A Short-Run Two-Sector Model with Immobile Capital

Standard IS-LM analysis assumes a single homogeneous commodity that can be purchased at a unique price and either consumed or added to the capital stock. Although the realism of this assumption is seldom discussed, it can be motivated by the argument that the relative prices of commodities are fixed in the short run. One possible rationale for this is that prices are exogenous; another is that some group of consumers, producers, or hoarders views the different commodities as perfect substitutes. A third explanation is that labor and capital are mobile and inelastically supplied with all sectors having the same capital intensities for all wage-rental ratios. A fourth possibility is that if labor and capital are immobile and wages and prices are flexible, then the equilibria of the separate labor markets will determine relative commodity prices independently of financial markets and the demand for commodities.

The realism of these rationalizations is considerably less persuasive than the practical argument that the simplicity of the single commodity price model greatly enhances its expositional value. There is, unfortunately, ample evidence that relative prices do change (often dramatically) and only a limited literature on the distortions introduced into IS-LM models by the pretense that relative prices do not change.

Most authors (such as [4, 8, 9]) who have distinguished between the prices of consumption and investment goods have followed the neoclassical tradition of treating capital as a perfectly mobile factor of production. Although this would seem to make them most relevant for long-run analysis, a short-run Keynesian flavor is sometimes provided by the assumption that the nominal wage is fixed at a level such that the supply of labor exceeds the demand.

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Recent articles by Benavie [1] and Mackay and Waud [7] have discussed two-sector fixed wage models in which capital is not mobile between industries. The demand for investment then arises from the divergence between the cost of new capital and the valuation of capital in the equities market. We are not aware of any two-sector immobile capital models in which wages are flexible. In this paper we analyze a two-sector model with immobile capital under conditions of both fixed and flexible wages.

Since our paper is closest in spirit to those of Benavie and Mackay and Waud we will note at the outset that they work with continuous models in which asset demands are constrained by current asset holdings. We, instead, use a discrete time framework in which the end-of-period asset demands are constrained by the beginning-of-period asset holdings plus saving during the period. We can consequently assume that all asset demands are positively related to income, whereas Benavie and Mackay and Waud must assume that the demand for bonds plus equities is negatively related to income when the demand for money is positively related to income. This positive relationship between income and the demand for interest-bearing financial assets turns out to be one of our key assumptions. We also introduce budget constraints for the corporate sector and the government sector. Firms are assumed to finance net investment purchases by issuing new equities, whereas government deficits must be financed by the issuance of new bonds or the printing of new money. In continuous models changes in household consumption, corporate investment, or government spending have no short-run effects on financial markets. As a consequence, the comparative static multipliers of, say, an increase in government spending do not depend upon whether the spending is financed by printing money or selling bonds. In a discrete model such a distinction can be made.

The structure of our model is detailed in section 1. There are three sectors: corporations, government, and households. Within the corporate sector there are two industries, one producing consumption goods and the other producing capital goods. Firms hire labor from the household sector for use with their endowment of immobile capital to produce output. All sales revenues net of depreciation are distributed to households as wages and dividends. Firms issue new equities to finance their net investment expenditures. The government prints money, sells bonds, and levies taxes to finance government purchases of consumption and investment goods. Since the government is not engaged in production, its purchases of investment goods are treated as tanks, monuments, and the like. The household sector provides labor services to the corporate sector in return for wages; receives dividends and interest income; pays taxes; purchases consumption goods; and accumulates money, bonds, and equities. We maintain the usual assumptions that expectations are static and that households consider bonds and equities to be perfect substitutes.

In the second section of this paper, we examine the current period effects of various government policies when all markets are cleared by freely moving prices. Here commodity and financial markets affect employment and production by altering the relative price of consumption and investment goods. In the third section we
examine the comparative statics when there is unemployment due to a fixed nominal wage rate. In this situation the transmission mechanisms are the price levels of consumption and investment goods.

1. THE BASIC MODEL

Corporate Sector

Both commodities are produced according to well-behaved constant returns to scale production functions using capital $K$ and labor $N$. Using subscripts $C$ and $I$ to denote the consumption good and the investment good, respectively, the production functions are given by (1).

$$Q_C = F^+_C[K_C, N_C], \quad Q_I = F^+_I[K_I, N_I].$$  \hspace{1cm} (1)

Thus, $Q_C$ is the output of the consumption good produced from the inputs of capital $K_C$ and labor $N_C$. Similar notation applies to the investment goods industry. Functional arguments are enclosed in brackets and the signs above the arguments indicate the sign of the partial derivative of the function with respect to that argument.

Both industries take nominal wages and output prices as given. Labor is perfectly mobile between industries, whereas capital is assumed to be fixed and completely immobile in the short run. The profit-maximizing firm will consequently adjust its labor demand so as to equate the marginal value product of labor to the money wage rate. Using $\omega$ as the nominal wage rate and $N^P_i$ and $P_i$ to represent the demand for labor and the price of output, respectively, in industry $i$ profit maximization implies\(^1\):

$$P_C F^i_C[K_C, N_C^P] = \omega, \quad P_I F^i_I[K_I, N_I^P] = \omega,$$  \hspace{1cm} (2)

and from the implicit function theorem

$$N_C^P = N_C^P[\omega/P_C], \quad N_I^P = N_I^P[\omega/P_I].$$  \hspace{1cm} (3)

The budget constraint for each industry is that total sales revenue plus revenue from new security issues must equal wage payments plus dividends plus investment. We make the simplifying assumption that firms distribute their total sales revenue net of depreciation to households in the form of wages and dividends and finance any net investment by issuing new equity. Consequently, dividends $D_i$ in industry $i$ can be written as

$$D_C = P_C Q_C - \omega N_C^P - P_I \delta C K_C = P_C F^i_C K_C - P_I \delta C K_C,$$

$$D_I = P_I Q_I - \omega N_I^P - P_I \delta I K_I = P_I F^i_I K_I - P_I \delta I K_I,$$  \hspace{1cm} (4)

\(^1\)The notation $F_i$ represents $\partial F_i/\partial N_i$. Similar notation is used for the partial derivatives of any function with more than one argument. For functions of one variable, the derivative will be denoted by a prime. For example, if $X = X[Z]$, then $dX/dZ = X'$; but if $X = X[Z,S]$, then $dX/dS = X_s$. \hspace{1cm}
where $\delta_c$ and $\delta_i$ are the constant rates of depreciation. The second equalities in (4) follow from the constant returns to scale assumption. Let $E_i$ and $E_i^*$ be the beginning- and end-of-period supplies of industry $i$'s equity shares and $P_{E_i}$ the current period per share price of equity. If industry $i$'s net investment demand is $I_i$, then the residual budget constraints can be written as

$$P_{I_i} = P_{E_i}(E_i^* - E_i), \quad P_{I_i} = P_{E_i}(E_i^* - E_i).$$

(5)

An industry's investment demand arises from the fact that there are two prices associated with capital. One is the price of new capital and the other is the market valuation of the equity claims to that capital. If firms want to maximize shareholder wealth, they will invest in new capital whenever its cost is less than the associated equity market valuation. Equivalently, firms will invest whenever the implicit rate of return on capital exceeds the rate of return required by shareholders on their equity [10].

With static expectations and dividends given by (4), the shareholder yield $r_{E_i}$ on the equity of industry $i$ is

$$r_{E_i} = \frac{(P_{I_i} F_{IK} - P_{I_i} \delta_i) K_i}{P_{E_i} E_i}, \quad r_{E_i} = \frac{(P_{I_i} F_{IK} - P_{I_i} \delta_i) K_i}{P_{E_i} E_i}$$

(6)

and the implicit rates of return on capital are

$$r_{KC} = \frac{P_{I_i} F_{IK} - P_{I_i} \delta_i}{P_{I_i}} \quad \text{and} \quad r_{KI} = \frac{P_{I_i} F_{IK} - P_{I_i} \delta_i}{P_{I_i}}.$$

(7)

Investment will occur within an industry when the implicit rate of return on capital in that industry exceeds the required rate of return on that industry's equity. From (6) and (7), we see that $r_{KI}$ will exceed $r_{E_i}$ if and only if the market value of the firm is greater than the replacement cost of its capital stock; i.e., if Tobin's $q$ (the ratio of these two values)

$$q_{C} = \frac{P_{E_i} E_i}{P_{I_i} K_i} = \frac{r_{KC}}{r_{EC}}, \quad q_{I} = \frac{P_{E_i} E_i}{P_{I_i} K_i} = \frac{r_{KI}}{r_{EI}}$$

(8)

is greater than one. An industry's investment demand $I_i$ can thus be written as a function of its $q_i$, and will be positive, zero, or negative as $q_i$ is greater than, equal to, or less than one:

$$I_c = I_c[q_c], \quad I_c[1] = 0$$

$$I_i = I_i[q_i], \quad I_i[1] = 0.$$
Total investment demand $I$ is then given by

$$ I = I_C[q^+_C] + I_I[q^+_I] = I[q^+_C, q^+_I]. \quad (9) $$

A familiar question in macro theory is whether changes in the current level of economic activity will significantly alter the profit expectations that underlie investment plans. In our model, the specific issue is the responsiveness of $q$ to employment-induced changes in the marginal physical product of capital. We will follow the general Keynesian tradition of assuming that there is no response, rationalized by the inelasticity of expectations and/or the marginal product itself.

**Government Sector**

The government prints money, sells single-period discount bonds each paying $1 after taxes, and collects income taxes in order to finance its purchases of consumption and investment goods. Its budget constraint is therefore

$$ P_I G_I + P_C G_C = M^S - M + B^S - B + tP_C Y, \quad (10) $$

where $M$ is the beginning-of-period money stock and $M^S$ is the end-of-period supply; $B$ is the amount of dollars paid to redeem the previous period's bonds and $B^S$ is the amount of dollars received for current period bonds; $t$ is the tax rate; $G_C$ and $G_I$ are the number of consumption and investment goods purchased by the government; and

$$ Y = (\omega N + D_C + D_I)P_C = Q_C + \rho (Q_I - \delta_C K_C - \delta_I K_I) \quad (11) $$

(using (4) with $\rho = P_I/P_C$) is real household net factor income. If the price of a bond is $P_B$, then the before-tax net rate of return on government bonds is implicitly

$$ r = (1 - P_B)[(1 - t)P_B] . $$

**Households**

Households are assumed to provide their labor services to the corporate sector in return for wages; to hold money, bonds, and equities; to receive dividends on their equity and interest on their bonds; to pay taxes; and to purchase the consumption good. Using equation (11), the household budget constraint is given by equation (12) where $N^S$ is the household labor supply; $YD$ is disposable factor income; $C$ is household consumption demand; and $M^P, B^D, and E^D_i$ represent nominal household end-of-period demands for money, bonds, and equities:

$$ P_C C + M^P - M + B^D - B + P_{eC}(E^D_C - E_C) + P_{eI}(E^D_I - E_I)$$

$$ = (1 - t)(\omega N^S + D_C + D_I) = P_C YD . \quad (12) $$
We will assume a sequential structure for household decisions in which the supply of labor is influenced only by the real wage measured in units of the consumption good

\[ N^S = N^S \left( \omega / P_C \right)^+. \]  \hspace{1cm} (13)

This is a powerful assumption that dichotomizes the model by sharply limiting the ways in which commodity demands and financial markets can affect employment and production. The introduction of the price level, interest rates, or wealth into the labor supply function would add considerable ambiguity to our results.

To simplify the analysis, it is assumed that households view bonds and equities as perfect substitutes. Equity prices must therefore adjust so that the required rates of return on equity \( r_{E} \), and \( r_{E} \), are equal to the rate of return on government bonds \( r \).

Thus, equations (6) can be rewritten to give

\[ P_{E^-} E_C = \frac{F_{ck} - \delta_c K_C}{r}, \quad P_{E^-} E_I = \frac{F_{ik} - \delta_i K_I}{r} \]

so that

\[ q_c = \frac{F_{ck} - \delta_c K_C}{P_c r} = q_c \left[ \frac{\bar{p}, \bar{r}}{r} \right] \]

\[ q_I = \frac{F_{ik} - \delta_i K_I}{r} = q_I \left[ \frac{\bar{r}}{r} \right] \]  \hspace{1cm} (14)

since \( F_{ck} \) and \( F_{ik} \) have been assumed inelastic. The beginning-of-period real financial wealth of households \( W \), valued at current period prices, is given by

\[ W = \frac{M + B + P_{E^-} E_C + P_{E^-} E_I}{P_c} \]

\[ = \frac{M + B}{P_c} \frac{\left( F_{ck} - \rho \delta_c \right) K_C}{r} + \frac{\rho \left( F_{ik} - \delta_i \right) K_I}{r} \]  \hspace{1cm} (15)

Because bonds and equities are perfect substitutes, the individual demand functions \( B^o, E_C^o \), and \( E_I^o \) can be merged into a real demand for bonds plus equities \( V \),

\[ V = \frac{B^o}{P_c} + \frac{P_{E^-} E_C^o}{P_c} + \frac{P_{E^-} E_I^o}{P_c} \]  \hspace{1cm} (16)

As shown in equations (17), we will assume that real household demands for consumption goods (\( C \)), money (\( L = M^o / P_c \)), and bonds plus equities (\( V \)) depend on the interest rate, real disposable income, and wealth.
\[
C = C[r, YD, W]
\]
\[
L = L[r, YD, W]
\]
\[
V = V[r, YD, W].
\]

Rearranging the household budget constraint (12) and substituting equations (15) and (17) gives

\[
C[r, YD, W] + L[r, YD, W] + V[r, YD, W] = YD + W, \tag{18}
\]

which implies the following adding up restrictions on the partial derivatives of the demand functions:

\[
C_r + L_r + V_r = 0
\]
\[
C_{YD} + L_{YD} + V_{YD} = 1
\]
\[
C_w + L_w + V_w = 1. \tag{19}
\]

Income appears in the asset equations as a transactions proxy and because of its influence on the desired level of financial wealth. In a continuous model the first reason usually motivates the assumptions that \(L_2 > 0\) and \(V_2 < 0\) (but see [5]). In a discrete time model, however, a positive value for \(L_2\) does not require \(V_2\) to be negative since the stocks of financial assets can be augmented by saving; the adding up restriction is instead that all of the income received during the period must be used to acquire consumption goods or financial assets, \(C_2 + L_2 + V_2 = 1\). The second reason is unimportant in a continuous model but argues for positive values for both \(L_2\) and \(V_2\) in a discrete model, since part of the increase in income will be used for asset accumulation. This will primarily involve the acquisition of interest earning assets though money could also be held as an asset or for facilitating increased asset transactions. We have chosen to stress in this paper the positive effect of income on the demand for bonds and equity. Our period is then implicitly sufficiently long (say a quarter) so that the amount of income that accrues is substantial.

From (17), the partial effect of an increase of the interest rate on consumption demand, holding income and wealth constant, is ambiguous. The adding up restrictions in (19) indicate that if \(V_r\) is positive and greater than \(L_r\) in absolute value, then \(C_r\) must be negative. Though some may find this argument persuasive, we will assume here only that \(dC/dr = C_r + C_w(\partial W/\partial r) \leq 0\).\(^2\)

\(^2\)In his two-asset model, Benavie assumes that this total effect is zero due to "the failure of [the interest rate] to appear as a significant predictor in studies of consumption spending" [1, p. 68]. Although this is true of direct effects, most contemporary consumption functions do include the market value of wealth and consequently involve a negative indirect effect of interest rates. In the widely respected MPS model, for example, the primary impact of monetary policy upon spending is through interest rate induced revaluations of wealth. In his three-asset model, Benavie leaves this indirect interest rate effect in his consumption function, although this is apparently an oversight.
The model can now be summarized by the equilibrium conditions in the markets for labor, consumption goods, investment goods, money and bonds plus equities:

\[ N_C^D\left[ \omega/P_C \right] + N_I^D\left[ \omega/P_I \right] = N^S\left[ \omega/P_C \right] \]
\[ C\left[ r, YD, W \right] + G_C = Q_C\left[ K_C, N_C \right] \]
\[ \delta_KK_C + \delta_IK_I + I\left[ q_C, q_I \right] + G_I = Q_I\left[ K_I, N_I \right] \] \hspace{1cm} (20)
\[ L\left[ r, YD, W \right] = M^S/P_C \]
\[ V\left[ r, YD, W \right] = \left( B^S + P_{E_C}E_C^S + P_{E_I}E_I^S \right)/P_C . \]

Adding the sectoral budget constraints (4), (5), (10), and (12) reveals that one of the above equilibrium conditions is redundant since

\[ \omega/P_C(N_C^D + N_I^D - N^S) + (C + G_C - Q_C) + \rho(\delta_KK_C + \delta_IK_I + I + G_I - Q_I) \]
\[ + (L - M^S/P_C) + (V - B^S/P_C - P_{E_C}E_C^S/P_C - P_{E_I}E_I^S/P_C) = 0 . \] \hspace{1cm} (21)

We will drop the equilibrium condition for bonds plus equities, leaving four equilibrium conditions and four potentially market clearing variables \( \omega, P_C, P_I, \) and \( r. \)

2. EQUILIBRIUM

This section examines the current period effects of various government policies when all markets are cleared by freely moving prices. The differentiation of the labor market equilibrium in (20) with respect to \( \omega/P_C \) and \( \rho = P_I/P_C \) gives

\[ \frac{d(\omega/P_C)}{d\rho} = \frac{-\rho(\omega/P_C)N_I^{d}}{N^S - N_D^{d} - (1/\rho)N_C^{d}} > 0 . \]

We know directly that an increase in \( \omega/P_C \) reduces \( N_C^D \) and increases \( N^S \). It follows that the equilibrium effects of a change in \( \rho \) are

\[ (\omega/P_C) = (\omega/P_C)\left[ \rho \right] \]
\[ N_C = N_C\left[ \rho \right] \] \hspace{1cm} (22)
\[ N_I = N_I\left[ \rho \right] . \]

Intuitively, at any level of \( \omega/P_C \) an increase in \( \rho \) reduces \( \omega/P_I \) and increases \( N_I^D \). This increased demand is met by an increase in \( \omega/P_C \) that reduces \( N_C^D \) and increases \( N^S \). Thus an increase in the relative price of the investment good results in a fall in
employment in the consumer goods industry, a rise in employment in the capital goods industry, and an overall increase in total employment due to the rise in the real wage. Substitution of (22) into (1) and (12) gives

\[ Q_c = Q_c[\rho], \quad Q_l = Q_l[\rho] \quad (1') \]

\[ YD = (1 - t)(Q_c[\rho] + \mu Q_l[\rho] - \delta_c K_c - \delta_l K_l) = YD[\rho]. \quad (12') \]

Equations (14) and (15) may also be rewritten as

\[ q_c = q_c[\rho, r], \quad q_l = q_l[r] \quad (14') \]

\[ W = W[P_c, \rho, r]. \quad (15') \]

The effects of a change in \( \rho \) on \( \rho l \) and \( W \) are ambiguous since

\[ \frac{\partial \rho l}{\partial \rho} = 1 + \rho l \rho \geq 0 \quad \text{as} \quad - (\rho l \rho) \leq 1 \]

\[ \frac{\partial W}{\partial \rho} = \frac{F_{hK} - \delta_l K_l - \delta_c K_c}{r}. \]

We will assume that these effects are not large enough to reverse the effects of \( \rho \) through \( YD \) on consumption, money, and bond demands.

In a discrete period model, with an unconstrained budgetary imbalance, the effects of price changes on the excess demands for financial assets are unfortunately ambiguous for two reasons: (a) The end-of-period asset supplies \( M^S \) and \( B^S \) cannot both be assumed close to the beginning-of-period stocks \( M \) and \( B \) and thus, for at least one asset, the price effects will be uncertain if there is some wealth effect on demand; and (b) the price effect on the supply of the residual policy instrument is wholly ambiguous. We will resolve these ambiguities by assuming that the policy changes originate from a position of budgetary balance with \( G = tQ_c \), \( G_l = t(Q_l - \delta_c K_c - \delta_l K_l) \), \( M^S = M \), and \( B^S = B \). It remains true that an increase in \( P_c \) will lower both the real supply and, through a reduction in wealth, the real

3The net effect of a change in \( \rho \) on \( YD \) is determined by differentiating \( YD \) using equation (1) for output as follows:

\[ \frac{\partial YD}{\partial \rho} = (1 - t) \left\{ F_{cN_c} \frac{\partial N_c}{\partial \rho} + \mu F_{ls} \frac{\partial N_l}{\partial \rho} + Q_l - \delta_c K_c - \delta_l K_l \right\}. \]

From the profit-maximization conditions in (2), we can write

\[ \frac{\partial YD}{\partial \rho} = (1 - t) \left\{ \frac{\omega}{P_c} \left( \frac{\partial N_c}{\partial \rho} + \frac{\partial N_l}{\partial \rho} \right) + Q_l - \delta_c K_c - \delta_l K_l \right\} > 0, \]

since an increase in \( \rho \) raises \( \omega/P_c \) and therefore aggregate employment.
demands for money and bonds. We will assume that \( L_W < M(M + B) \) and \( V_W < B(M + B) \) so that a rise in \( P_C \) will increase the excess demands for both assets.

With these assumptions, differentiation of the four remaining equilibrium conditions in (20), along with the government budget constraint (10) to facilitate the switch from bond- to money-financed policies, gives

\[
\begin{bmatrix}
C_{w}W_{pC} & C_{r}YD' + C_{w}W_{p} - Q'_{c} & C_{r} + C_{w}W_{r} & 0 \\
0 & \rho l_{p} - \rho Q'_{l} & \rho l_{r} & 0 \\
L_{w}W_{pC} + M/P_{c} & L_{r}YD' + L_{w}W_{p} & L_{r} + L_{w}W_{r} & 0 \\
V_{w}W_{pC} + B/P_{c} & V_{r}YD' + (V_{w} - 1)W_{p} - \rho l_{p} - I - V_{r} + (V_{w} - 1)W_{r} - \rho l_{r} - 1 & 0 & \mathbf{t}(Q'_{c} + \rho Q'_{l})
\end{bmatrix}
\begin{bmatrix}
\Delta P_{c} \\
\Delta \rho \\
\Delta r \\
(1/P_{c})\Delta B^{s}
\end{bmatrix} = \begin{bmatrix}
-\Delta G_{c} \\
-\rho \Delta G_{l} \\
(1/P_{c})\Delta M^{s} \\
0
\end{bmatrix}, \quad (24)
\]

which can be rewritten as

\[
\begin{bmatrix}
-a_{11} & a_{12} & -a_{13} & 0 \\
0 & -a_{22} & -a_{23} & 0 \\
a_{31} & a_{32} & -a_{33} & 0 \\
a_{41} & a_{42} & a_{43} & -1 \\
0 & a_{52} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta P_{c} \\
\Delta \rho \\
\Delta r \\
(1/P_{c})\Delta B^{s}
\end{bmatrix} = \begin{bmatrix}
-\Delta G_{c} \\
-\rho \Delta G_{l} \\
(1/P_{c})\Delta M^{s} \\
0
\end{bmatrix}
\]

(25)

where each \( a_{ij} \) is positive. The five equations in (24) and (25) are linearly dependent since the derivatives with respect to any argument sum across equations to zero; in particular, \( \sum a_{ij} = 0 \). (This can be directly verified by recalling the adding up restrictions (19) and the balanced budget condition \( G_{I} = \mathbf{t}(Q_{r} - \delta_{r}K_{C} - \delta_{I}K_{I}) \).)

We will consequently delete the bond equation when solving the model. The bond equation is temporarily displayed here because the signing of its terms provides information about the parameters that we will be explicitly using; for example, the positive sign for \( a_{41} \) implies that \( a_{11} \) is larger than \( a_{31} \).

To analyze bond-financed policies we can delete the bond equation from (25)
\[
\begin{bmatrix}
-a_{11} & a_{12} & -a_{13} & 0 \\
0 & -a_{22} & -a_{23} & 0 \\
a_{31} & a_{32} & -a_{33} & 0 \\
0 & a_{32} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta P_C \\
\Delta \rho \\
\Delta r \\
(1/P_C) \Delta B^e
\end{bmatrix}
= 
\begin{bmatrix}
-\Delta G_C \\
-\rho \Delta G_1 \\
\Delta G_C + \rho \Delta G_1 - (1/P_C) \Delta M^e
\end{bmatrix}
\] (26)

and solve for the changes in the equilibrating variables and the residual policy instrument. The Jacobian determinant \(|A|\) is negative and premultiplication by \(A^{-1}\) yields

\[
\begin{bmatrix}
\Delta P_C \\
\Delta \rho \\
\Delta r \\
\frac{1}{P_C} \Delta B^e
\end{bmatrix}
= 
\begin{bmatrix}
a_{22} a_{33} + a_{23} a_{32} & a_{12} a_{33} - a_{13} a_{32} & a_{12} a_{23} + a_{13} a_{22} \\
-a_{23} a_{31} & a_{11} a_{32} + a_{13} a_{31} & a_{11} a_{23} \\
a_{32} a_{31} & a_{11} a_{32} + a_{12} a_{31} & -a_{11} a_{22} \\
a_{31} a_{32} - \frac{1}{|A|} \left( a_{23} a_{31} + a_{21} a_{32} \right) - \frac{1}{|A|} \left( a_{22} a_{31} + a_{21} a_{32} \right)
\end{bmatrix}
\] (27)

The signs of these bond-financed multipliers are shown in Table 1. These qualitative results are quite similar to those of the mobile capital model of Foley and Sidrauski. The main differences are that their labor is inelastically supplied and that their interest rate effects are ambiguous.

A bond-financed purchase of consumption goods raises \(P_C\) relative to \(P_1\), lowering \(\rho\) so that consumption output is stimulated. An increase in \(r\) reconciles investment demand to the lower output due to the fall in \(\rho\). The increase in \(P_C\) reduces the real money supply to a level consistent with the reduced demand arising from the higher \(r\) and lower \(\rho\).

A bond-financed purchase of investment goods raises the relative price ratio, increasing the production of investment goods and reducing that of consumption goods. The higher level of interest rates crowds out some private investment, but not enough to fully offset the increased government purchases. With both income and interest rates higher, it is uncertain what the net effect will be on the demand for money and hence \(P_C\). If the demand for money is interest inelastic, then \(P_C\) will decline so as to increase the real supply of money.
An increase in the money supply through open market operations lowers interest rates, stimulating consumption and investment spending. A higher relative price ratio $\rho$ restores investment goods equilibrium by stimulating production and depressing demand. However, the rise in $\rho$ also reduces consumer goods output in the face of increased demand. Consumer goods equilibrium is attained by a higher price level $P_C$, which lowers consumption demand (through the Pigou effect) to a level consistent with the reduced supply. Real money demand has increased since interest rates have fallen and income risen. Therefore, $P_C$ does not increase by as much as $M^S$. If there are no Pigou effects ($C_W = 0$), then $r$ and $P$ are uniquely determined by the consumer and investment goods markets, and cannot be influenced by open market operations. If, in addition, $L_W = 0$ then the demand for money will be unaffected by open market operations and the price level will rise by the same percent as the nominal money supply.

If consumption were interest- as well as price-inelastic, there would be a complete sequential determination of the endogenous variables. Equilibrium in the consumption goods market would depend only upon the relative price ratio and would therefore determine $\rho$. Investment equilibrium depends upon the interest rate and the relative price ratio; with the latter determined by consumption goods equilibrium, this market would determine $r$. Given $\rho$ and $r$, the demand and supply of money would fix the level of prices. In such a situation, not only would monetary policy be neutral but government purchases of investment goods would no longer affect the level or composition of output; instead, they would fully crowd out an equal amount of private purchases of investment goods.

The solution of the system with money financing is obtained by solving (26) with $\Delta B^S$ exogenous and $\Delta M^S$ endogenous.
\[
\begin{bmatrix}
-a_{11} & a_{12} & -a_{13} & 0 & \Delta P_C \\
0 & -a_{22} & -a_{23} & 0 & \Delta \rho \\
-a_{31} & a_{32} & -a_{33} & -1 & \Delta r \\
0 & a_{52} & 0 & 1 & \frac{1}{P_C} \Delta M^S \\
0 & 0 & 0 & 0 & \Delta G_C + \rho \Delta G_I - \frac{1}{P_C} \Delta B^S
\end{bmatrix} =
\begin{bmatrix}
-\Delta G_C \\
-\rho \Delta G_I \\
0 \\
0 \\
0
\end{bmatrix}.
\] (28)

The Jacobian determinant \( |A| = -a_{11}a_{22}a_{33} \) is again negative, and using \( a_{11} > a_{33} \) and \( a_{22} > a_{12} + a_{32} + a_{52} \), the signs of the money-financed multipliers displayed in Table 1 can be obtained. Since a bond-financed expenditure increases \( B^S \) and a bond-financed expansion of \( M^S \) reduces \( B^S \), a money-financed government expenditure can be viewed as a bond-financed expenditure accompanied by an expansionary open market operation. The open market operation turns out to be more powerful than the fiscal operation in that when their effects are conflicting the monetary effects prevail. The assumption that \( a_{41} \) and \( a_{42} \) clearly underlie these results. The particularly interesting conclusions are that a money-financed purchase of consumption goods increases investment output and reduces consumption output, and that a money-financed purchase of either commodity lowers interest rates.

The one-commodity version of our model is

\[
N^0 \left( \omega / P \right) = N^S \left( \omega / P \right) \\
C \left[ r, YD, W \right] + I \left[ r \right] + G = Q \left[ K, N \right] \\
L \left[ r, YD, W \right] = M^S / P \\
(V \left[ r, YD, W \right] = B^S / P + F_k K / r + I \left[ r \right] ) \\
(M^S - M + B^S - B) / P = G - t \bar{Q} \left[ K, N \right],
\]

where

\[
YD = (1 - t) \bar{Q} \left[ K, N \right] \\
W = \frac{M + B}{P} = \frac{F_k K}{r}.
\]

The straightforward solution of this model yields the qualitative multipliers displayed in Table 1. The most striking contrast is that, unlike this one-commodity model, demand management policies can affect aggregate income and employment in a two-commodity world by altering relative prices. This result does not require the complete omission of \( P_t \) from the labor supply function. If \( P_t \) enters the price deflators of some firms and households, an increase in \( \rho \) with \( \omega / P_C \) fixed will reduce
real wages measured by these price deflators, increasing $N^D$ and reducing $N^S$. Holding $\rho$ constant, an increase in $\omega/P_C$ will have the opposite effects if $P_C$ enters the price deflators of some firms and households. Thus an increase in $\rho$ will require an increase in $\omega/P_C$ to maintain labor market equilibrium. Employment will increase if the relative importance of $\rho$ to $\omega/P_C$ is greater for firms than households in the specific sense that for those increases in $\omega/P_C$ and $\rho$ that leave the supply of labor unchanged, the demand for labor will increase. A particular example of this is the model

$$N^D \left[ \frac{-\omega}{\alpha P_C + (1 - \alpha) P_I} \right] = N^S \left[ \frac{\omega}{\beta P_C + (1 - \beta) P_I} \right]$$

or

$$N^D \left[ \frac{\omega}{P_C} \frac{1}{\lambda_1} \right] = N^S \left[ \frac{\omega}{P_C} \frac{1}{\lambda_2} \right], \quad \lambda_1 = \alpha + (1 - \alpha) \rho, \quad \lambda_2 = \beta + (1 - \beta) \rho.$$

Differentiation yields

$$\frac{\Delta(\omega/P_C)}{\Delta \rho} = \frac{\omega}{P_C} \left( \frac{-\lambda_1 \lambda_2 N^D + (1 - \beta) \lambda_2 N^S}{-\lambda_1 \lambda_2 N^D + \lambda_2 \lambda_1 N^S} \right) > 0$$

so that

$$\frac{\Delta N^S}{\Delta \rho} = - \frac{\omega}{P_C} \frac{N^D N^S \lambda_2}{\lambda_1 \lambda_2} \left( \frac{-\lambda_1 - (1 - \beta) \lambda_2}{-\lambda_2 N^D + \lambda_1 N^S} \right) \equiv 0 \quad \text{as} \quad \beta \equiv 0.$$

This same moral would apply with more realistically detailed price indices. The government can in a wide variety of ways alter taxes and relative commodity and asset prices so as to differentially affect employers' and employees' perceptions of real wages nationally and in specific sectors of the economy.

3. FIXED NOMINAL WAGES

This section examines the current period effects of various government policies when the nominal wage is exogenously fixed at some level $\omega_0$ at which the supply of labor exceeds the demand. The labor market is assumed to be cleared by the actual employment equaling the total labor demanded

$$N = N^D + N^D.$$

(29)

With the nominal wage fixed, the labor demand in each industry (3) can be expressed as an increasing function of the price of output in that industry.
\[ N_c^D = N_c^D \left[ P_c \right], \quad N_I^D = N_P \left[ P_I \right]. \] (30)

The substitution of (30) into (1) with capital fixed gives the output of each commodity as an increasing function of its price

\[ Q_c = Q_c \left[ P_c \right], \quad Q_I = Q_I \left[ P_I \right]. \] (31)

Since employment and output depend upon price levels, we will express aggregate demands in terms of these arguments also. The substitution of equations (31) into (11) gives realized disposable income as a function of the commodity price levels

\[ \bar{YD} = (1 - t)Y = (1 - t)(Q_c \left[ P_c \right] + P_I (Q_I \left[ P_I \right] - \delta_c K_c - \delta_I K_I)/P_c \]
\[ = \bar{YD} \left[ P_c, P_I \right]. \] (32)

Although \( \partial \bar{YD} / \partial P_c \) is formally ambiguous, we would expect it to be positive.\(^4\)

We must also modify our household demand equations to take into account spillovers from the labor market disequilibrium. Since actual labor income is less than notional, household effective demands for commodities, money, and bonds (\( \bar{C}, \bar{L}, \) and \( \bar{V} \)) will be less than their notional demands (\( C, L, \) and \( V \)),

\[ \bar{C} + \bar{L} + \bar{V} = W + (1 - t) \frac{D_1}{P_c} + \frac{D_2}{P_c} + \frac{\omega_0}{P_c} N \]
\[ < W + (1 - t) \frac{D_1}{P_c} + \frac{D_2}{P_c} + \frac{\omega_0}{P_c} N^s \frac{\omega_0}{P_c} \]
\[ = C + L + V. \] (33)

We will specify the effective demand functions as

\[ \bar{C} = \bar{C} \left[ r, \bar{YD}, \ W \right]; \]
\[ \bar{L} = \bar{L} \left[ - r, \bar{YD}, \ W \right]; \] (34)

\(^4\)With a Cobb-Douglas production function \( Q_c = AK_c^{\alpha}N_c^{1-\alpha}, \)

\[ \frac{\partial Y}{\partial P_c} = \frac{(1 - \alpha)/(1 - \alpha) P_c Q_c - P_I (Q_I - \delta_c K_c - \delta_I K_I)}{P_c^2}. \]

The common presumptions that consumption is larger than investment and that labor's share of consumption income is larger than capital's share imply a positive value for this expression.
\[ \bar{V} = \bar{V}[r, \frac{\bar{YD}}{r}, W^+] . \]

Although these are of the same general form as the notional demands (17), the interpretation (and hence the magnitudes) of the parameters are of course different, since one reflects voluntary and the other involuntary variations in the employment of presumably different individuals. Some may consequently believe that \( \delta C/\delta YD \) is likely to be larger than \( \partial C/\partial YD \).

Since corporations and the government are able to realize their notional demands, it is not necessary to modify their behavioral equations. Now substituting (30), (31), and (34) into (20) and replacing the labor market equilibrium condition with (29) yields

\[
\begin{align*}
N_c[P_c] + N_l[P_l] &= N \\
\bar{C}[r, \bar{YD}, W] + G_c &= Q_c[P_c] \\
\delta_cK_c + \delta_lK_l + I[q_c, q_l] + G_l &= Q_l[P_l] \\
\bar{L}[r, \bar{YD}, W] &= M^\delta/P_c \\
\bar{V}[r, \bar{YD}, W] &= (B^\delta + P_cE^\delta + P_lE^\delta)/P_c .
\end{align*}
\]

The addition of the sectoral budget constraints (4), (5), (10), and (33) and the substitution of the identities (9), (11), (15), and (29) gives Walras’s law for a disequilibrium model, that the excess effective demands in the nonrationed markets sum to zero:

\[
(\bar{C} + G_c - Q_c) + \rho(\delta_cK_c + \delta_lK_l + I + G_l - Q_l) + (\bar{L} - M^\delta/P_c) + (\bar{V} - B^\delta/P_c - P_cE^\delta/P_c - P_lE^\delta/P_c) = 0 .
\]

We will consequently delete the bond market and solve the three remaining equilibrium conditions for the market clearing values of \( P_c, P_l, \) and \( r \). We will, however, again initially display the derivatives of all four equilibrium conditions and the government budget constraint

\[
\begin{bmatrix}
\bar{C}_r\bar{YD}_{P_c} + \bar{C}_wW_{P_c} - Q'c & \bar{C}_r\bar{YD}_{P_l} + \bar{C}_wW_{P_l} & \bar{C}_r + \bar{C}_wW_r & 0 \\
\rho l_{P_c} & \rho l_{P_l} - \rho Q'_l & \rho l_r & 0 \\
\bar{L}_r\bar{YD}_{P_c} + \bar{L}_wW_{P_c} + M^\delta/P_c^2 & \bar{L}_r\bar{YD}_{P_l} + \bar{L}_wW_{P_l} & \bar{L}_r + \bar{L}_wW_r & 0 \\
\bar{V}_r\bar{YD}_{P_c} - \rho(l_{P_c} - l/P_c) & \bar{V}_r\bar{YD}_{P_l} - \rho(l_{P_l} - l/P_l) & \bar{V}_r + (\bar{V}_w - 1)W_r & -1 \\
+ (\bar{V}_w - 1)W_{P_c} - M^\delta/P_c^2 & + \bar{V}_w(F_{sk} - \delta_lK_l - \delta_cK_c)/rP_c & -\rho l_r & 0 \\
tQ'_c & t\rho Q'_l & 0 & 1
\end{bmatrix}
\]
\[
\begin{bmatrix}
\Delta P_C \\
\Delta P_I \\
\Delta r \\
\Delta B^s \\
\frac{\Delta B^s}{P_c}
\end{bmatrix} =
\begin{bmatrix}
-\Delta G_c \\
-\rho \Delta G_I \\
(1/P_c) \Delta M^s \\
0 \\
\Delta G_C + \rho \Delta G_I \\
-(1/P_c) \Delta M^s
\end{bmatrix}
\]

(37)

in order to assist our imposition of parametric restrictions. Again assuming an initially balanced budget and that the income effects on asset markets dominate the wealth and investment effects, (37) can be rewritten in the form

\[
\begin{bmatrix}
-b_{11} & b_{12} & -b_{13} & 0 \\
-b_{21} & -b_{22} & -b_{23} & 0 \\
-b_{31} & b_{32} & -b_{33} & 0 \\
-b_{41} & b_{42} & b_{43} & -1 \\
-b_{51} & b_{52} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta P_C \\
\Delta P_I \\
\Delta r \\
\frac{1}{P_c} \Delta B^s \\
\frac{1}{P_c} \Delta B^s
\end{bmatrix} =
\begin{bmatrix}
-\Delta G_c \\
-\rho \Delta G_I \\
0 \\
\Delta G_C + \rho \Delta G_I - \frac{1}{P_c} \Delta M^s \\
\Delta G_C + \rho \Delta G_I - \frac{1}{P_c} \Delta M^s
\end{bmatrix}
\]

(38)

where each \( b_{ij} \) is positive. Since the interest rate is inversely related to the price of bonds, our sign pattern reflects gross substitutability. The five equations in (38) are linearly dependent (\( \sum b_{ij} = 0 \)) and we will consequently delete the bond equation in order to solve the system with bond-financed policies:

\[
\begin{bmatrix}
-b_{11} & b_{12} & -b_{13} & 0 \\
-b_{21} & -b_{22} & -b_{23} & 0 \\
-b_{31} & b_{32} & -b_{33} & 0 \\
-b_{41} & b_{42} & b_{43} & -1 \\
-b_{51} & b_{52} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\Delta P_C \\
\Delta P_I \\
\Delta r \\
\frac{1}{P_c} \Delta B^s \\
\frac{1}{P_c} \Delta B^s
\end{bmatrix} =
\begin{bmatrix}
-\Delta G_c \\
-\rho \Delta G_I \\
0 \\
\Delta G_C + \rho \Delta G_I - \frac{1}{P_c} \Delta M^s \\
\Delta G_C + \rho \Delta G_I - \frac{1}{P_c} \Delta M^s
\end{bmatrix}
\]

(39)

The Jacobian determinant

\[
|B| = -b_{13}(b_{21}b_{32} + b_{22}b_{31}) - b_{13}(b_{11}b_{22} + b_{12}b_{21}) - b_{23}(b_{11}b_{23} - b_{13}b_{21})
\]
is negative since \( b_{11} > b_{12} \) and \( b_{22} > b_{23} \) follow from \( b_{41} > 0 \) and \( b_{42} > 0 \). A weaker sufficient condition, \( b_{11}b_{22} > b_{12}b_{23} \), states that the product of the own excess demand effects of consumption and investment goods price changes is greater than the product of the cross effects. After some manipulation it can be shown that a sufficient condition for this is that

\[
(1 - t)C_{YB} \leq \frac{P_cQ_c'}{P_cQ_c' + (P_t/P_c)P_tQ_t'}
\]

which states that equal percentage increases in \( P_c \) and \( P_t \) raise consumption output by more than demand. If the price elasticities of output in the two sectors are the same, then this is a requirement that the household marginal propensity to consume is less than society's average propensity to consume:

\[
(1 - t)C_{YB} \leq \frac{Q_c}{Q_c + (P_t/P_c)Q_t'}
\]

Now premultiplying (39) by \( B^{-1} \) and using \( b_{22} > b_{12} + b_{22} \) and \( b_{11} > b_{21} + b_{31} \) gives the bond-financed multipliers displayed in Table 2. A bond-financed increase in government spending increases the interest rate and the price (and therefore the output) of the commodity purchased and has an ambiguous effect on the price (and output) of the other commodity. For an intuitive explanation of these results, consider first an increase in government expenditures on the consumption good. The excess demand for consumption drives up the price of consumption goods, stimulating employment and production in this sector. The rise in consumption goods prices raises the \( q \) for that industry, causing an increase in investment demand. At the same time, the real demand for money has increased and the real supply has fallen, putting upward pressure on interest rates and discouraging investment demand. If the demand for money has a low interest elasticity (\( b_{33} \) is small), then interest rates may rise sufficiently to on balance reduce investment. Similarly a bond-financed purchase of investment goods stimulates production in that sector, and hence income and consumption demand. An excess demand for money again pushes up interest rates, which discourages consumption. A low interest elasticity of the demand for money again makes it more likely that the increase in interest rates will be substantial enough to on balance reduce consumption. (Benavie's assumption that consumption demand has no interest elasticity insures an increase in consumption.) An open

\[\text{Mackay and Waud sign their analogous Jacobian by making the inexplicable assumption that } P_c \text{, } ( = b_{31}) \text{ is zero. Benavie instead appeals to the correspondence principle and assumes that the time rates of change of } P_c, P_t, \text{ and } r \text{ are increasing functions of the excess demands for the consumption good, the investment good, and money, respectively. However, the motivation for these auxiliary equations is unclear since the real time paths of the endogenous variables are presumably given by the model itself together with the obvious dynamic equations for } K_c, K_t, E_c, E_t, M, B, \text{ and } N \text{ (see, for example, [2, 11]). In addition, if Benavie's excess demand adjustment equations were to hold in real time, then he would need to take into account the spillovers that necessarily result from unfulfilled demands. Benavie's auxiliary equations can be alternatively interpreted as a description of tentative prices in a tatonnement process when the fictitious auctioneer follows a particular pricing strategy. Stability is then concerned with whether or not a particular imaginary auctioneer would find the market-clearing set of prices.}\]
market operation increases the prices and outputs of both commodities and lowers the interest rate. As the government prints money to purchase bonds, it creates an excess demand for bonds that lowers the required rate of return, raises the value of equities, and stimulates both consumption and investment demand. Both \( P_c \) and \( P_f \) rise, each reducing the own and increasing the cross excess demands. Since the own effects are greater than the cross effects (\( b_{11}b_{22} > b_{12}b_{21} \)), the new equilibrium is with a higher \( P_c \) and \( P_f \) and a lower interest rate.

To analyze money-financed policies we can solve (39) with \( \Delta B^g \) exogenous and \( \Delta M^g \) endogenous. The Jacobian determinant

\[
|J| = |B| = b_{51}(b_{13}b_{23} + b_{13}b_{22}) - b_{23}(b_{31}b_{23} + b_{31}b_{22})
\]

is negative. Using \( b_{11} > b_{21} + b_{31} + b_{51} \) and \( b_{22} > b_{12} + b_{22} + b_{52} \), the sign pattern for the multipliers is displayed in Table 2.

A money-financed expenditure can again be viewed as a bond-financed expenditure accompanied by an expansionary open market operation. As in the previous section the later monetary operation is more powerful than the fiscal action in that its effects prevail when there is a conflict. It is again noteworthy that a money-financed expenditure lowers interest rates. The primary difference between the flexible and fixed wage case is that in the former a demand-induced expansion of one industry necessarily contracts the other industry, even though total employment is not fixed. This is, of course, because production in each industry is related (in opposite ways) to the same variable, the relative price ratio. In the fixed wage model, however, sectoral outputs are related to the separate commodity price levels, and can be expanded or contracted together by government policies.
The one-commodity version of this model is the standard IS-LM model with fixed wages and flexible prices:

\[ N^D[P] = N \]
\[ \bar{C}[r, \bar{YD}, W] + I[r] + G = Q[K, N] \]
\[ L[r, \bar{YD}, W] = M^S/P \]
\[ (\bar{Y}[r, \bar{YD}, W]) = B^S/P + F_rK/r + I[r] \]
\[ \frac{M^S - M + B^S - M}{P} = G - rQ[K, N], \]

where

\[ \bar{YD} = (1 - \tau)Q[K, N] \]
\[ W = \frac{M + B}{P} + \frac{F_rK}{r}. \]

The familiar multipliers are shown in Table 2. A comparison of the two-commodity model with the one-commodity model reveals that the short-run effects of bond-financed expenditure on aggregate income and employment become ambiguous when one moves from one commodity to two. In addition, an increase in government expenditures on the investment good may reduce the level of prices.

In the single-commodity IS-LM model, mention is made frequently of two extreme cases \((L_r = 0 \text{ and } I_r = -\infty)\) that emasculate fiscal policy and two opposite extremes \((L_r = -\infty \text{ and } I_r = 0)\) in which monetary policy is impotent. One of the interesting features of the two-commodity model is that this simple contrast is considerably more clouded. The only qualitative effect of an interest-insensitive demand for money \((L_r = 0)\) is to insure that \(\Delta P_r/\Delta G_r\) and \(\Delta P_f/\Delta G_c\) are unambiguously negative; in either case the change in income need not be zero. (With flexible wages, the only effect is to insure \(\Delta P_c/\Delta G_f < 0\).) In both the fixed and flexible wage cases, an infinitely interest elastic investment demand \((I_r = -\infty)\) does fix the interest rate and emasculate government investment purchases; however, government consumer goods purchases will stimulate consumption output and retard investment. A liquidity trap \((L_r = -\infty)\) would also fix the interest rate. Here monetary policy is impotent in both models and, interestingly, \(\Delta P_r/\Delta G_r = 0\) in the flexible wage model. With fixed wages government purchases stimulate production in both sectors. Finally, interest-insensitive investment \((I_r = 0)\) renders monetary policy and consumer goods purchases impotent in the flexible wage model. In the fixed wage case, the only qualitative effect is to make \(\Delta P_f/\Delta G_c\) unambiguously positive.
LITERATURE CITED


