

Dynamic Models of Portfolio Behavior: More on Pitfalls in Financial Model Building

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In an important article in this *Review*, William Brainard and James Tobin have emphasized the role played by the wealth constraint in systems of asset demand equations. The wealth constraint gives rise to consistency conditions which must be satisfied by the demand functions when such a system is specified and estimated. As Brainard and Tobin caution, care must be taken to ensure that unrealistic coefficients are not inadvertently imposed on omitted equations by failure to recognize the consistency conditions.¹ Noting that the wealth constraint applies out of, as well as in, portfolio equilibrium, Brainard and Tobin focus attention on systems in which actual and desired stocks of assets differ. They specify a multivariate stock adjustment model wherein the desired change in holdings of any asset depends in general upon all asset stock disequilibria; the existence of such stock disequilibria can be implicitly rationalized on the basis of costs of adjustment which impinge on the rate of change of at least some assets. In this framework they show that the stock adjustment coefficients must also satisfy certain consistency conditions to ensure that the wealth constraint is satisfied.

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¹This also has implications for the common practice in macro-economic models of leaving the bond market as implicit. Care must be taken to ensure that silly behavior is not inadvertently attributed to bondholders. William Silber, Tobin, and Alan Blinder and Robert Solow have initiated research which "reintroduces" the bond market into macroeconomic models.

An important feature of their analysis is that the total change in wealth (savings plus capital gains) is treated as exogenous to the financial sector, and the asset flow demands described above are *conditional* upon the exogenously given change in wealth. This strategy of separating the portfolio balance decision from the consumption-saving decision is one that Tobin has explicitly used and justified in his 1969 article (especially pp. 15-16), and is one that has been widely and effectively used in modern macro-econometric models.

The central argument of the present paper is that this separation of flow-allocation and stock-allocation decisions is not legitimate in the presence of adjustment costs attached to changing the level of individual asset holdings. The existence of adjustment costs means that there is no portfolio balance problem per se (in the sense of allocation of a given level of wealth), but rather a (longer run) problem of determining an optimal time path for each asset and for the level of consumption. Thus a natural extension of the Brainard-Tobin model is to treat saving and portfolio decisions in an integrated fashion.²

Note that the Brainard-Tobin model is perfectly consistent with *any* model of savings behavior and hence no logical con-

²It appears to be a fairly general result that the existence of adjustment costs leads to integrated behavior. M. Ishaq Nadiri and Sherwin Rosen have established a similar result for the theory of the firm, and Robin Mukherjee and Edward Zabel have recently shown that the "separation theorem" prominent in the finance literature on the mean-variance approach to optimal consumption-portfolio behavior fails to hold when transactions costs are introduced. In my 1975 paper (Appendix), I have argued that the integration of saving and portfolio balance decisions also applies in continuous-time models, even though such models are characterized by separate stock and flow budget constraints.

tradition with an integrated approach is necessarily involved. However, by the very fact that their analysis is consistent with *any* model of saving behavior, we observe that it fails to incorporate information contained in the implicit assumptions about adjustment costs, and this information plays a central role in the integrated model presented in detail in Section I. Then, in Section II the Brainard-Tobin model is re-examined in light of the results of Section I. Section III offers some concluding comments on how the integrated model presents a framework in which one can discriminate between monetarist and Keynesian views about the transmission of monetary policy.

I. An Integrated Model of Consumption and Asset Accumulation

As noted above, the wealth constraint plays a central role in the Brainard-Tobin analysis. However, it is worth noting that in the discrete time framework generally used there is no wealth constraint per se, but rather one single budget constraint relating assets held at the beginning of the period plus income received during the period to assets held at the end of the period plus consumption during the period. The budget constraint is given by

$$(1) \quad \sum_r y_r + c = \sum_r y_r(-1) + X_0$$

where $y_r(-1)$ and y_r are, respectively, beginning and end of period holdings of the r th asset, c is consumption, and X_0 is income.³

To specify asset demands subject to a wealth constraint in such a model is to assume that the household makes its saving decision thereby determining end of period wealth ($X_m = \sum_r y_r(-1) + X_0 - c$), and *independently* chooses the desired allocation

³Throughout I will use the following conventions: X_m is total wealth; X_0 is income; subscripts (i, j) will be used to refer to explanatory variables (X) and unless otherwise noted will take on values from 0 to $m - 1$ in summation; and subscripts (r, s) will refer to assets (y) and take on values 1 to n in summation.

of that wealth amongst the alternative assets. This specification is reasonable when there are no adjustment costs pertaining to the reallocation of a given level of wealth. For then the household's initial position is fully described by the level of income X_0 and the *total* of assets inherited from the previous period $\sum y_r(-1)$. Saving and portfolio balance decisions may be influenced by common variables, but in this case both will be independent of the composition of inherited wealth.⁴ However, when such adjustment costs are present, knowledge of the household's initial position requires knowledge of its income and the values of individual assets $y_r(-1)$ for *each* r . The rational household would, in such circumstances, formulate consumption and asset flow demands dependent upon income, current holdings of individual assets, and long-run asset considerations. For simplicity I assume that the latter are captured by steady-state demands which are independent of initial asset holdings. It is assumed that the desired asset positions y_r^* depend only on the expected values of income and rates of return $X_i^e (i \neq m)$, which in turn, assuming static expectations, equal actual $X_i (i \neq m)$. Throughout, I abstract from uncertainty so that all changes in wealth are planned. Consumption and the asset flow demands are given, respectively, as⁵

$$(2) \quad c = \sum_i b_i X_i + \sum_s e_s y_s(-1)$$

⁴In the absence of adjustment costs, asset demands would be short-run in nature (contingent upon total wealth) and would also be *achieved* in the short run. This is what Lewis Johnson has called a portfolio balance model as opposed to the portfolio adjustment model used in the present paper.

⁵As the referee has pointed out, this and the Brainard-Tobin models are versions of what Duncan Foley has called an end-of-period equilibrium model. Note however that Foley's discussion is somewhat misleading: the end-of-period specification does have an equilibrium. This can be seen by observing that $K^D(0)$ in his first equation on p. 312 is the short-run demand for capital while in equation (4a), where it appears multiplied by k , $K^D(0)$ is the long-run demand; in both, the short-run excess demand disappears as the time period goes to zero.

$$(3) \quad \Delta y_r = \sum_s \gamma_{rs} [y_s^* - y_s(-1)]$$

for $r = 1, \dots, n$

The target values y_s^* are *long-run* steady-state demands,

$$(4) \quad y_s^* = \sum_i \beta_{si} X_i \quad \text{for } s = 1, \dots, n$$

Note that by assumption current wealth (X_m) does *not* enter the long-run asset demand functions (4). However, the n long-run asset demand functions together do imply a target value for total wealth X_m^* , and long-run equilibrium, given X_m^* , will be independent of whether the composition of wealth matters in the short run. Substituting from (4) into (3) yields the n reduced form asset-demand equations for estimation

$$(5) \quad \Delta y_r = \sum_i \alpha_{ri} X_i - \sum_s \gamma_{rs} y_s(-1)$$

for $r = 1, \dots, n$

where the impact multipliers given by $\alpha_{ri} = \sum_s \gamma_{rs} \beta_{si}$ denote the total impact effect of a change in the i th explanatory variable on the flow demand for the r th asset.

The $(n + 1)$ equations (2) and (5) determine the $(n + 1)$ endogenous variables $\Delta y_r (r = 1, \dots, n)$ and c . There are n predetermined variables $y_s(-1)$, and m exogenous variables $X_i (i = 0, \dots, m - 1)$. Total wealth does not appear explicitly since it is the composition of wealth which is important in determining the optimum time paths. There are only n independent endogenous variables since consumption is linked to the asset flow demands by the budget constraint given by (1) and rewritten below in flow terms as

$$(6) \quad X_0 = c + \sum_r \Delta y_r$$

This introduces a linear dependency among the $(n + 1)$ equations which is reflected in the following $(m + n)$ consistency conditions:

$$(7a) \quad b_0 = \sum_r \alpha_{r0} = 1 \quad (\text{since } X_0 \text{ is income})$$

$$(7b) \quad b_i + \sum_r \alpha_{ri} = 0$$

for $i = 1, \dots, m - 1$

$$(7c) \quad e_s - \sum_r \gamma_{rs} = 0 \quad \text{for } s = 1, \dots, n$$

These consistency conditions ensure that the budget constraint is always satisfied: (7a) simply reflects the fact that income gets allocated to either consumption or asset accumulation; (7b) and (7c) show that changes in nonincome explanatory variables (rates of return or initial stocks) cause a change in the allocation of income, consistent with the total $(c + \sum \Delta y_r)$ being constant.

Of particular interest is (7c) which states that the column sums of the spillover matrix $\{\gamma_{rs}\}$ must vary to reflect any variation in the coefficients of the lagged stocks in the consumption equation. Variations in these latter coefficients indicate how the composition of assets influences consumption; variations in the column sums of the spillover matrix are the "mirror images" which indicate how the composition of assets influences saving. Saving, found by summing over the asset flow demand functions, using (7c) is given by

$$(8) \quad \Delta X_m = \sum_s e_s [y_s^* - y_s(-1)]$$

The special case where only total wealth influences consumption, which obtains in the absence of adjustment costs or possibly under restrictive assumptions about the underlying structure of such costs or behavior, arises when $e_s = \bar{e}$ for all s . From (7c) we can see this means that the column sums $\sum_r \gamma_{rs}$ are constant at \bar{e} for all s , the multivariate analogue of a constant speed of adjustment.⁶

The model outlined above is similar in structure to that presented by Brainard and

⁶Saving, in this case, can be represented as the adjustment of actual towards desired wealth,

$$\Delta X_m = \bar{e} [X_m^* - X_m(-1)] \quad \text{where } X_m^* = \sum y_s^*$$

Tobin, but, as a result of explicit consideration of the existence of adjustment costs, differs in several respects, especially with regard to the role played by the wealth constraint. However, it is still true that at any point in time, the sum of asset holdings must equal the value of financial wealth. That fact is incontrovertible; only the experiment giving rise to "consistency conditions" is changed since the change in wealth during any period is now treated endogenously. Furthermore, the basic Brainard-Tobin message remains intact: the potential pitfalls awaiting the researcher who does not take full account of the implications of the budget constraint are severe.

II. The Brainard-Tobin Pitfalls Model

In terms of the notation set out above, the Brainard-Tobin model can be illustrated in terms of the n asset equations given by

$$(9) \quad \Delta y_r = \sum_s \hat{\gamma}_{rs} [y_s^* - y_s(-1)] + \delta_r \Delta X_m$$

for $r = 1, \dots, n$

where the asset targets y_s^* are given by

$$(10) \quad y_s^* = \sum_{i=0}^m \beta_{si} X_i \quad \text{for } s = 1, \dots, n$$

and where $X_m = \Sigma y_s = \Sigma y_s^*$. There are three basic differences between the model described by (9) and (10) and the model outlined in Section I above. These are: the inclusion of the term $\delta_r \Delta X_m$ on the right-hand side of (9); the lack of an equation for saving; and the inclusion of end of period wealth X_m as an explanatory variable in (and constraint on) the asset target demand functions given by (10).

The first point has been adequately discussed by Gary Smith; we only need to note that since the $\delta_r \Delta X_m$ term is completely redundant we can, without loss of generality, drop it. As Smith notes, this is equivalent to using the substitution $\Delta X_m = \Sigma [y_s^* - y_s(-1)]$ to derive new stock adjustment coefficients $\gamma_{rs} = \hat{\gamma}_{rs} + \delta_r$. This is only one of many possible substitutions, all

of which are equivalent, as shown by Smith on page 512.⁷

The second point is more important for the comparative interpretation of the two models. By analyzing the consumption-saving decision separately, the Brainard-Tobin model treats the change in wealth, that is, the sum over r of the left-hand side of equation (9), as exogenous to the financial sector. This gives rise to the consistency conditions shown by (11):

$$(11a)$$

$$\sum_r \alpha_{ri} = 0 \quad \text{for } i = 0, 1, \dots, m-1$$

$$(11b)$$

$$\sum_r \alpha_{rm} = 1$$

$$(11c)$$

$$\sum_r \gamma_{rs} = 1 \quad \text{for } s = 1, \dots, n$$

where the alphas are impact multipliers as described above. Conditions (11) reflect both the fact that asset demands must satisfy the wealth constraint *and* the treatment of wealth as exogenous. Thus (11a) shows that changes in nonwealth explanatory variables lead only to a reshuffling of assets since wealth is explicitly held constant; similarly (11b) shows that an exogenous increase in wealth must induce an equivalent increase in asset holdings. Condition (11c) implies that any reshuffling of initial assets $y_s(-1)$, holding $X_m(-1)$ constant, does not *necessarily* influence current wealth X_m . This of course must be the case since X_m is exogenous. In contrast to the role played by the column sums in the integrated model of Section I (i.e., condition

⁷Brainard-Tobin substitute for the excess demand of equities. They originally introduced the redundancy shown in equation (9) to allow for differential effects of alternative sources of wealth changes (i.e., planned savings vs. capital gains) and hence *two* terms were added to the stock adjustment terms. Since equations such as (9) don't make this distinction, the additional term is completely redundant; as Smith's clarification shows, the confusions in the literature that he addressed were completely unnecessary.

(7c)), this condition tells us nothing about the influence of the composition of wealth on consumption.

It is important to emphasize the different nature of the "flow-asset demand" equations (3) of the integrated model compared to the "contingent-asset-flow" equations (9). If the latter (assuming $\delta_r = 0$) were interpreted as asset-flow demands like the former, and hence their sum were used to represent the *planned* change in wealth, then conditions (11) would imply that saving not only be independent of the composition of wealth (11c), but also of income and rates of return (11a). This, of course, is *not* the way these equations were meant to be interpreted. The point is that, due to the exogeneity of wealth, conditions (11) are only sufficient conditions; if a saving relation explaining ΔX_m is added to the model depicted by (9) and (10), then the sufficient conditions (11) can be replaced by a (possibly less restrictive) set of necessary conditions which will depend upon the saving behavior postulated, as illustrated in the integrated model set out above.⁸

The second point also arises when it comes to implementing the Brainard-Tobin model. There will be econometric problems involved in using wealth as an explanatory variable and hence treating it as predetermined, if it is in fact systematically related to the other explanatory variables. In order to deal with this, one would likely resort to limited information techniques, such as instrumental variables; the natural instruments to choose would be the variables expected to influence saving. Hence the econometric problem is really a reflection of the specification; the integrated model which explicitly includes the consumption-saving choice would lend itself to full-information estimation procedures.

The third basic difference between the

⁸Note that this issue arises independently of whether saving depends on the composition of wealth. If one is working with only a model of the financial sector, conditions (11) are appropriate. If, however, one is working with a complete macro model including a saving function, then to continue to impose conditions (11) will in general result in ignoring information present elsewhere in the system.

models is that Brainard-Tobin constrain the target values by end of period wealth. Mark Ladenson presents this as "... a sort of rational desires hypothesis [since it] constrains the *desired* or equilibrium values of the financial assets and liabilities to obey the balance sheet identity" (p. 179). Such a statement is misleading since the target demand y_s^* is not in general achieved at the end of the period and hence is not an "effective demand"—there is no reason for it to be constrained by end of period wealth. Obviously what must be constrained are the actual quantities y_s . Of course, the model is all conditional upon X_m , and the y_s^* are the quantities that ultimately would be held if the values of X_m were constant for a long enough time. But, there is no reason for (expected) X_m to remain constant, and the decision rules (9) are very myopic in that they don't take into account possible changes in the size of total wealth when planning the adjustment paths of individual assets. In the integrated model, the desired paths for assets do incorporate the accumulation decision, and total wealth at the end of period is as much a result of as it is a constraint on asset demands.⁹

Hence the Brainard-Tobin pitfalls model is formally consistent with the integrated model studied in Section I; when combined with a consumption-savings relationship such as (2), the Brainard-Tobin model will in principle give rise to exactly the same short- and long-run behavior as the integrated model.¹⁰ However, the research

⁹Dropping X_m as an explanatory variable for y_s^* also eliminates the redundancy in equation (9). However we continue to omit the $\delta_r \Delta X_m$ terms since ΔX_m is endogenous. Constraining the y_s^* by X_m is the feature of the Brainard-Tobin model which renders the univariate stock adjustment model ($\gamma_{rs} = 0$ for $r \neq s$) inconsistent. In the integrated model of Section I, univariate adjustment is possible ($\gamma_{rr} = e_r$; $\gamma_{rs} = 0$, $r \neq s$) as is the special case of constant speed of adjustment ($\gamma_{rr} = \bar{e}$).

¹⁰In practice, the results may differ since the "familiar" pitfalls model would maintain a deterministic relationship between $\Sigma \Delta y_r$ and ΔX_m but, presumably, a stochastic relationship between ΔX_m and $X_0 - c$. The integrated model would exclude the concept ΔX_m and posit a deterministic relationship between $\Sigma \Delta y_r$ and $X_0 - c$.

strategies underlying the two approaches are very different, and care must be taken to interpret the results of the models accordingly.

III. Concluding Comments

The portfolio balance models common in the literature have been based on a dichotomized approach to household decisions; as a result the coefficients of the asset demands were not only constrained to satisfy the wealth constraint but also to be consistent with the exogenous wealth assumption. I have presented an integrated approach to household behavior and developed a model which was more general but no more complicated than the portfolio balance model; in fact, the latter model is in some sense a special case of the integrated model.

These results are important for the analysis of the linkage between the monetary and real sectors of the economy. Tobin has used the dichotomized model to provide an insightful analysis of how changes in relative asset supplies alter the structure of equilibrium rates of return (especially the supply price of capital) which in turn induce expenditure flows. Thus monetary policy operates in the usual Keynesian fashion by influencing interest rates; since total wealth is unchanged, no direct effect on expenditure is postulated. Monetarists, on the other hand, suggest that changes in relative asset supplies may elicit direct expenditure effects so that an open market purchase, for example, may affect expenditure other than via interest rates. That is, the monetarist view maintains that there will be a "real balance effect" even in the presence of a multiasset portfolio. In terms of my model the monetarist view of the transmission process is that the coefficient of money in the consumption equation would exceed that on other assets, a possibility generally ruled out in the dichotomized Keynesian model.¹¹ The integrated model

¹¹This is an empirical rather than a theoretical proposition since there is nothing in my model to distinguish money from any other asset. Milton Friedman has argued that the monetarist view of the

presents a framework in which the monetarist view and the Keynesian view arise as special cases, but the special cases can be tested for rather than imposed a priori.

In the integrated model, consumption would in general be affected by the process of the adjustment of actual toward desired asset holdings; a change in asset stocks may well alter current consumption in a manner independent of changes in current or desired wealth (the latter arising out of interest rate changes) by altering the "speed" with which current adjusts to desired wealth. The monetarist view of the transmission process suggests, in terms of my model, that a substitution of money for any other asset increases consumption; that is, it slows the adjustment of actual towards desired wealth. This "liquidity" effect is a short-run dynamic effect since in my model no effect on long-run wealth and consumption is postulated. But in the short run there is a close relationship between money and expenditure, and in this sense, velocity is relatively constant in the short run.¹²

A more general result from the considerations in this section is that consistency conditions derived from examining the portfolio balance decision can be treated only as "sufficient" conditions. Once systematic relationships between the explanatory variables are introduced in other equations—for example, the savings decision—then the sufficient conditions can be replaced by a less restrictive set of "necessary" conditions. Such conditions allow not only for more general models of consumption behavior as emphasized above, but also give rise to an estimation procedure in which asset substitution relationships may

transmission process does not confine the substitution effects to a limited range of financial assets, but supposes that individuals seeking "to dispose of what they regard as their excess money balances . . . will try to pay out a larger sum for the purchase of securities, goods and services, for the repayment of debts, and as gifts than they are receiving from corresponding sources" (p. 910).

¹²See also Friedman's comment, p. 316, in Jerome Stein's volume, on the relative stability of stock-flow and flow-flow relationships.

appear very different from those resulting in the standard approach since the constraints on such relationships are now relaxed somewhat (compare (7a) and (7b) with (11a) and (11b)).

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