Phillips curve renders the long run dynamics of this model hopelessly ambiguous. I will

\(^2\)Fortunately, the use of the market value of capital only complicates the dynamics without qualitatively altering any of the results presented here or in Tobin–Bulte. The introduction of flexible expectations about \( q = \gamma Y + \pi K \) would however increase the ambiguity of the dynamics.
therefore concentrate on two special cases which are analogous to the noninflationary scenarios analyzed by Tobin-Blitter. The first case is a situation of persistent unemployment \((g = \beta = 1 - \Psi = 0)\) while the second involves permanent full employment \((g \rightarrow \infty)\).

The short run comparative statics for the general Phillips curve model coincide with those for the persistent unemployment model. With \(\Psi = 1\), the long run comparative statics are identical to the permanent full employment model. For \(\Psi \neq 1\), the long run analysis is ambiguous with \(\tilde{m} = 0\) or \(\tilde{h} = 0\) policies since the sign of the Jacobian determinant is uncertain. Interestingly, with a \(G^*\) spending rule and a mixed financing policy \((6)\), the long run equilibria are not qualitatively affected by \(\Psi < 1\). With a \(G^*\) rule and mixed financing, only \(dK/d\gamma, -\gamma r/\gamma^*,\) and \(-\gamma(m + b)/\gamma^*\) depend upon \(\Psi\); these are zero for \(\Psi = 1\) and positive for \(0 < \Psi < 1\).

3. The unemployment model \((g = \beta = 1 - \Psi = 0)\)

In the familiar short run IS–LM model, wages and prices are set at other than market clearing levels and both income and employment are demand determined. In a short run analysis it is not necessary to go beyond the general assumption that prices are predetermined. The precise time path of prices is important, however, in the long run. Tobin–Blitter assumed a constant price level. I am here assuming a constant rate of change of prices. Neither assumption is particularly realistic but both are helpful in tracing out the logical implications of a sluggish price model.

With \(g = \beta = 1 - \Psi = 0\), the last two equations of \((11)\) become

\[
\pi = \pi^* \quad \text{and} \quad \pi^* = 0,
\]

and these can be readily substituted out of the remaining equations in \((11)\). The short run IS–LM analysis of the model \((11)\) is well known. An increase in \(G^*\) (or \(G^*\)) raises the temporary equilibrium values of \(Y\) and \(r\). An asset swap of \(m\) for \(b\) raises \(Y\) and lowers \(r\). A change in the deficit financing parameter \(\gamma\) has no immediate effect on the equilibrium.

With a \(G^*\) policy, the long run steady state equilibrium is obtained by substituting \((7), (8), \) and \(K = \pi^* - \pi = 0\) into \((11)\),

\[
G' + \gamma Y = \pi(m + b) = \pi(\hat{\mu}Y - K),
\]

\[
\gamma \left[ r + \pi, \frac{1}{\hat{\mu}} \right] \hat{\mu} = \frac{m}{Y} = \frac{m + b}{Y} = \gamma \left( \hat{\mu} - \frac{z}{r} \right),
\]

\[
zY = rK,
\]

\[
\hat{\mu}Y = m + b + K.
\]
Differentiating,

\[
\begin{bmatrix}
0 & -\pi \hat{\mu} - \tau & \pi & 0 \\
\frac{l_1 \hat{\mu} - \frac{2\gamma}{r^2}}{r^2} & 0 & 0 & 0 \\
-k & \alpha & -r & 0 \\
0 & \hat{\mu} & -1 & -1
\end{bmatrix}
\begin{bmatrix}
dr \\
dY \\
dK \\
d(m+b)
\end{bmatrix}
= \begin{bmatrix}
-dG' \\
\left(\hat{\mu} - \frac{\alpha}{r}\right) dy
\end{bmatrix}
\]

(13)

The Jacobian determinant

\[
J = \left(l_1 \hat{\mu} r - \frac{2\gamma}{r}\right)\left(\tau + \pi \left(\hat{\mu} - \frac{\alpha}{r}\right)\right) < 0
\]

is negative for positive rates of inflation \(\hat{\mu} - \alpha/r = (m+b)/Y > 0\). Since a deflation turns out to be unstable, I will only analyze the case \(\pi > 0\) here.

Premultiplying (13) by the Jacobian inverse gives

\[
\begin{bmatrix}
dr \\
dY \\
dK \\
d(m+b)
\end{bmatrix}
= \begin{bmatrix}
-\left(\hat{\mu} - \frac{\alpha}{r}\right) r \left(\tau + \pi \left(\hat{\mu} - \frac{\alpha}{r}\right)\right) dy \\
\left(\frac{2\gamma}{r} - l_1 \hat{\mu} r\right) dG' + \pi K \left(\hat{\mu} - \frac{\alpha}{r}\right) dy \\
\left(\frac{2\gamma}{r^2} - l_1 \hat{\mu}\right) dG' + \left(\hat{\mu} - \frac{\alpha}{r}\right) K \left(\tau + \pi \hat{\mu}\right) dy \\
\left(\hat{\mu} - \frac{\alpha}{r}\right) \left(\frac{2\gamma}{r} - l_1 \hat{\mu} r\right) dG' - \left(\hat{\mu} - \frac{\alpha}{r}\right) \tau K dy
\end{bmatrix}
\]

(14)

The signs of these multipliers are displayed in table 1. The second equation in (12) shows that for a given interest rate both the demand for and supply of money are proportional to \(Y\). The interest rate is consequently uniquely determined by the equality of the money demand and supply. That is to say, the long run LM curve (LLM) is horizontal. The first and third equations of (12) give a negative (GT) relationship between \(Y\) and \(r\). An increase in \(Y\) raises tax revenue and the stock of government debt, providing capital gains to the government. An increase in \(r\) also yields greater inflationary capital gains by reducing the supply of capital. With \(\pi = 0\) or \(G'\) policies, the GT curve is vertical. Using these relations figure 1 depicts the impact and long run effects of expansionary fiscal and monetary policies.

An increase in \(G'\) initially raises both \(Y\) and \(r\) and so has an ambiguous effect on investment. In the initial steady state the government was running a budget deficit with the nominal stocks of both money and bonds growing at the same rate as prices. The increase in \(G'\) worsens the deficit, causing both-
Table 1
Long run implications of the unemployment model.

<table>
<thead>
<tr>
<th>( P ) constant</th>
<th>( \pi &gt; 0 ) constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Tobin–Buiter)</td>
<td>( \frac{M}{B} )</td>
</tr>
<tr>
<td>( M = 0 )</td>
<td>( \pi = 0 )</td>
</tr>
<tr>
<td>( B = 0 )</td>
<td>( b = 0 )</td>
</tr>
</tbody>
</table>

\[
\begin{array}{ccccccc}
\text{d}G & \text{d}M & \text{d}G' & -\text{d}B & \text{d}G' & \text{d}m & \text{d}G' & -\text{db} & \text{d}G' & \text{dy} & \text{d}G' & -\text{db} & \text{d}G' & \text{dy} \\
\text{dr} & + & - & - & - & + & - & - & 0 & - & + & - & - & 0 & - \\
\text{dk} & ? & + & + & + & ? & + & + & + & 0 & + & + & + & + & + \\
\text{dy} & + & 0 & + & 0 & + & + & + & + & 0 & 0 & 0 & 0 & + & 0 \\
\text{dm} & 0 & 1 & + & + & 0 & 1 & + & + & + & 0 & 1 & + & + & + \\
\text{db} & + & - & 0 & -1 & + & - & 0 & -1 & + & - & 0 & -1 & + & - \\
\text{Stability} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} & \text{stable} \\
\end{array}
\]
real asset stocks to increase. Since $\gamma = m/(m+h) > 1 - l_z/\hat{\mu} = \hat{\iota}/c(m+h)$, this shifts both the IS and LM curves rightward as long as the deficit is larger than $\pi(m+h)$, i.e. as long as the economy is to the left of the GT curve. In

Fig. 1. Policy effects with unemployment ($\tilde{M}/\tilde{B} = \gamma/1-\gamma$).

the new long run equilibrium the higher level of $G^*$ is financed by the tax revenue on a larger national income and the capital gains on a larger real stock of debt. The interest rate returns in the long run to the unique value consistent with the government’s debt financing policy. With income larger, the capital stock expands so as to maintain a constant marginal product of
capital. The symmetry with the policies $\dot{m}=0$ and $\dot{h}=0$ (or with $\dot{M}=0$ and $\dot{B}=0$ in Tobin–Baiter) is particularly interesting. When the real money supply is pegged, the LLM curve is steeper than the LM and the excessive deficit shifts the LM leftward to permanently raise the interest rate. When the bond supply is pegged, the LLM curve is negatively sloped and the rightward shifting LM curve eventually drives the interest rate below its initial level.

In an inflationary scenario a one shot open market operation (figure 1b) has no lasting effect on the economy when the eventual steady state composition of government debt is fixed by the government's deficit financing parameter $\gamma$. The impact effects of an expansionary open market operation are to raise $Y$ and lower $r$. The growing capital stock shifts the LM curve leftward and has an ambiguous effect on the IS curve. The larger tax revenue reduces the deficit, leading nominal asset stocks to grow more slowly than prices. Both the IS and LM curves consequently shift leftward until the original deficit is re-established with asset stocks growing at the same rate as prices.

In contrast, a change in the composition of deficit financing has no impact effects but is important in the long run. An increase in $\gamma$ causes money to grow faster (and bonds more slowly) than prices, continuously shifting the LM curve to the right. As income rises the deficit shrinks, causing nominal government debt to grow more slowly than prices and the IS curve to shift leftward. In the new long run equilibrium the financing of $G'$ is accomplished by the increase in tax revenue offsetting the fall in capital gains resulting from the reduced stock of debt. The real stock of money will have increased, and a reduction in the interest rate is required for this to be willingly held. Since income has increased, an increase in the capital stock is necessary to attain the lower marginal product of capital consistent with the lower interest rate. As indicated in table 1, the same qualitative effects can be obtained by an expansionary open market operation that is followed by the pegging of either $m$ or $h$.

To analyze the stability of these equilibria, the characteristic equation of (11) in the neighborhood of long run equilibrium is given by

$$0 = \begin{vmatrix} -s-\dot{\lambda} & \pi-s & \pi-s & s\dot{\mu}+\tau & 0 \\ -L_w & 1-L_w & -L_w & -l_2 & -l_2\dot{\mu}Y \\ -i-\dot{\lambda} & 0 & 0 & i\gamma/r & -i\dot{\gamma}Y/r^2 \\ 0 & -\pi-\dot{\lambda} & 0 & -\gamma r & 0 \\ 0 & 0 & -\pi-\dot{\lambda} & -(1-\gamma)r & 0 \end{vmatrix},$$

where $L_w=1-l_2/\dot{\mu}$. After some manipulation, the characteristic equation
reduces to

\[
0 = (\lambda + \pi) \left\{ l_1 \hat{\mu} Y \left[ i\tau + i\pi \left( \hat{\mu} - \frac{z}{\tau} \right) + \left( i\tau + i\pi \left( \hat{\mu} - \frac{z}{\tau} \right) \right) \right. \\
+ \pi \left( \tau + s\hat{\mu} - \frac{z}{\tau} \right) + \tau (s - \pi) \right\} \lambda + \left( \tau + s\mu - \frac{z}{\tau} \right) \lambda^2 \\
- \frac{izY}{r^2} \left[ s\gamma \tau + s\pi l \hat{\mu} + (\gamma \tau + \pi l^2 + sj\pi) \lambda + l^2 \lambda^2 \right],
\]

which is of the form \( 0 = (\lambda + \pi)(-a - b\lambda - c\lambda^2) \), where \( a, b, \) and \( c \) are positive. A necessary and sufficient condition for stability is consequently that the rate of change of prices \( \pi \) not be negative. This critical dependence upon the sign of \( \pi \) here and throughout this paper\(^3\) can be explained as follows. In the dynamic model (11), the subtraction of the fifth equation multiplied by \( \gamma \) from the fourth equation multiplied by \( 1 - \gamma \) yields

\[
\dot{z} = -\pi z,
\]

where \( z = m - \gamma(m + b) \) is the discrepancy between actual real money holdings and what real money holdings would be if they were at their long run equilibrium fraction of debt. Eq. (15) reveals that \( z \) will converge to zero for \( \pi > 0 \) but will explode for \( \pi < 0 \). At a given level of prices, a budget deficit or surplus has no effect on \( m - \gamma(m + b) \) (or \( b - (1 - \gamma)(m + b) \)). However, an inflation will reduce these real gaps to zero and a deflation will increase them without limit. Alternatively, consider the dynamic equation for \( m/(m + b) \),

\[
\left( \frac{\dot{m}}{m + b} \right) = -\frac{G' - \tau Y}{m + b} \left( \frac{m}{m + b} - \gamma \right).
\]

Near a long run equilibrium, \( m + b = 0 \) and \( G' - \tau Y = \pi(m + b) \) so that

\[
\left( \frac{\dot{m}}{m + b} \right) = -\pi \left( \frac{m}{m + b} - \gamma \right).
\]

Clearly \( m/(m + b) \) will converge to \( \gamma \) if and only if \( \pi > 0 \). An increase in prices has no direct effect on \( m/m + b \). However, near an inflationary equilibrium the government is running deficits and the expansion of asset supplies will drive \( m/(m + b) \) to \( \gamma \). By contrast, in a deflation the

\(^3\)With \( \dot{m} = 0 \) and \( b = 0 \) financing policies, \( \pi = 0 \) is a sufficient but not necessary condition for stability.
government's surplus and contraction of asset supplies will exacerbate any deviation of $m/(m+b)$ from $\gamma$.

If the government instead follows a $G''$ policy, then the balanced budget condition in (12) is replaced by $G'' = \tau Y$, so that the GT curve is vertical. A steady state increase in $G''$ requires an increase in the tax base, and monetary policy is neutral. The straightforward multipliers are displayed in table 1. The pegging of a real asset supply yields the same qualitative multipliers as in the Tobin–Butter analysis of the pegging of a nominal asset supply when the price level is constant. The characteristic equation with $G''$ policies is given by

$$0 = \begin{vmatrix}
-s-\lambda & -s & -s & \tau + s\mu & 0 \\
-L_w & 1-L_w & -L_w & -l_2 & -l_1\mu Y \\
-1-\lambda & 0 & 0 & i\gamma/r & -i\gamma Y/r^2 \\
0 & -\pi(1-\gamma)-\lambda & \pi\gamma & -\gamma\tau & 0 \\
0 & \pi(1-\gamma) & -\pi\gamma-\lambda & -(1-\gamma)\tau & 0 \\
\end{vmatrix}
= (\lambda + \pi)(\lambda + \pi) \left[ ist + \lambda \left( is \left( \frac{\pi}{r} - i\tau + s\tau \right) + \lambda^2 \left( \tau + s\mu - \frac{i\gamma Y}{r} \right) \right) \right]$$

$$- \frac{i\gamma Y}{r^2} \left[ \pi\tau + \lambda (\pi\tau + s\pi) + \lambda^2 l_2 \right].$$

This is again of the form $0 = (\lambda + \pi)(-a + b\lambda - c\lambda^2)$, with $a$, $b$, and $c$ positive. The model is consequently again stable for inflations and unstable during deflations.

4. A model with full employment ($g \to \infty$)

The usual full employment interpretation of the IS–LM model assumes that output is supply determined with employment set by the equilibration (through the real wage rate) of the labor market. With an unchanging relationship between the supply of labor and the real wage, output will depend only upon the capital stock,

$$Y = F[K].$$

This special case of (10) closes the model by endogenously determining the commodity price. A discontinuous shock to the economy will cause a discrete jump in the price level. Continuous changes in the state variables will give a continuous time path for prices. With an endogenous rate of inflation, a behavioral relation such as the last equation in (11) is needed to describe inflationary expectations, $\pi^e = \beta(\pi - \pi^e)$. The complexity of the
dynamic analysis will force us to pay particular attention to the special cases \( \beta \rightarrow 0 \) (static expectations) and \( \beta \rightarrow \infty \) (perfect foresight).

The short run analysis of (11) using (16) is well known. An increase in either \( G' \) or \( G'' \) will increase both \( r \) and \( P \). An asset swap of \( M \) for \( B \) raises \( P \) and lowers \( r \). Income is fixed in the short run by the given capital stock.

The long run equilibrium with \( G' \) policies can be obtained by substituting (16) into (11) to determine \( r, K, \pi, \) and \( m+b \):

\[
G' - \pi F[K] = \pi(\hat{\mu} F[K] - K),
\]
\[
l(r + \pi, 1/\hat{\mu})\hat{\mu} = \gamma(\hat{\mu} - \xi/r),
\]
\[
x F[K] = r K,
\]
\[
m + b + K = \hat{\mu} F[K].
\]

(17)

Differentiating,

\[
\begin{bmatrix}
0 & \pi(1 - \hat{\mu} r) - \tau r & K - \hat{\mu} Y & 0 \\
1, \hat{\mu} - \frac{\gamma}{r^2} & 0 & l_1, \hat{\mu} & 0 \\
-K & (\alpha - 1) r & 0 & 0 \\
0 & 1 - \hat{\mu} r & 0 & 1
\end{bmatrix}
\begin{bmatrix}
dr \\
dK \\
d\pi \\
d(m+b)
\end{bmatrix}
=
\begin{bmatrix}
dG \\
(\hat{\mu} - \frac{\gamma}{r}) dy
\end{bmatrix}

\]

The sign of the Jacobian determinant

\[ |J| = \left| l_1, \hat{\mu}(1 - \alpha) r (\hat{\mu} Y - K) + K (\tau r - \pi (1 - \hat{\mu} r)) \right| - \alpha (1 - \alpha) \gamma (\hat{\mu} Y - K) / r \]

depends upon the sign of \( \tau r - \pi (1 - \hat{\mu} r) \), which describes the effect of an increase in \( K \) on the funds available to support \( G' \)-increased tax revenue plus capital gains on a changed stock of debt. The solution of (18) is given by

\[
\begin{bmatrix}
dr \\
dK \\
d\pi \\
d(m+b)
\end{bmatrix}
= -1
\]

\[
\left| J \right|
\times
\begin{bmatrix}
(1 - \alpha)(1 - \tau)r l_1, \hat{\mu} dG' - (1 - \alpha) r Y (\hat{\mu} - \xi/r)^2 dy \\
- (1 - \tau) l_1, \hat{\mu} K G' + K Y \left( \hat{\mu} - \frac{\gamma}{r} \right)^2 dy \\
(1 - \alpha)(1 - \tau) l_1, \hat{\mu} r + \frac{\gamma r}{r} dG' - K \left( \hat{\mu} - \frac{\gamma}{r} \right)^2 dy \\
(1 - \beta \hat{\mu}) K dG' - (1 - \beta \hat{\mu}) K Y \left( \hat{\mu} - \frac{\gamma}{r} \right)^2 dy
\end{bmatrix}
\]

(19)
With the reasonable assumption that \( \tau r - \pi(1 - \mu r) \) is positive, \( |J| \) is negative and the signs of the multipliers are as displayed in table 2. If \( \tau r - \pi(1 - \mu r) \) is negative with \( |J| \) still negative, then \( \Delta \pi / \Delta y \) would be positive rather than negative. If \( |J| \) is positive, then \( \Delta \pi / \Delta y \) would be negative and the signs of the remaining multiplier would be the opposite of those in table 2. The most striking result is probably that an increased monetization of deficits raises the steady state rate of inflation only in a rather restricted region of the parameter space. Also, the signs of the long run multipliers tend to be largely independent of the deficit financing rule employed.

In the full employment model, a graphical analysis is more difficult and less useful than in the unemployment model because the short run equations determine \( r \) and \( P \) while the long run equations determine \( r \) and \( \pi \). The familiar short run multipliers are depicted in figures 2a and 2b. In a steady state inflation both the IS and LM curves are continuously shifting rightward with \( r \) constant.

The long run LM curve is obtained from the second equation in (17). In the long run, with the private sector at its desired wealth-income position, the demand for \( m \) relative to \( m + b \) is negatively related to both \( r \) and \( \pi \) while the supply is equal to \( \gamma \). Thus, for a given \( \gamma \) the rate of inflation is inversely related to the real interest rate. The first equation in (17) gives the long run balanced budget (GT) relation. An increase in \( \pi \) increases the government’s capital gains, while an increase in \( r \) decreases the capital stock and this is assumed to reduce the resources that support \( G' \).

The impact effects of an increase in \( G' \) are to raise both \( r \) and \( P \). Investment is discouraged and the capital stock and hence production decline. The increase in \( G' \) and the decline in tax revenue exacerbate the government deficit, shifting the IS and LM curves rightward with uncertain implications for the interest rate. The system may be unstable with ever higher interest rates leading to the erosion of the capital stock and collapse of the economy. If the system is stable it will converge to a new long run equilibrium with a lower interest rate and a larger tax base. The resultant increase in the demand for \( m \) relative to \( m + b \) must be offset by a decline in \( \pi \).

An increase in \( \gamma \) shifts the LM curve rightward, continuously lowering \( r \) and raising \( P \) higher than it would be otherwise (i.e. increasing \( \pi \)). The capital stock increases, enlarging the tax base and reducing real government debt which puts leftward pressure on the IS and LM curves. An unending expansion might ensue. If stable, the economy will converge to a new long run equilibrium with lower values of \( r \) and \( \pi \) than initially.

For stability analysis the characteristic equation of (11), using (16) to replace \( Y \) is
Table 2
Long run implications of the full employment model.

<table>
<thead>
<tr>
<th>( P ) endogenous (Tobin-Butler)</th>
<th>( \pi &gt; 0 ) endogenous</th>
<th>( G' ) policies ((\tau + \pi(\delta r - 1) &gt; 0))</th>
<th>( G'' ) policies</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{M} = 0 ) ( \dot{B} = 0 )</td>
<td>( \dot{m} = 0 ) ( \dot{b} = 0 )</td>
<td>( \dot{M}/\dot{B} = \gamma/1 - \gamma )</td>
<td>( \dot{M}/\dot{B} = \gamma/1 - \gamma )</td>
</tr>
<tr>
<td>( d\dot{G} ) ( d\dot{M} ) ( d\dot{G} ) ( -db ) ( d\dot{G} ) ( d\dot{m} ) ( d\dot{G} ) ( -db ) ( d\dot{G} ) ( d\dot{m} ) ( d\dot{G} ) ( -db ) ( d\dot{G} ) ( d\dot{m} ) ( d\dot{G} ) ( -db ) ( d\dot{G} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( d\dot{r} )</td>
<td>( -) ( 0 ) ( -) ( 0 )</td>
<td>( -) ( -) ( -) ( -) ( -) ( -) ( 0 ) ( -) ( 0 ) ( -) ( 0 )</td>
<td></td>
</tr>
<tr>
<td>( d\dot{k} )</td>
<td>( +) ( -) ( +) ( 0 )</td>
<td>( +) ( +) ( +) ( +) ( +) ( +) ( 0 ) ( +) ( 0 ) ( +) ( 0 )</td>
<td></td>
</tr>
<tr>
<td>( d\dot{c} )</td>
<td>( +) ( 0 ) ( +) ( 0 )</td>
<td>( +) ( +) ( +) ( +) ( +) ( +) ( 0 ) ( +) ( 0 ) ( +) ( 0 )</td>
<td></td>
</tr>
<tr>
<td>( d\dot{m} )</td>
<td>( +) ( 0 ) ( +) ( 0 ) ( 0 ) ( 1 )</td>
<td>( \dot{m} ) ( +) ( +) ( +) ( +) ( +) ( +) ( 0 ) ( 1 ) ( +) ( ? ) ( 0 )</td>
<td></td>
</tr>
<tr>
<td>( db )</td>
<td>( ? ) ( 0 ) ( -) ( 0 )</td>
<td>( ? ) ( ? ) ( 0 ) ( -1 ) ( ? ) ( ? ) ( ? ) ( -) ( 0 ) ( -1 ) ( ? )</td>
<td></td>
</tr>
<tr>
<td>( d\pi )</td>
<td>( ? ) ( ? ) ( ? )</td>
<td>( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? )</td>
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<tr>
<td>( d\rho )</td>
<td>( ? ) ( ? ) ( ? )</td>
<td>( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? ) ( ? )</td>
<td></td>
</tr>
<tr>
<td>( d\dot{B} )</td>
<td>( -) ( \rho/\dot{M} ) ( +) ( -) ( \rho/\dot{B} )</td>
<td>( -) ( \rho/\dot{B} ) ( +) ( -) ( \rho/\dot{B} )</td>
<td>( -) ( \rho/\dot{B} ) ( +) ( -) ( \rho/\dot{B} )</td>
</tr>
<tr>
<td>( d\dot{m} )</td>
<td>( 0 ) ( 1 ) ( +) ( -) ( \dot{M}/\dot{B} )</td>
<td>( 0 ) ( 1 ) ( +) ( -) ( \dot{M}/\dot{B} )</td>
<td>( 0 ) ( 1 ) ( +) ( -) ( \dot{M}/\dot{B} )</td>
</tr>
<tr>
<td>( d\dot{B} )</td>
<td>( -) ( \dot{B}/\dot{M} ) ( 0 ) ( -1 )</td>
<td>( -) ( \dot{B}/\dot{B} ) ( 0 ) ( -1 )</td>
<td>( -) ( \dot{B}/\dot{B} ) ( 0 ) ( -1 )</td>
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<tr>
<td>( \beta \rightarrow 0 )</td>
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<td>( \beta \rightarrow 0 )</td>
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<tr>
<td>stability</td>
<td>ambiguous</td>
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<td>ambiguous</td>
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<tr>
<td>Stability</td>
<td>unstable</td>
<td>stable</td>
<td>ambiguous</td>
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<tr>
<td>( \beta \rightarrow \infty )</td>
<td>( \beta \rightarrow \infty )</td>
<td>( \beta \rightarrow \infty )</td>
<td>( \beta \rightarrow \infty )</td>
</tr>
<tr>
<td>ambiguous</td>
<td>ambiguous</td>
<td>ambiguous</td>
<td>unstable</td>
</tr>
</tbody>
</table>
Fig. 2. Policy effects with full employment ($\bar{M}/\bar{Y} = \gamma / (1 - \gamma)$).

\[
0 = \begin{vmatrix}
\tau r + s\mu r - s - \lambda & \pi - s & \pi - s & m + b & 0 \\
-r_l - L_W & 1 - L_W & -L_W & -l_1 \mu Y & -l_1 \mu Y \\
(\alpha - 1)\mu - \lambda & 0 & 0 & 0 & -iK/r \\
-\tau \gamma r & -\pi - \lambda & 0 & -m(1 + \lambda / \beta) & 0 \\
-\tau (1 - \gamma) r & 0 & -\pi - \lambda & -b(1 + \lambda / \beta) & 0 \\
\end{vmatrix}
\]

\[
= (-\lambda - \pi)\left(-l_1 W(m + b - iK/r)\right)^2 \\
+ [iK(m + b) (l_2 + \gamma / r) - l_1 W iK (\tau + \pi (\mu - 1 / r))] \\
+ (s - \pi)\mu - l_1 W(m + b)(1 - \alpha) - l_1 W - l_1 W s(1 + \lambda / \beta) \\
- l_1 W [i(1 - \alpha)(m + b) + iK (\tau + \pi (\mu - 1 / r))] \\
+ iK m(1 - \alpha) / r - (\lambda + \pi) (m + b) \lambda / \beta \\
\times \{ -l_1 W(s - \pi)(\lambda + i(1 - \alpha)) + iK/r [(\gamma - L_W) \lambda] \\
+ (L_W - \gamma)(\pi + \pi \mu r - \pi) + (s - \pi) \gamma (1 - \alpha) \}.
\]
Again, one of the characteristic roots is \(-\pi\), so that a long run deflationary equilibrium is unstable. With \(\beta\) unknown, the remaining roots are formidably ambiguous. As \(\beta \to 0\) the system is stable if \(\tau r + \pi \mu r - \pi \geq 0\), \(\gamma > L_p\), and \((1 - \alpha)\delta Y > rG\). The first of these sufficient conditions represents our assumption that an increase in the capital stock raises the government revenue available for financing \(G\). The second sufficient condition states that the fraction of the deficit that is money financed is greater than the marginal demand for money out of an increase in wealth. This implies that the direct effect of a growing federal debt will be to shift the short run LM curve rightward. The third condition comes from

\[
(1 - \alpha)(s - \pi) - \tau r - (\mu \pi r - \pi + \alpha - 1)\pi = (1 - \alpha)s - rt - \psi \frac{m + b}{r} G' - \tau Y - \psi \frac{m + b}{r} G' - \psi \frac{m + b}{Y} \pi = (1 - \alpha)s - \psi \frac{m + b}{Y} \pi,
\]

but unfortunately is rather uncertain.

For \(\beta \to \infty\), stability requires \(m + b > iK/r\). This assumption is difficult to interpret but does not seem very plausible. If we consider an increase in \(\pi\) offset by a fall in \(r\) so that the nominal interest rate and the demand for money are unchanged, then \(m + b > iK/r\) states that saving will rise relative to investment presumably reducing the upward pressure on prices.

\(G''\) policies are more restrictive and yield more definite results. The long run equilibrium can be described by

\[
G'' = \tau F[K],
\]

\[
\frac{d}{dt} \left[ r + \pi, \frac{1}{\mu} \right] \tilde{\mu} = \gamma \left( \hat{\mu} - \frac{2}{r} \right),
\]

\[
\alpha F(K) = rK,
\]

\[
\hat{\mu} F[K] = m + b + K.
\]

Differentiating and solving, the multipliers are given by

\[
\begin{bmatrix}
\frac{dr}{dt} \\
\frac{dK}{dt} \\
\frac{d\pi}{dt} \\
\frac{d(m + b)}{dt}
\end{bmatrix}
= - \frac{1}{J}
\begin{bmatrix}
(1 - \alpha)\tau l_1 \hat{\mu} dG'' \\
-l_1 \hat{\mu} K dG'' \\
-(1 - \alpha)(\tau l_1 \hat{\mu} - \alpha \pi/r) dG'' - r\tau K(\hat{\mu} - \alpha/r) d\gamma \\
(1 - \mu \pi) l_1 \hat{\mu} K dG''
\end{bmatrix},
\]

where this Jacobian determinant is

\[
|J| = - r\tau l_1 \hat{\mu} K > 0.
\]
The signs of these multipliers are displayed in table 2. The level of \( G^* \) fixes the needed tax revenue and hence \( K \). A higher \( K \) implies a lower marginal product of capital and real interest rate; financial market equilibrium requires a higher rate of inflation to offset the lower real return. The change in debt depends upon whether the demand for debt rises or falls as the capital stock increases. Since \( K \) and \( r \) are fixed by \( G^* \), a change in \( \gamma \) only affects the rate of inflation. An increased monetization of deficits leads to a larger stock of money, which will be held only at a lower nominal yield and hence a lower \( \pi \) since \( r \) is constant. The \( dG^* \) multipliers are qualitatively identical to the \( dG^* \) multipliers. Monetary policy is however neutral in a \( G^* \) regime and the stability of the model is clarified.

The characteristic equation with \( G^* \) policies is

\[
0 = \begin{vmatrix}
    s(\mu r - 1) + rt - \lambda & -s & -s & -(m+b)\lambda/\beta & 0 \\
    -2r - L_w & 1 - L_w & -L_w & -l_1\mu Y & -l_1\mu Y \\
    (\alpha - 1)\mu - \lambda & 0 & 0 & 0 & -IK/r \\
    -\tau r & -(1-\gamma)\pi - \lambda & \gamma \pi & 0 & 0 \\
    -\tau(1-\gamma) r & (1-\gamma)\pi & -\gamma \pi - \lambda & 0 & 0 \\
\end{vmatrix}
\]

\[
= (\lambda + \pi) \left( l_1\mu Y \frac{IK}{r} \left[ r\tau + \lambda(\tau + s(\mu r - 1)) - \lambda^2 \right] \right.
\]

\[
- \lambda \frac{m+b}{\beta} \left[ tK(L_w - \gamma) - \hat{K}(l_2 + L_w) \right]
\]

\[
\times \left[ r + l_2\mu Y \left( i(1-\gamma)\lambda + \lambda^2 \right) \right).\]

Once again one of the characteristic roots is \( -\pi \). With perfect foresight (\( \beta \rightarrow \infty \)), the characteristic equation reduces to

\[
0 = (\lambda + \pi) l_1\mu Y \frac{IK}{r} \left[ r\tau + \lambda(\tau + s(\mu r - 1)) - \lambda^2 \right].
\]

Since \( r\tau \) is positive, the system is unstable.

With static expectations (\( \beta \rightarrow 0 \)) the characteristic equation is dominated by

\[
0 = (\lambda + \pi)(m+b) \left( i\pi K(L_w - \gamma) + \lambda \left[ -i(1-\gamma)l_1\mu Y \right.ight.
\]

\[
+ iKl_2 + iKL_w/r] - \lambda^2 l_1\mu Y),
\]
which is of the form

\[ 0 = (\lambda + \pi) (a + b\lambda + c\lambda^2), \]

where \( b \) and \( c \) are positive and \( a \) is proportional to \( \gamma - L_w \), the difference between the fraction of a deficit that is money financed and the marginal demand for money out of an increase in wealth. The model will be stable if \( \pi > 0 \) and \( \gamma \geq L_w \). In this model \( L_w = (m/W) - (l_2/\mu) < m/(m + b) = \gamma \), so that the model is stable for inflations and unstable for deflations. As might be expected, with bond financing \((m = 0)\) the model is always unstable while with money financing \((b = 0)\) the model is stable with static expectations.

Appendix

This appendix displays the formidable ambiguity of the Phillips curve pricing model. The specific policies considered here are fixed levels of \( G^* \) and \( b \). This is one of the simpler cases and also one that is more likely to be stable. The behavioral equations are

\[ \dot{K} = s(\mu Y - m - b - K) + \tau Y - G^* + (\pi^e - \pi) (m + b), \]

\[ I \left[ r + \pi^e, \frac{Y}{m + b + K} \right] (m + b + K) = m, \]

\[ \dot{K} = I \left( \frac{aY}{r} - K \right), \]

\[ \dot{m} = G^* - \tau Y, \]

\[ \pi = g(Y - F[K]) + \Psi \pi^e, \]

\[ \dot{\pi}^e = \beta (\pi - \pi^e). \]

Replacing \( \pi \) by the implication of the price expectations equation that \( \pi = \pi^e + \dot{\pi}^e/\beta \), the characteristic equation in \( Y, K, m, \pi^e \), and \( r \) is given by

\[
0 = \begin{vmatrix}
\lambda & -s - \dot{\lambda} & -s & -(m + b)\lambda/\beta & 0 \\
-\lambda & -L_w & 1 - L_w & -l_1 W & -l_1 W \\
-\lambda & -l_1 W & 0 & 0 & -iK/r \\
-\tau & 0 & -\dot{\lambda} & 0 & 0 \\
g & -gF[K] & 0 & \Psi - 1 - \dot{\lambda}/\beta & 0 \\
\end{vmatrix}
\]
\[
\begin{align*}
g \left\{ (iKl_1W/r) \lambda^2 + \lambda(s - s\hat{\mu}F[K] - \tau F[K]) - \tau rF[K]) \right. \\
- (g(m + b)\lambda/\beta)(\lambda^2(-l_1W) + \lambda[l_1W(1 - s\hat{\mu}F[K]/r) \\
+ ilK(L_W + l_2F[K])/r] + F[K](1 - L_W)\tau rK/r] \\
+ [(1 - \Psi) + \lambda/\beta] \lambda^2[-l_1W(\tau + s\hat{\mu} - \tau x/r) + iKl_2/r] \\
+ \lambda[-l_1W(\tau + s\hat{\mu} - \tau x/r) + (s\hat{\mu} + \tau)K/r] \\
- istl_1W + istK/r].
\end{align*}
\]

This is of the form

\[
0 = g[-a_1\lambda^2 + a_2\lambda + a_3] + [g\lambda(m + b)/\beta] [-b_1\lambda^2 - b_2\lambda - b_3]
+ (1 - \Psi + \lambda/\beta)[c_1\lambda^2 + c_2\lambda + c_3],
\]

where each \(a_i\), \(b_i\), and \(c_i\) (except \(a_2\)) is unambiguously positive. In the special case of exogenous inflation \((g = 0, \beta = 0, \Psi = 1)\), the equation collapses to \(0 = c_1\lambda^2 + c_2\lambda + c_3\) which is unambiguously stable. With continuous full employment, \(g = \infty\) and the equation

\[
0 = -a_1\lambda^2 + a_2\lambda + a_3 + [\lambda(m + b)/\beta] [-b_1\lambda^2 - b_2\lambda - b_3]
\]

is stable for inelastic expectations \((\beta \to 0)\) and unstable for perfect foresight \((\beta \to \infty)\). More generally, it can be seen that the perfect foresight model becomes more likely to be stable as the price adjustment becomes more sluggish \((\Psi \text{ declines})\). If we spurn the simplifications offered by such extreme parametric assumptions, stability is quite uncertain and unfortunately dependent upon the specific values of such controversial parameters as \(\beta\) and \(g\).

References


