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Miscellany

An Example of a Trading Economy with Three Competitive Equilibria

This brief note is presented merely as a convenience for those who wish to see what an actual numerical example of a smooth trading economy with multiple equilibria looks like when depicted in an Edgeworth box diagram. We present a two-trader two-commodity economy in terms of a fanciful exchange between two kinds of money. The example is robust, in that its qualitative features would survive small perturbations in the data.

The tourists.—Ivan has R 40 in his pocket and wants some dollars; John has \$50 to spare and would be happy to exchange some of it for rubles. Their utility functions (x = rubles, y = dollars) are

$$\begin{cases} u^1(x, y) = x + 100(1 - e^{-y/10}) & \text{(Ivan, in rubles)} \\ u^2(x, y) = y + 110(1 - e^{-x/10}) & \text{(John, in dollars).} \end{cases} \tag{1}$$

Note that these functions are not only concave and smooth (C^∞) but additively separable, with one good entering linearly in each. It is well known (but virtually ignored in many textbook treatments of competitive uniqueness)[1] that the competitive equilibrium is unique if the *same* good is linear and separable in all utility functions, provided only that this good is in sufficient supply and the preference sets are smooth (C^1) and strictly convex, as they are here. The present example shows that this "transferable utility" or "welfare maximization" approach to uniqueness does not allow even a modest tinkering with the hypotheses.

In figure 1 the indifference curves are indicated by the broken lines. The locus of points of tangency is the straight line D^1D^2, given by

$$y = x + 50 - 10 \log 110 = x + 2.995. \tag{2}$$

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[1] See, e.g., Arrow and Hahn 1971, chap. 9.

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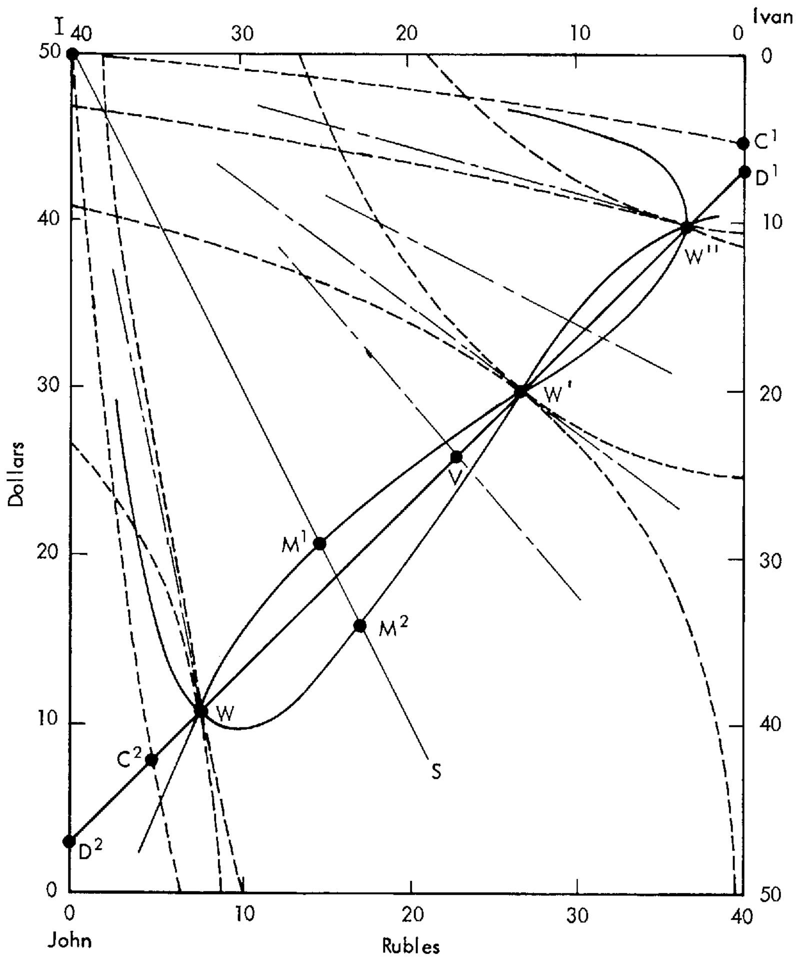

FIG. 1.—Three competitive equilibria

Edgeworth's "contract curve" C^1C^2 runs along this line and a short piece of the boundary. The conditions for a competitive allocation reduce by elementary calculus to the following transcendental equation:

$$x(1 + 11e^{-x/10}) = 10 \log 110, \qquad (3)$$

which has three roots in the region of interest. These lead to the three solutions indicated by W, W', W'' in figure 1 and given numerically in table 1. Their relation to the two response curves (solid lines) is also shown in figure 1.[2]

[2] To illustrate the definition of "response curve," suppose the price ray IS is given exogenously. Then Ivan's best trade is M^1, John's M^2.

TABLE 1

NUMERICAL DATA FOR FIGURE 1

	ALLOCATION (TO JOHN)*		EXCHANGE RATIO	UTILITY PAYOFFS		
	Rubles	Dollars	(Dollars: Rubles)	Ivan	John	
I	0.00	50.00	...	40.00	50.00	Initial point
C^1	40.00	44.89	0.20:1 (m) 0.13:1 (a)	40.00	152.88	Endpoints of core
C^2	4.83	7.83	6.79:1 (m) 8.73:1 (a)	133.69	50.00	
W	7.74	10.74	5.07:1	130.29	70.01	Competitive solutions
W'	26.83	29.82	0.75:1	99.88	132.30	
W''	36.78	39.77	0.28:1	67.27	146.99	
V	23.00	25.99	1.10:1 (m) 1.04:1 (a)	107.94	124.96	Game value

NOTE.—(m) = marginal, (a) = average.
* For Ivan, subtract from (40, 50).

If we take a contract point between W and W', the direction of common tangency (dot-dash line) passes above the initial point I; if we take one between W' and W'', it passes below I. This means that the equilibrium prices associated with W' are dynamically unstable, in the sense that raising the price of either good would create a positive excess demand for that good. This in turn (in a suitable dynamic model) would tend to drive that price up still further. The two other solutions, W and W'', are dynamically stable (see Gale's article [1963], where another simple example of nonuniqueness will be found).

In table 1 we have also indicated the core and value solutions of the trading game in order to suggest outcomes alternative to those of the competitive equilibrium (see e.g., Shapley and Shubik 1969).

L. S. SHAPLEY

RAND Corporation

M. SHUBIK

Yale University

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