Cowles Foundation Paper 444

Reprinted from

THE JOURNAL OF FINANCE · VOL. XXXII, NO. 1 · MARCH 1977

COMPETITIVE EQUILIBRIUM CONTINGENT COMMODITIES AND INFORMATION*

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TWO SIMPLE EXAMPLES ARE presented to illustrate a problem in the interpretation of competitive equilibrium with contingent goods.

Consider an economy with two individuals, each of which has the same utility function of the form

$$\Pi_i = \sum_{j=1}^{m} p_j \log(1 + x_j^i)$$

where i is the "name" of the trader, $i = 1, 2$ and j is the commodity consumed by i when the system is in state j. We assume that there is only one basic noncontigent commodity which appears in the j states of the system.

For simplicity let us assume that the system reaches 3 states each with probability of $\frac{1}{3}$. The distribution of the commodity in state j is given by (x_j^1, x_j^2). In particular in:

State 1 the distribution is (0, 1),
State 2 the distribution is (1, 0), and in
State 3 the distribution is (1, 1).

We now consider two economies which differ only in their information content. They are illustrated in Figure 1a and 1b.

In the first economy there is no uncertainty. Nature randomizes first and the traders are informed before they go to market. Thus we have essentially three separate markets without uncertainty, with one market for each state. There will be no trade in any of the three markets and the expected payoff to each trader in this economy will be:

$$\Pi_1 = \Pi_2 = \tfrac{1}{3}\log(1) + \tfrac{2}{3}\log(2) = .20066.$$

In the second economy, at the point of trade, there is uncertainty. Following the Arrow-Debreu [1, 2] way of dealing with this we may consider that the first trader has an endowment of the three contingent goods of (0, 1, 1) and the second trader has (1, 0, 1). There will be an exchange of (contingent) commodity 1 for 2 and the expected payoff that each will obtain in the competitive market is:

$$\Pi_1 = \Pi_2 = \tfrac{2}{3}\log(1.5) + \tfrac{1}{3}\log(2) = .21773.$$

* The research was supported by the Office of Naval Research. The research was also partially supported by a grant from the Ford Foundation.

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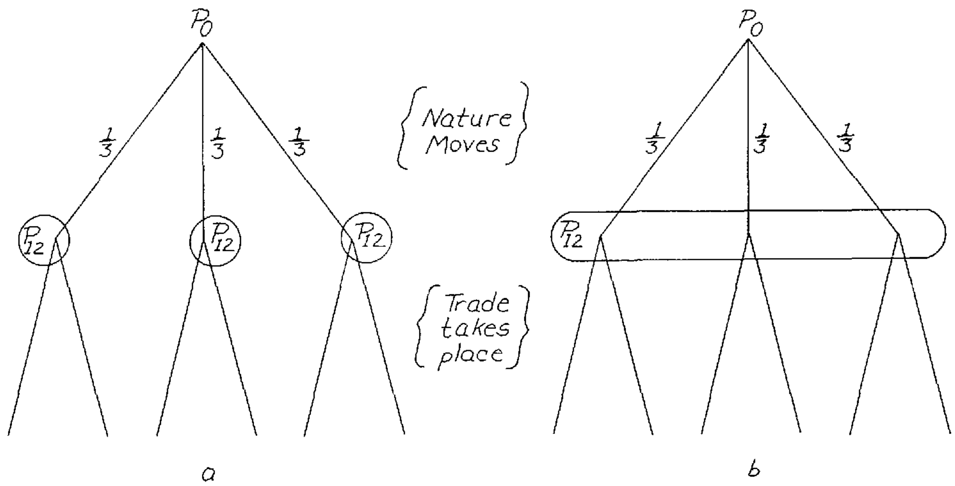

FIGURE 1

We note that this is greater than in the first case where they had more information.

Figure 2 shows the (*ex ante*) Pareto optimal surface for the traders. c_1 is the payoff associated with the markets without uncertainty and c_2 the markets with uncertainty. We thus have higher expected payoffs for greater ignorance.

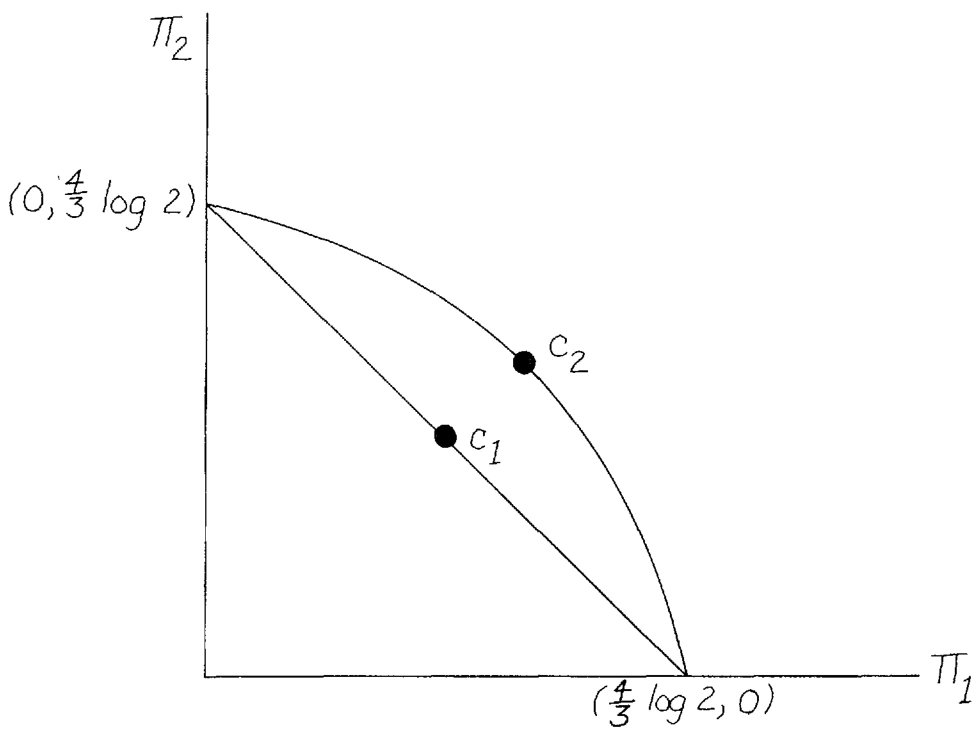

FIGURE 2

We can improve the expected payoffs for the first market by giving the traders the option of trading in ignorance of Nature's move, i.e. the option of throwing away the (unwelcome) additional information. But this appears to require a somewhat detailed rationale in terms of the actual mechanisms of the order of trade, information, communication and types of insurance contract available. This can be done by modeling the markets as noncooperative games in extensive form [5].

The basic point in this paper has been made in a somewhat less formal manner by Hirshleifer [3] who states: It can be shown that in the circumstances assumed here, risk-averse individuals will prefer that the information not be released. For, the anticipation of public information becoming available in advance of trading adds a significant "distributive risk" to the underlying "technological risk" (as to which state will obtain). A community of such individuals would actually pay something to a knowledgeable outsider not to reveal, in advance of market trading, which state will obtain!

This example concerning the availability of information is presented here more formally in a general equilibrium context to stress that the general equilibrium model is both noninstitutional and inadequate in its ability to cope with problems of trade posed by most different information conditions. Instead of specifying financial instruments such as money, futures contracts and insurance contracts, and financial institutions such as banks and insurance companies, the extension of the Arrow-Debreu model to cope with uncertainty was an ingenious device to extend the already unsatisfactory general equilibrium nonfinancial barter model to one with barter trade among contingent commodities.[1]

In order to stress this, we can combine the two examples from Figure 1 into a single example which brings out the essential feature that the information conditions appear naturally as an externality in an enterprise economy.

Case 1. Each individual must independently decide whether or not he wishes to be informed of nature's move. Before the traders go to market, they are informed to each others decision. This is shown in Figure 3. The traders select their state of information then trade knowing what the other knows. The two nonsymmetric information conditions (where one knows the outcome of the random move and the other does not) cannot be handled by general equilibrium analysis but can be handled when considered as a noncooperative or cooperative game of strategy. The failure of the competitive equilibrium analysis for this type of information condition has already been noted by Radner [4]. The type of noncooperative game considered here is not fully defined until the rules concerning what type of contingent trade contracts are permitted. If we assume that the trader A with more information can still offer to buy or sell contingent commodities it is clear that he should do so only when nature is against him. However as the other trader B knows A is informed, B can deduce that if a contingent contract is offered it is not worth buying, thus the decision to select what information to know can be portrayed by the single 2×2 matrix game shown in Figure 4. This has a nonco-

1. It might be thought that a contingent commodity is a futures contract—it is only in an extremely limited manner—it is a barter contract whose single condition is always fulfilled.

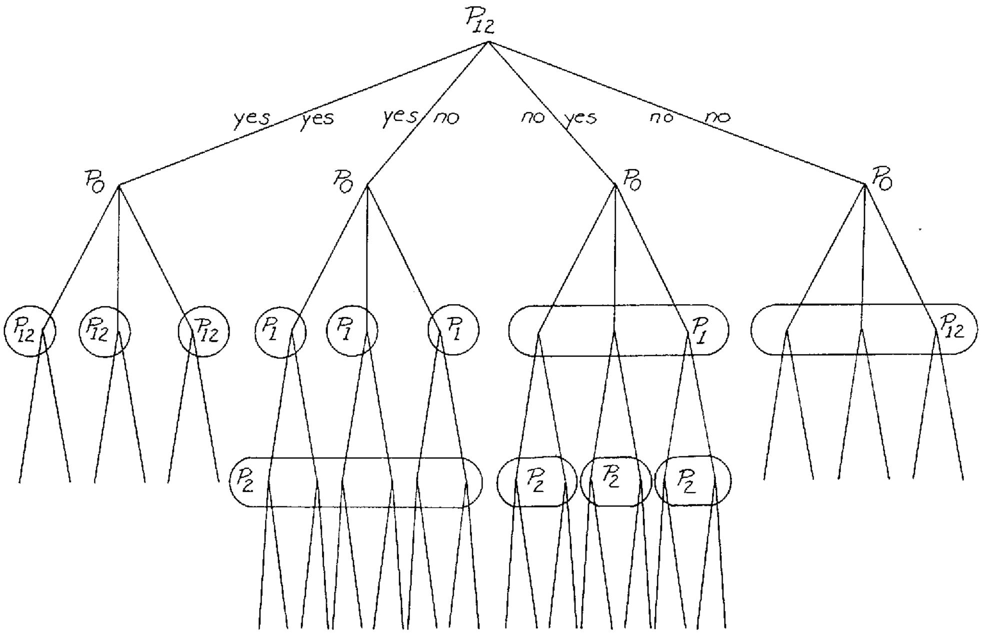

FIGURE 3

operative equilibrium point at which both decide not to be informed of nature's move.

Case 2. A slight modification of case 1 illustrates the full need for a cooperative decision concerning disclosure. Suppose that each individual must independently choose whether he wishes to be informed of nature's move. However, after he has done so he is *not* informed of the choice of information state of the other. This condition would cause a relatively complex change in the information sets shown in Figure 3. In particular, if trader A is offered a contingent commodity contract by B, not knowing B's state of information, A must consider that there is some probability that the contract is offered by an already informed individual and (1-p) that the individual is uninformed. The individuals will not necessarily choose no disclosure in this instance.

This type of problem is closely related to adverse selection in insurance. If those who have cancer are known prior to trading insurance policies there is a market reason to not have cancer insurance. In a risk averse world however there is a joint positive externality to set up an insurance scheme cooperatively prior to giving everyone a medical examination. The incentive to do this is not within the general equilibrium model but must come from adding institutional rules of the game.

	NO	YES
NO	.218, .218	.201, .201
YES	.201, .201	.201, .201

FIGURE 4

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