CHAPTER 15

Theories of political processes

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1. Introduction

Social choice theory, though sometimes referred to as "mathematical politics" (Samuelson (1967)), differs in emphasis in several respects from the work considered in this survey. Its central concern is normative, focusing on the question of whether there exists a "reasonable" democratic method for aggregating individual preferences into a collective choice or social ordering. If such a method should exist (and the tenor of much of the literature is that it does not), it would still be an open question whether it could ever be realized in an institutional sense. Indeed, governmental institutions as such are essentially absent in that literature, the role of government being confined to mechanically applying the vote-counting or aggregation procedure. Moreover, with few exceptions, noted below, the social choice tradition proceeds on the assumption of perfect information as to voter preferences, or "sincere" voting, and thus ignores the possibility of strategic behavior on the part of voters and of the agents who comprise the government. In this paper we shall be primarily concerned with work which deals with these institutional and strategic aspects of social choice, and which analyzes the operation of governmental and political institutions from a positive or descriptive point of view.

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2. Social optima vs. social equilibria

Wilson's (1970) game-theoretic interpretation of Arrow's (1963) theorem shows one connection between normative social choice theory and the positive analysis of collective political processes. Consider a fixed set $A$ of alternatives and $N$ of individuals (both containing three or more elements). A society or profile $(R_i)_{i \in N}$ is an assignment of a (complete, transitive) preference ordering $R_i$ on $A$ to each individual. In the social choice literature a social welfare function (or method) is a rule or function $f$ which assigns to each profile a binary social preference relation $R$ on $A$.

In Arrow's work the method of aggregation $f$ is required to satisfy the (weak) Pareto principal (PP) and the Independence of Irrelevant Alternatives (IIA) condition, and the social preference relation $R$ is assumed to be complete and transitive (CT). Given any method $f$, a set $D \subseteq N$ of individuals is decisive for a particular pair $x, y \in A$ if $xPy$, all $i \in D$ implies $xPy$. If the method satisfies CT, PP and IIA, a set $D \subseteq N$ is either decisive for every pair of alternatives, or for none (Blau (1972)); hence any such method defines a unique family $\mathcal{D}$ of sets which are decisive for all pairs of alternatives.

From a more descriptive point of view, the process of social choice can be viewed as an $n$-player strategic game. In this framework, one can then ask whether the game has a solution or equilibrium, for example a core. A particular type of cooperative game introduced by von Neuman and Morgenstern (1953) to model voting-type processes is that of a simple game (see also Shapley (1962)). In such a game each of the various player coalitions is either all-powerful and capable of achieving any outcome (these are the ‘winning’ coalitions) or are essentially powerless. Thus the distribution of strategic power in a simple game is described by specifying the set $\mathcal{W} \subseteq 2^N$ of winning coalitions, $\mathcal{W}$, where $\mathcal{W}$ is assumed to be nonempty, monotonic ($C \subseteq C' \subseteq N$, $C \in \mathcal{W}$ implies $C' \in \mathcal{W}$) and proper ($C \in \mathcal{W}$ implies $N - C \not\in \mathcal{W}$). (If, in addition, $C \not\in \mathcal{W}$ implies $N - C \in \mathcal{W}$ then the game is strong; there are no blocking or tie-producing coalitions.)

If for some $x, y \in A$, there exists a winning coalition $C \in \mathcal{W}$ such that $xPy$, all $i \in C$, then $x$ dominates $y$ (or blocks it, in the usual terminology of the theory of the core). A simple game has a core if and only if there exists some undominated alternative, i.e. some $x \in A$ such that for no $C \in \mathcal{W}$ and $y \in A$ is $yPx$, all $i \in C$.

Wilson's (1970) results show that under any social choice procedure which satisfies CT, PP, and IIA, the class of decisive sets of citizens has precisely the properties of a class $\mathcal{W}$ of winning coalitions of a strong
simple game. Moreover, if all individual preferences are strict, the (strict) social preference relation $P$ will be identical to the dominance relation of the game. Similar relations hold under various weakenings of the Arrow axioms: in effect, there is a very close connection between the class of "reasonable" or "democratic" methods of social choice of Arrow's sense, and the class of simple games. The various social choice impossibility theorems, in effect, show that the only voting methods (strong simple games) which can guarantee the existence of a cooperative equilibrium, or core, under every profile of individual preferences are the dictatorial ones (such that for some $i^* \in N$, $R = R_{i^*}$ for every profile); under a non-dictatorial voting procedure a core will often not exist.

3. Strategy and voting

Recent results on "strategy-proof" voting procedures by Gibbard (1973), Satterwaite (1975), and others provide another link between the collective choice framework and the more descriptive study of political processes and behavior. Consider again a set of alternatives $A$ and of individuals $N$. Each individual $i \in N$ has a set of available strategies, consisting of the set of possible weak orders on $A$; thus he "votes" by naming or reporting a ranking of the alternatives. Voter $i$'s sincere strategy is the ordering which coincides with his true preference ordering $R_i$; if he votes insincerely by choosing some other strategy, then some degree of strategic distortion or misrepresentation of preferences is present. A voting procedure is a function which maps the set of $n$-tuples of strategy choices onto the set of alternatives. A strategy-proof procedure is one which never confronts any voter with an incentive to vote insincerely; or, more precisely, one such that the sincere voting $n$-tuple is always (for every society or profile of individual preferences) a Nash equilibrium point. The basic Gibbard-Satterwaite result shows that no (non-dictatorial) strategy-proof procedure exists. (This is essentially equivalent to showing that there is no reasonable voting procedure under which sincere voting is a dominant strategy, or is straightforward in Farquharson's (1969) sense, for every voter.) Similar (negative) results have been obtained by Gibbard (1975) for a wider class of stochastic procedures. These studies deal with voter strategy in an individualistic or non-cooperative framework. Pattanaik (1973, 1975) has examined the problem of strategic voting from a more cooperative or collusive point of view. His results show essentially that with any reasonable procedure some groups
or coalitions of voters will have incentives *collusively* to distort their preferences; this analysis is based on a variant of the game-theoretic *strong equilibrium point* notion, first used by Dummett and Farquharson (1961) in the analysis of voting processes.

4. Voting in committees and legislatures

One interpretation of these various impossibility and vulnerability results is that no procedure or voting method is completely satisfactory and general in all circumstances. Whatever the procedure, situations can always arise which make the outcome valuable to manipulations of the agenda or order of voting or to collusion or strategic voting by some voters or groups. There is a substantial theoretical literature analyzing the effects of these factors in committees and similar voting bodies. An important early contribution is Black’s *Theory of Committees and Elections* (1958), which also contains a survey of earlier contributions by Condorcet, Nansen, and Dodgson (see also Black (1969)). Black introduced the notion of single-peaked preferences, and showed that when voter preferences satisfy this restriction the cyclical majority problem cannot arise and that any of a variety of sequential voting procedures will lead to final selection of the “majority winner” alternative.

Black did not consider strategic aspects of voting. However, an insightful game-theoretic analysis of strategic voting under one important class of voting procedures has been presented by Farquharson (1969). He defines the class of *binary* voting procedures (roughly speaking, the class of sequential voting procedures which proceed by pairwise or binary votes) and analyzes them under various (non-cooperative) strategic assumptions. One such assumption is that of *sophisticated* voting: a sophisticated voting strategy is one which is undominated in the wide sense, in more standard game theoretical terminology (Luce and Raiffa (1957)). Although sophisticated strategies are defined in abstract game-theoretic terms, they seem quite plausible as choices for an intelligent and informed voter in a sequential voting situation. In effect, they are computed by a sort of backward induction on the voting tree, with the voter using his knowledge of others’ preferences to forecast the outcomes of the various contingent final rounds of voting, then using these forecasts to decide how best to cast his own vote on the next-to-last rounds, and so forth. Farquharson shows that (if all voter preferences are strict) sophisticated voting leads to a unique outcome (which does depend on the
procedure, however). A further interesting fact is that the sophisticated voting outcome necessarily belongs to the core, if the latter exists. This fact suggests that binary voting procedures share with competitive markets the property that individual (non-cooperative) optimization leads to a result which is also a collective or social optimum, a property which is certainly not true of many social institutions or social choice procedures.

"Logrolling" or "vote-trading" in committees and legislatures is an extensively studied but still poorly understood phenomenon (for a critical survey, see Ferejohn (1974)). Various claims concerning its effects have been advanced, for example that it (i) leads to overspending on certain types of government services, e.g. "pork-barrel" projects (Buchanan and Tullock (1962), Niskanen (1971)), or, more generally, leads to Pareto inefficient outcomes (Riker and Brams (1973)); (ii) leads to "welfare increases" or net social gains (Coleman (1966); Tullock (1970); Mueller, Philpotts and Vanek (1972)); (iii) permits expression of intensities of preference (Coleman (1966)); (iv) is stabilizing in the sense that it circumvents the voter's paradox problem and leads to a deterministic outcome (Tullock (1970)); (v) is destabilizing (Bernholz (1974), Koehler (1975)); or (vi) has no effect, in the sense that, whenever the effects of logrolling are well defined, the resulting outcome is identical to that which would have occurred without logrolling (Bernholz (1974), Kadan (1972)).

The absence of a common terminology and lack of explicitness in definitions and assumptions in much of this work make it difficult to assess these various claims. In much of the literature "vote-trading" seems essentially to mean strategic collusion among voters. With this interpretation it would usually be destabilizing, since, in general, no cooperative equilibrium (core) exists in an n-person voting game, because of the presence of cyclical majorities (Dummert and Farquharson (1961)). If it is destabilizing and there is no equilibrium outcome then little can be concluded about overspending or efficiency. (Of course, in those cases where a cooperative equilibrium or core does exist, it is Pareto optimal by definition.) Whether the same outcome would also occur in the absence of logrolling (i.e. under sincere voting) would seem to be sensitive to the particular voting procedure employed, in general. With the procedures employed in common parliamentary practice, however, the sincere voting outcome generally does belong to the core (whenever it exists), so it is the same as the vote-trading result (Kadan (1973), Kramer (forthcoming)).

More restrictive formulations of the vote-trading process have been proposed. Ferejohn (1974), for example, has investigated the effects of successive pairwise vote trades (essentially the formulation used also by
Riker and Brams (1973)). It turns out, however, that such trades may be either stabilizing or destabilizing, and may (or may not) lead to outcomes different from both the core and the sincere voting outcome, i.e. their effects are nearly as indeterminate as those of collusion in general.

Many perceptive observers of legislative bodies such as the U.S. Congress believe that much public works and other expenditure results from some sort of pervasive logrolling or quid pro quo arrangement among legislators, and so widely shared a belief certainly deserves to be taken seriously. However, the various rigorous models of the logrolling process proposed so far do not provide an adequate analytical explanation of the phenomenon.

5. Direct democracy and representative government

In the work discussed above, and in the social choice literature in general, the central focus is with the aggregation of individual preferences directly into a social ordering or decision, without intervening agents or institutions. Arrow’s rationality and Independence of Irrelevant Alternatives conditions (and others, such as Plott’s (1973) Path Independence condition) have the effect, if not the intent, of rendering the social choice process independent of such institutional features as the order of voting or the manner in which certain alternatives are selected and structured or grouped into issues. The “government,” to the extent that it appears at all in this tradition, is an essentially passive entity whose role is confined to mechanically applying the social decision rule to translate citizen’s preferences into a collective decision. The theory is thus that of a direct democracy, such as a New England town meeting or an Athenian assembly (or perhaps a committee or legislative assembly). Most democratic societies, however, are of the indirect or representative type, in which the citizenry votes only sporadically, to elect representatives or perhaps to decide an occasional issue put to a referendum. The vast majority of detailed policy and resource allocation decisions are made not by the citizenry as a whole, but by these elected representatives and by a permanent government bureaucracy. These agents are not passive entities which mechanically translate the citizenry’s desires into policy. They have interests and objectives of their own, which may be related to, but are by no means identical to, those of the populace at large; and their decisions are made in the face of informational, institutional, and other constraints.
The extensive political science literature on these subjects (e.g. Key (1958), Schattschneider (1942)) is primarily descriptive and empirical, with little by way of systematic theoretical development. There have been several interesting attempts by economists (e.g. Breton (1974), Downs (1967), Niskanen (1971), Tullock (1965)) to develop analytical theories of interest groups and bureaucracies. This work, however, is still of a fairly intuitive level and cast in a partial-equilibrium framework and thus difficult to relate to broader preference-aggregation or general-equilibrium questions.

The more rigorous and explicit theoretical work has been largely concentrated on voting and electoral competition. Most of this has been set in an essentially partial-equilibrium framework. One exception, which considers both political and economic processes in a rigorous general-equilibrium framework, is Slutsky (forthcoming). He assumes taxation and allocation decisions on social goods within the public sector are decided by a competitive voting process, and he establishes conditions for the existence of a general equilibrium in public and private goods, taxes, and private goods market prices. A rather special type of voting mechanism is needed to achieve this equilibrium, however.

6. Electoral competition

In the classical work on electoral competition (e.g. Hotelling (1929), Downs (1957)), the underlying space of alternatives (policies or social states) is taken to be a continuum, with the alternatives variously interpreted as different positions along some underlying ideological (liberal-conservative) spectrum of policies, or varying amounts of a single public good or service, or perhaps alternative possible sizes of the government sector as a whole. Each citizen is assumed to have a strictly convex preference ordering $R$, over the alternatives (strict convexity and unidimensionality are equivalent to Black’s (1958) single-peakedness condition). The other agents in the process are two political parties I and II, who compete for the electorate’s votes by advocating particular policies or alternatives. In every period an election is held, with each voter voting “sincerely,” for the party whose platform he prefers. Whichever party receives a majority is then elected and puts its policies into effect. The parties are interested only in winning elections, not in governmental policies as such; thus each party is motivated to adopt policies which maximize its prospects in the next election. The original
Downs–Hotelling analysis argued that under these conditions, both parties will eventually converge on their platform choices, on the “majority winner” policy, which will be the median of the distribution of voters’ most-preferred points.

From a more rigorous game-theoretic point of view, the competition for votes can be viewed as a two-player game. As before, $A$ is the set of underlying alternatives or policies, and $N$ the (finite or infinite) set of voters, each with a (transitive, complete) preference ordering on $A$. For any $x, y \in A$, define $h(x, y)$ as the fraction of the electorate which prefers $x$ to $y$ (the measure of $\{i \in N : x_P y\}$ in the infinite-vector case). Clearly $h(x, y) + h(y, x) \leq 1$, with equality holding if no voters (or only a set of zero measure) are indifferent between $x$ and $y$. A policy $x^o$ is a Condorcet or majority winner if it cannot be defeated in any pairwise majority vote, in the sense that $h(x^o, y) \leq h(y, x^o)$ for all $y \in A$. The active players are the two political parties $I$ and $II$; each chooses as a strategy some alternative in $A$ so as to maximize its support in the election. The payoff functions $v_I$ and $v_{II}$ can be defined in various ways, most simply in terms of each player’s plurality, or margin over its opponent. Thus, for any pair of strategy choices $s_I, s_{II} \in A$, let $v_I(s_I, s_{II}) = h(s_I, s_{II}) - h(s_{II}, s_I) = -v_{II}(s_I, s_{II})$. (Other objectives are sometimes assumed in the literature, e.g. that the parties strive to maximize their total vote shares, or their probabilities of winning by a positive margin of any size; these are essentially equivalent with respect to the main propositions given below, though for certain purposes – e.g. showing the existence of an “abstention” equilibrium – the precise specification of the objective function does become important.) Given this structure, it is not difficult to establish the following: the two-player zero-sum electoral game will have a pure strategy equilibrium if, and only if, the electorate’s preferences are such that some policy is a Condorcet winner (in such an equilibrium, if it exists, each party will adopt a majority-winner policy as its strategy).

This condition is satisfied in the Downs–Hotelling case, since the single-peakedness of voter preferences (implicitly assumed by Downs and Hotelling) ensures the existence of a Condorcet winner. Thus, under these conditions there exists a well-defined equilibrium policy or level of public services. Moreover, since the equilibrium policy is the “majority winner” which defeats every other policy, there is a sense in which the “hidden hand” of two-party competition results in maximization of the abstractly defined majority preference relation, which (under single-peakedness) is an essentially Arrowian social welfare function.

The single-peakedness assumption, which effectively requires the
space of alternatives to be one-dimensional (Wagstaff (forthcoming)) is clearly a restrictive one, however, particularly if the alternatives are interpreted as budget allocations, or quantities or levels of public services. There has been a great deal of work on extending the analysis of electoral competition to multi-issue elections and to the analysis of majority voting on multi-dimensional choice spaces. (In view of the relation noted above between the existence of a pure strategy electoral equilibrium and the existence of a "majority winner" policy, these are equivalent.) In the multi-dimensional case, where the alternatives are represented by points in $R^k$, $k > 1$, and voter preferences satisfy the normal continuity and convexity conditions, the usual social choice conditions for transitivity of majority rule (e.g. Sen (1966), Inada (1969), Sen and Pattanaik (1969)) fail, in general (Kramer (1973)). A variety of rather different conditions for the existence of a "majority winner" in such situations have been proposed, however (Davis and Hinich (1966); Tullock (1967); Plott (1967); Davis, de Groot, and Hinich (1972); Wendell and Thorson (1974)).

When the number of voters is finite, the Plott (1967) conditions (and their extensions, e.g. by Sloss (1973)) apply; these assume voter preferences to be representable by strictly quasi-concave, continuously differentiable utility functions, and are expressed in terms of the gradient vectors, $\nabla$, of these functions. The conditions are satisfied at a point $\theta$ if some voter $i$ is satiated at $\theta$ (so $\nabla_i(\theta) = 0$), and the remaining voters $N - \{i\}$ can be partitioned into pairs $(j, j')$, $(k, k')$, ... in such a way that each voter $j$ is paired with a voter $j'$ such that $\nabla_j(\theta) = -\lambda \nabla_i(\theta)$, $\lambda > 0$ (or equivalently, the contract curve between $j$ and $j'$ contains $\theta$). Each individual preferring a change from $\theta$ in one direction is thus offset by another who prefers a change in the opposite direction. The conditions thus amount to a kind of symmetry requirement on the distribution of voter preferences. These conditions are sufficient for the point $\theta$ to be a Condorcet winner. If $n$ is odd and all voters' satiation points are distinct, they are necessary as well. It is clear that the Plott conditions are very restrictive, however, and will not be satisfied by most distributions of voter preferences. Even if they should happen to hold for a particular distribution, they would not be preserved under arbitrarily small perturbations of some voters' preferences. In this sense, the existence of a Condorcet winner is an exceptional and unstable occurrence in multi-dimensional voting problem, at least in the finite-electorate case.

Several authors, notably Tullock (1967), have argued that majority cycles are less likely if the number of voters is very large, and that in the
limiting case of a continuum of voters, Condorcet winners will exist quite
generally. Examples supporting this thesis have been exhibited by Tul-
lock Davis and Hinich (1966), and others, and Arrow (1969) has given a
general condition for the existence of a Condorcet winner in the contin-
um-of-voters case. Subsequent work, however, has shown that this
“law of large numbers” hypothesis is not valid. Most work on infinite
electorates (Arrow’s excepted) has assumed voter preferences to be of
special form, in which each voter $i$ is satiated at a unique point $s^i$, and his
utility for any point $x$ decreases monotonically with the distance (by some
metric, such as the Euclidean) from $x$ to $s^i$. (We shall refer to this as a
Type I utility function henceforth.) The distribution of voters satiation
points over $R^4$ is described by a continuous density function $f$. These
assumptions are not sufficient for a Condorcet winner; the Tullock and
Davis-Hinich examples are, in fact, rather special, since each uses a
density function which is radially symmetric about some point $\theta$ (in the
sense that $f(x) = f(\theta - (x - \theta))$, for any $x$). The (very strong) radial
symmetry condition is sufficient, though not quite necessary, for the
existence of a majority winner. McKelvey, Ordeshook, and Ungar (1975)
have shown an only slightly weaker condition to be both necessary and
sufficient. Their condition is that there exists a point $\theta$ such that for any
$z \neq \theta$, the integral of the density satisfy $\int_{L^+} f = \int_{L^-} f$, where $L^+$ and $L^-$
are the rays from $\theta$ through $z$ and $\theta - (z - \theta)$, respectively. This condition, a
natural analog of Plott’s conditions for the finite-voter case, is clearly also
a very strong one, which holds only on a subset of measure zero (of the
set of Type I, continuous $f$ societies). Relaxing the Type I or continuity
conditions would not affect the basic conclusion, however, for there is no
reason to expect more complex preference structures of voter distribu-
tions to be any more likely to yield a majority equilibrium.

It is thus clear that majority rule is not transitive and does not yield a
majority winner in multi-dimensional voting problems, irrespective of the
size of the electorate. It is still conceivable that it might be “well behav-
ed” in some weaker sense, however, Buchanan (1968), for example,
has suggested that when majority rule does cycle in such problems, the
cycles will be confined to the Pareto Optimal set. Tullock (1967) has gone
further and argued (at least for Type I preferences) that the cycling will
tend to move toward a central area in the interior of the Pareto set and
remain there. There have been related proposals in the social choice
literature to define a social preference relation by the transitive closure of
the majority preference relation, so that any two alternatives belonging
to the same cycle are socially indifferent (Kadane (1972), Good (1971)).
Presumably, these proposals one based on the hope that one of the cycles will be confined to a relatively small subset of the alternatives, akin to Tullock's "central region." However, a result by McKelvey (1975) shows that in multi-dimensional voting problems, there will typically by a single majority rule cycle extending over the entire feasible region. McKelvey's result is as follows: if all voters have Type I preferences and no majority winner exists, then, for any two alternatives \( x, y \), a sequence of points \( (x, x', x'', \ldots, y) \) can be found, which begins with \( x \) and ends with \( y \), such that each point is preferred by a majority to the preceding point. Though this result invokes the assumption of Type I preferences, it seems clear that the conclusion is a general one: except under very special conditions or restrictions on voters' preferences, in multi-dimensional choice problems majority rule can, quite literally, wander from anywhere to anywhere in the space.

With respect to electoral competition, the import of these rather pessimistic results is that no purely strategy equilibrium for the parties will exist, in general, and the competition for votes may lead the parties to "chase" each other over the entire feasible region. In one attempt to circumvent these difficulties, Shubik (1970) and Mc Kelvey and Ordeshook (1975) have explored the possibility and nature of a mixed-strategy equilibrium for the parties. It has been shown that the set of policies which are played with positive probability (or density) in such an equilibrium constitutes a subset of the Pareto Optimal policies. No sharp characterization of this set of policies (or of the equilibrium probability mixture itself) has yet been given, however. In fact, it is an open question whether such a mixed strategy equilibrium exists at all: the usual versions of the min-max theorem do not apply, since the pure strategy sets are infinite and (as is easily shown) the payoff function is neither continuous nor concave in each party's strategies. There also interpretative difficulties with this mixed-strategy approach. It is tempting to interpret mixed strategies as ambiguous or uncertain policy commitments by the parties. If they were so interpreted, however, voters would be in the position of voting on lotteries over policies. The relevant pure strategies for each party would then be the set of possible lotteries, and within these expanded strategy sets no pure strategy equilibrium will exist, in general (Ordeshook (1971)). Thus one must interpret mixed strategies as requiring the parties simultaneously and randomly to choose unambiguous or "pure" policies in advance of the electoral campaign. However, either party could benefit by postponing its choice until its opponent commits itself to a policy, and then choosing (non-stochastically) a policy which
ensures its victory. Hence, both parties have strong incentives to abandon their equilibrium mixed strategies, so it is not clear that a mixed strategy equilibrium (even if it does exist) is operationally or descriptively meaningful in the context of electoral competition.

A rather different approach proceeds from the premise that some citizens will (stochastically) abstain from voting under certain conditions, for example, if they dislike both parties' policies. Particular versions of this type of assumption yield a pure strategy equilibrium for the parties (Hinich, Ledyard, and Ordeshook (1972, 1973); Riker and Ordeshook (1973)). However, the existence of this type of "abstention" equilibrium is sensitive to the specification used, and the particular formulations needed to ensure existence seem somewhat ad hoc and not particularly consonant with the available empirical evidence on voter abstention (Slutsky (1975)). In any event, such an equilibrium would presumably fail to exist whenever compulsory voting laws or the salience of the election resulted in high voter turnout.

A more dynamical approach to the analysis of electoral competition has been proposed by Kramer (1975). The two parties are assumed to compete for votes repeatedly, over an infinite series of elections. In each period one of the parties is elected and enacts the policy it advocated. In the next election the incumbent must defend this policy, while the "out" party, which may adopt any policy it wishes, is assumed to choose one which maximizes its vote against the incumbent. A sequence of successively-enacted "winning" policies will be generated by this process, which depends on the policy choices of the parties and the outcomes of the successive elections. It is shown (under the assumption of Type 1 preferences) that every such sequence of policies converges on a particular subset of the feasible policies, the min-max set (Simpson (1969)). Typically this equilibrium set is a small subset of the Pareto set which decreases in size as the number of voters increases. In this sense a "law of large numbers" does hold for this type of equilibrium, and the process becomes more determinate in large electorates.

From a normative or social welfare point of view, one conclusion suggested by these analyses is that two-party electoral competition is generally Pareto efficient: in the mixed strategy approach, the equilibrium strategies of the parties assign zero probability to the Pareto inoptimal policies, while the dynamical min-max equilibrium set is also a subset of the Pareto set. The dynamical equilibrium, in fact, is a social optimum in a stronger sense, for it can be shown to consist of the maximal elements of an essentially Arrowian social preference relation (which is, roughly
speaking, the sharpest or most decisive of the anonymous and acyclic relations on A (Kramer (1975)).

7. Multi-party competition

The preceding results apply to two-party electoral competition. The theory of multi-party competition is in a much more rudimentary state, Dow's (1957) pronouncements on the subject notwithstanding. Competition among three or more parties seems intrinsically unstable - even under the strongest equilibrium does not exist. In general, when more than two parties or candidates are competing for votes, as is easily shown. Moreover, with three or more players there are also opportunities for collusive or non-competitive behavior, which is a further source of instability.

Another serious complication is the problem of voter strategy. This is not an issue in the two-party case, since incentives for strategic voting or misrepresentation of preferences do not arise when there are only two alternatives. In that case the analysis can proceed on the assumption of sincere voting, the parties' vote shares will be uniquely determined by their platform choices, and thus the payoff functions for the two-player electoral game are well-defined. This conclusion no longer holds when there are three or more parties, however, since, as has been long realized, under most electoral rules some voters or groups will have incentives to vote strategically, in ways not necessarily reflecting their real preferences for the alternatives. While there do exist certain "strategy-proof" voting methods under which such distortions do not arise, they are rather special. Stochastic mechanisms which make essential use of randomization in selecting a winner, for example, are unlikely to be widely accepted for deciding important electoral contests, even though some such schemes are strategy-proof (Gibbard (1975)). If voter preferences are single-peaked then parliamentary elections under a proportional representation electoral rule are invulnerable to (individual or collusive) strategic voting (Kramer (1974)). This result does not extend to non-single-peaked preferences, however, or to other types of electoral rules or systems. Thus, in general, sincere voting cannot be assumed in multi-party elections, and voter behavior must be analyzed in terms of strategic game-theoretical notions of rationality. The obvious cooperative (e.g. core or strong equilibrium point) and non-cooperative (e.g. Nash equilibrium) formulations of strategic rationality do not lead to unique strategy
choices, however, and voter behavior is, to some degree, indeterminate. In this case the parties' vote shares are not well-defined functions of their strategy choices, so it is difficult to see how the Downs-Hotelling framework can provide useful or convincing analysis of multi-party competition.

Comparison of what is known about the two-party and multi-party cases does suggest one interesting difference between political and economic competition, however. In electoral competition, the duopolistic two-party case is stable, efficient, and socially optimal. These virtues are not assured as the number of competitors increases, and (though one hesitates to draw inferences from a theory which is largely non-existent at present) may well fail completely in the multi-party case. This is in contrast to a private-goods economic market where the benefits of competition arise when the number of firms is large.

8. Extensions and limitations of the theory of electoral competition

In concluding, let us briefly note some areas in which further work could lead to a more realistic and useful theory. One concerns the nature of the political parties themselves, with respect to their internal organization, structure, and motivation. The Downsian type of party is completely pragmatic, interested solely in electoral gain, and willing and able to commit itself to any policies which further this aim. In fact, the active membership of most parties has strong ideological and policy preferences, and the internal structure of some parties (e.g. the British Labor party, or the U.S. Democratic party, particularly since the recent reforms of the nominating process) permits these preferences to influence its electoral strategy. Davis and Hinich (1966) and Hirschman (1970) have explored the effects of nominating primaries in special cases, but there has as yet been no comprehensive analysis of electoral competition under more general assumptions, in which the parties are assumed to be pursuing both policy and electoral objectives. This case may be of particular importance in developing a theory of multi-party competition, since as noted earlier, pure vote maximization is unstable in that case.

A somewhat different issue concerns the role of information and uncertainty. There have been several studies in the abstract voting theory literature (e.g. Zeckhauser (1969), Fishburn (1972), Shepsle (1972a)) in which lotteries over alternatives are voted on, along with the pure alternatives themselves, and Shepsle (1972b) has investigated the effects
of this type of uncertainty in electoral competition. In general uncertainty
tends to be destabilizing: even if a Condorcet winner or electoral
equilibrium exists in the underlying set of “pure” policies, there generally
will not exist such an equilibrium in the domain of probability mixtures
over this same set of policies. In the context of electoral competition,
greater variability in a party’s policy commitments (or variance, if a
party’s platform is interpreted as a distribution over a single policy
variable) may either hurt, or in some cases, help the party.

It seems important to extend this work, for in several ways it could lead
to a more realistic and usable theory of elections. One natural interpreta-
tion of campaign advertising, for example, is that the parties spend
resources to reduce (or conceivably increase) the uncertainty with which
they are viewed by the electorate. A better understanding of the role of
uncertainty might thus have direct policy implications in the area of
campaign finance reform. Another possible application concerns recent
econometric studies of the electoral effects of macroeconomic conditions
(e.g. Kramer (1971), Lepper (1974), Tufte (1975), Bloom and Price
(1975)), which show that the electorate generally tends to respond by
voting for or against the incumbent party according to its success (or luck)
in managing the economy. This is open to the obvious objection (Stigler
(1973)) that it is hardly rational for a voter to vote against an incumbent
unless the opposition party can be expected to do better. However, voters
face an obvious asymmetry in information concerning the incumbent and
opposing parties, since the incumbent’s policies and personnel have been
put to the test of very recent practice at the time of election, while the
oppositions’ probable performance can only be inferred from its state-
ments of intention and its previous performance in office, which may have
occurred long ago and under quite different conditions. Thus, the incorpo-
ration of uncertainty (and perhaps also some “rational expectations”
ideas) into the voters’ decision model may well reconcile these two views
and provide a needed link between empirical studies of voting and the
more abstract theory of electoral competition.

References

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