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A Noncooperative Model of a Closed Trading Economy with Many Traders and Two Bankers*

By

Martin Shubik, New Haven, Conn., U. S. A.

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1. Introduction

In previous papers Shapley and Shubik have formulated a noncooperative game model of a closed trading economy where one commodity is used as a means of payment. The existence of a noncooperative equilibrium was proved; the role of cash flow and the need for credit was examined and the relationship between the noncooperative equilibria and the competitive equilibria was illustrated.

The basic model is as follows:

Consider n traders in a market with $m+1$ goods. We assume that initially each trader i has a bundle of goods $(a^i_1, a^i_2, \ldots, a^i_m, a^i_{m+1})$. We assume that the $m+1^{st}$ good is the means of payment and that the convention of this market is that all traders are required to offer all their holdings of the first m goods for sale. This means that individuals must buy back items they own for their own consumption. It is as though the farmer sells all of his milk to the market and buys his consumption milk from the store.

This condition amounts to requiring that all goods are monetized — it is a useful accounting convention and is mathematically easy to analyze. Other conventions can be used which modify this condition.

We may consider the market as consisting of m trading posts where all of each of the m commodities is offered for sale. Each

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trader i allocates an amount of his means of payment b^i_j to the j^{th} market. If there is no credit mechanism we require that

$$\sum_{j=1}^{m} b^i_j \leqq a^i_{m+1} \quad \text{for } i=1,2,\ldots,n. \tag{1}$$

In this model price is given by:

$$p_j = b_j/a_j \ \text{ where } b_j = \sum_{i=1}^{n} b^i_j \tag{2}$$

and the amount that an individual obtains upon bidding b^i_j for good j is:

$$x^i_j = b^i_j/p_j \ \text{ if } p_j > 0 \quad (\text{where } a_j = \sum_{k=1}^{n} a_j{}^k) \ \text{ for } j=1,\ldots,m \tag{3}$$
$$= 0 \qquad \text{if } p_j = 0$$

and

$$x^i_{m+1} = a^i_{m+1} - \sum_{k=1}^{m} b_k{}^i + \sum_{k=1}^{m} a_k{}^i p_k.$$

Thus the payoff function to the traders can be expressed both in terms of their strategic variables explicitly or the final bundles of goods which implicitly reflect the strategic behavior

$$\Pi^i(b^1, b^2, \ldots, b^n) = u^i(x_1{}^i, \ldots, x^i_{m+1}). \tag{4}$$

Given the nature of the market mechanism, all traders pay the same price for the same good. Furthermore the markets and price system constitute an externality to all traders in the sense that all trade is interlinked through the markets.

The specification of a mechanism for trade and price formation may limit the attainable set of economic outcomes.

2. On Modeling the Strategic Choices of Economic Actors

In the modeling of a trading economy as a noncooperative game two separate problems are posed. They are (1) the specification of a mechanism for the generation of prices and (2) the examination of any constraints placed on attainable outcomes by the mechanism. In discussion among Scarf, Shapley and Shubik it was observed that if all commodities can be used as a means of payment then it is possible to formulate a noncooperative game model of a closed trading economy with some restrictions on the feasible set of outcomes but with no need for credit. If fewer than all commodities are used as a means of payment then not only are restrictions placed on feasible outcomes but credit may be needed.

The above comments can be made somewhat less cryptic by means of simple examples. In a simple barter model of a trading economy where any two goods can be bartered by any group of individuals owning them, it is *feasible* (though not optimal) for different groups to simultaneously exchange two commodities at different rates of exchange. If all individuals are required to deal through a formal market which aggregates bids and offers there will be restrictions on attainable trades. In particular a common restriction is that all traders may end up paying the same price for the same commodity.

When all goods serve as a means of payment, all of one's assets are immediately available in the markets and no cash flow problem is created, in the sense that an individual might have less means of payment immediately available than he has assets. When fewer, for example, one or two goods serve as the means of payment then a cash constraint (such as condition (1) in Section 1) may appear. This constraint can block optimal trades unless a credit mechanism is introduced which enables the individuals to obtain short term credit to enable them to carry out exchanges which could conceivably involve payments as high as the total market value of any individual's initial assets.

Shapley and Shubik introduced a credit mechanism *ex machina;* one can merely relax condition (1) and assume that short term credit or "trust chips" are available from a mechanism. These chips must be paid back after trade. Otherwise if an individual fails to repay, then the worth of insolvency must be specified and even bankruptcy rules may need to be specified.

There is another way to handle the supply of credit. This involves introducing a new special class of economic agents known as "bankers". These individuals are distinguished from other only by the set of actions they can take in the game model of the economy. Two formulations of the game with credit are given, under one formulation only two bankers are needed for the noncooperative game to yield efficient equilibria; under the other, many bankers are needed to attenuate their oligopolistic influence.

3. Bankers as Special Economic Actors

In order to define the model with bankers we introduce credit formally as the $m+2^{\text{nd}}$ commodity. The means of payment is the $m+1^{\text{st}}$ commodity which may or may not have an intrinsic value. The $m+2^{\text{nd}}$ commodity does not enter into the utility functions. We may imagine the credit lines of the banks to consist of a pile of chips they can lend out, or we can consider them as a set of I. O. U. notes bearing the banker's name. The unit for the $m+2^{nd}$ commodity is the same as that for the $m+1^{st}$ commodity. Thus on payback a unit of one is interchangeable with the other.

In this model there is a money (the $m+1^{st}$ commodity) which is fiat or has consumer worth, which can be regarded as having been created by community custom, law or "rules of the game". The credit (the $m+2^{nd}$ commodity) has a different issuer, to wit: the banks. If we wish to be careful with the doubleentry book-keeping, we must include an $m+3^{rd}$ commodity which consists of the I. O. U. notes of the borrowers which the bank holds in return for the I. O. U. notes it issues to serve as currency. Thus in the model we have:

Commodity $m+1$ = Society's accepted means of payment.

Commodity $m+2$ = Banker's I. O. U. notes or guaranteed credits which circulate on par with the means of payment.

Commodity $m+3$ = Individuals' I. O. U.s or promissory notes which are nonnegotiable, held by the banks in return for quantities of $m+2$ and are contracts for payments of $m+1$ or $m+2$ at the end of the period.

In our extended model with $m+3$ commodities, of which only $m+2$ are negotiable and 2 or 3 do not appear in the utility functions we have as the initial holdings for the $n+2$ traders the following bundles:

For Traders:

$$(a_1{}^i, a_2{}^i, \ldots, a^i{}_m, a^i{}_{m+1}, 0, 0) \text{ for } i=1, \ldots, n. \tag{5}$$

For Bankers:

$$(0, 0, \ldots, 0, a^i{}_{m+1}, a^i{}_{m+2}, 0) \text{ for } i=n+1, n+2. \tag{6}$$

The consumption goods ownership and trading aspects of the bankers has been removed by giving them no other consumer good than the means of payment. We need extra rules of the game to separate banking from commercial activities. In particular a bank is not permitted to extend credit to itself for its own consumption purposes. The bank may only spend for consumption purposes up to the amount $a^i{}_{m+1}$ which can be regarded as representing its capital.

Perhaps a more satisfactory way of representing a bank's initial holdings than (6) above we might consider it to be:

$$(0, 0, \ldots, 0, a^i{}_{m+1}, 0, 0) \tag{7}$$

but that there is a special rule of the game permitting bank i to create or issue an amount $C^i{}_{m+2}$ of the $m+2^{\text{nd}}$ commodity in exchange for individual trader's promissory notes. This convention has the $m+2$ commodity "in being" until it is issued. The convention in (6) gives it a physical existence, but it is "sterilized" as a means of exchange when it is held by a bank.

The interpretation of the limits $C^i{}_{m+2}$ is that by the rules of the game (set, for example by some, as yet, unmodeled central bank) the credit creation powers of the banks are limited.

We note that the making of a loan by the bank involves two creating agencies. The bank creates its I. O. U.s or $m+2$ and the trader creates his I. O. U.s or $m+3$. Three sets of doubleentry bookkeeping statements for a bank and a trader can be made up for this one period game. They are before lending, at the point of loan, before trade and after trade when the loan is due. The books are kept in terms of $m+1$. The books only show the relevant parts of the banker-trader relationship.

	Bank			Trader		
Before Loan	Assets	Liability		Assets	Liability	
	0	0		0	0	
Loan	Assets	Liability		Assets	Liability	
Traders IOU for 120	100	100	Loan of $m+2$	100	100	Present Value of IOU for 120
	Assets	Liability		Assets	Liability	
	120	100 Loan	Value of Assets after Trade	A	120	IOU due in $m+1$, or $m+2$
IOU Due		20 Earned			E	Value of Equity after Trade
				$A = 120 + E$		

We note that the bank issues units of $m+2$ subject to a ceiling, but so far in this model no ceiling on the issue of units of $m+3$

by the borrowing traders has been specified. Thus for example a trader might offer a promissory note of a billion for one unit of $m+2$! At the point of the loan the promissory note is carried on the bank's books at the amount in terms of $m+2$ (or $m+1$ as they are on par) advanced by the bank.

When the trader's note is due, after trade then the "face value" of the promissory note is translated into terms of $m+1$ at par. Thus in the example above an I. O. U. note of 120 was written in return for the bank's note of 100. After trade when the note is due the bank has "earned" the extra 20 and this is reflected in the books.

If a bank announces as its strategy a premium, or rate of interest it wishes to charge for a loan of a unit of $m+2$ it exercises one type of control over the borrowers different from if it lets borrowers bid quantities of $m+3$ for a fixed supply of $m+2$. In either instance if the borrowers create a volume of $m+3$ which is larger than the supply of $m+1$ plus $m+2$ then insolvency must be forced on some borrowers as the supply of "NOW" money or money acceptable to repay the bank loan is insufficient to cancel all trader I. O. U. notes.

3.1. "Price" or Rate of Interest Naming Bankers

First we consider an economy where there are two banks who each name a price for their extension of credit. After they have named their prices, loans are made, at which point all individuals make their bids in the markets for the first m commodities. After this the game is over and individuals automatically are called upon to settle accounts.

The tree diagram in Fig. 1 shows the sequencing of moves and the information conditions. In this instance it is assumed that the banks set their rates each without information on the act of the other. A slight modification could have them move sequentially. It is assumed that all traders are informed of the two bank rates before they simultaneously bid for loans. If the total amount of loans requested is larger than the limits $C^1{}_{m+2}$ and $C^2{}_{m+2}$ then a rationing scheme for loans must be described. In this model we will limit our concern to the case where the $C^i{}_{m+2}$ are sufficiently large that rationing is no problem.

After all loans have been secured it is assumed that all traders are completely informed and all traders (including the bankers) bid in the markets for the first m goods. The bankers however are forbidden from using bank credit.

After the market has functioned there is a "settlement" which is essentially automatic; loans must be repaid or insolvency must be handled.

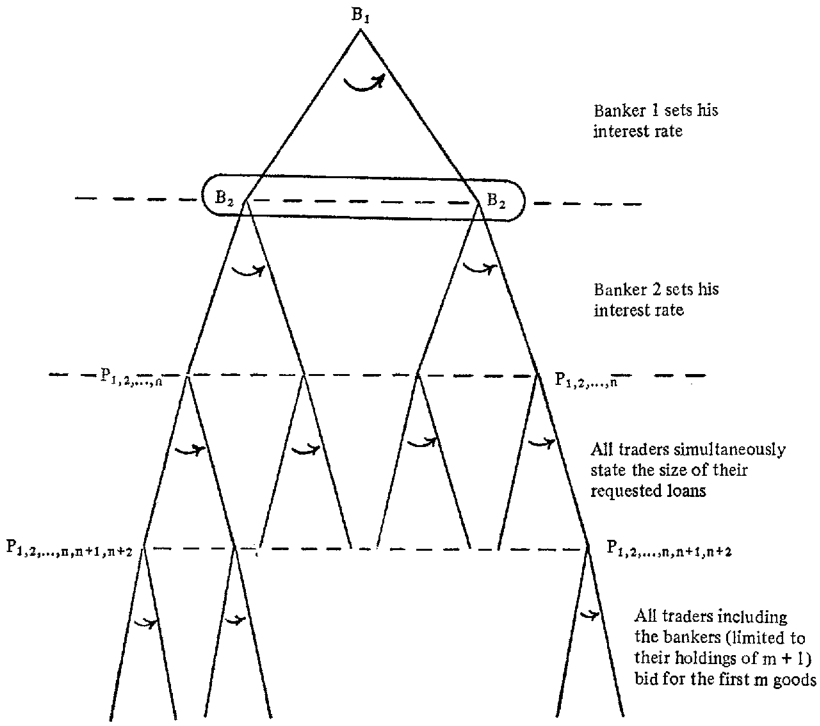

Fig. 1

The existence and nature of the equilibrium point

Shapley and Shubik have shown that a noncooperative equilibrium exists for the last stage. If we solve for a perfect equilibrium point, i. e. one which is not only an equilibrium point in the game as a whole, but also in every subgame, then we may work a backward induction replacing the ultimate branches of the tree by equilibrium payoffs at the penultimate branches.

In the games consisting of selecting the size of a loan the payoffs to each individual are concave and continuous hence these games will have an equilibrium point.

We are now back to the "bankers' game" of setting the interest rate. If we assume that each banker has a credit line so large that he could service the loan requirements of the whole community under any circumstances then our bankers' game is the classical Bertrand duopoly model which has an equilibrium point where the rate fo interest charged on short term credit is zero. This depends on the relatively weak assumption that *ceteris paribus*, borrowers will borrow from the bank charging the lowest rate of interest. When they charge the same rate we may assume that borrowers are indifferent and each obtains half of the loans.

The proof is simple if one bank undercuts the other it obtains all of the market. As each has a costless credit line that is sufficient for all customers (for instance we could merely remove the bounds $C^1{}_{m+2}$ and $C^2{}_{m+2}$) as long as there is a positive rate of interest charged by one bank there is an incentive for the other to just undercut.

In this formulation the oligopoly power of the banks is attenuated as soon as there are two banks.

If we now replicate the number of traders so that we have an economy with $kn+2$ participants this economy will have noncooperative equilibria which approach (as k becomes large) the same relative prices and distribution of the first $m+1$ commodities (assuming that the $m+1^{st}$ commodity has a worth in consumption) as the competitive equilibria solutions.

3.2. Quantity or Credit Rationing Bankers

A different mechanism would be to have the bankers simultaneously select an amount q^{n+1}_{m+2} and q^{n+2}_{m+2} which are the limits on the credit they are willing to extend. The traders in turn simultaneously and with knowledge offer $s^i{}_{m+3}$ for their loans. Using this mechanism the amount of credit the i^{th} trader obtains is

$$c^i = \frac{s^i{}_{m+3}}{s_{m+3}} \left(q^{n+1}_{m+2} + q^{n+2}_{m+2}\right), \quad s_{m+3} = \sum_{i=1}^{n} s^i{}_{m+3}. \tag{8}$$

The rate of interest or "price of short term credit" which evolves from this model is given by:

$$1 + \text{the rate of interest} = \frac{s_{m+3}}{\left(q^{n+1}_{m+2} + q^{n+2}_{m+2}\right)}. \tag{9}$$

We assume that the rate of interest at equilibrium can never be less than zero because if it were it would pay a bank to reduce the amount of credit offered. This is not quite covered in the model above where credit granting is absolutely costless, but there are several minor modifications which would produce this condition.

This model is the Cournot type model of the market for loans, and like the Cournot oligopoly model may still have a marked oligopolistic force when there are only two firms in control of a market. It appears that for the attenuation of the effect of oligopolistic banking when the banks can control the quantity of credit we need many banks.

4. A Simple Example

A simple example involving $2n$ traders and then 2 or more bankers is computed for illustrative purposes.

Suppose there are n traders with initial endowments $(A, 0, 0, 0)$ where the first commodity is to be put up for auction, the second is the means of payment $(m+1)$, the third bank credit $(m+2)$ and the fourth trader promissory notes $(m+3)$. Let these traders have utility functions $\varphi_i (q_1^i, q_2^i) = q_1^i q_2^i$.

There are n other traders with the same utility functions and endowments $(0, A, 0, 0)$.

There are two bankers with endowments $(0, 0, \infty, 0)$ and utility functions $\Phi_i (q_1^i, q_2^i) = \Phi (q_1^i, q_2^i)$.

Competitive Equilibrium

Traders of Type 1: 2; $(A/2, A/2, 0, 0)$ = final allocation

$p_1 = p_2 = 1$ = prices.

Noncooperative Equilibrium without Credit

For Type 1

$$\Pi_1^i = \left(\frac{n x_i A}{x+y}\right)\left(\frac{x+y}{n}\right) = x_i A = 0 \text{ as } x_i = 0. \tag{10}$$

For Type 2

$$\Pi_2^j = \left(\frac{n y_j A}{y}\right)(A - y_j). \tag{11}$$

From (11) we obtain:

$$y_j = \left(\frac{n-1}{2n-1}\right) A \tag{12}$$

which for $n = 2$ gives $y_j = A/3$

$n \to \infty$ gives $y_j = A/2$

hence for Traders of Type 1: $(0, A/3, 0, 0)$ = final allocation $n=2$

$(0, A/2, 0, 0)$ = final allocation $n \to \infty$

and for Traders of Type 2: $(A, A/3, 0, 0)$ = final allocation $n=2$

$(A, A/2, 0, 0)$ = final allocation $n \to \infty$

with prices $p_1 = 1/3,\ p_2 = 1$ for $n = 2$,
$p_1 = 1/2,\ p_2 = 1$ for $n \to \infty$.

Noncooperative Equilibrium with Credit Cost of $1+\varrho$

Suppose that an outside agency charges $(1+\varrho)$ for a unit of $m+2$, (or bank credit) then the payoff for traders of Type 1 is:

$$\Pi_1^i = \left(\frac{n\,x_i\,A}{x+y}\right)\left[\frac{x+y}{n} - (1+\varrho)\,x_i\right] \tag{13}$$

and for traders of Type 2:

$$\Pi_2^j = \frac{n\,y_j\,A}{x+y}\,(A - y_j). \tag{14}$$

After a certain amount of manipulation from (13) we obtain

$$(x_i + y_j)^2 = \left(2\,(x_i + y_j)\,(x_i) - \frac{x_i^2}{n}\right)(1+\varrho) \tag{15}$$

and from (14) we obtain

$$A\left(x_i + y_i - \frac{y_j}{n}\right) = 2\,y_j\,(x_i + y_j) - \frac{y_j^2}{n}. \tag{16}$$

For a mass market, from (16) with $n \to \infty$ we obtain

$$y_j = A/2. \tag{17}$$

Substituting (17) into (16) and considering $n \to \infty$ we obtain:

$$x_i = \frac{A}{2}\left(\frac{1}{1+2\varrho}\right). \tag{18}$$

Thus we have for traders of Type 1 a final allocation of

$$\left[\frac{A}{2\,(1+\varrho)},\ \frac{A}{2}\left(\frac{1+\varrho}{1+2\varrho}\right),\ 0,\ 0\right]$$

For Type 2

$$\left[\frac{A}{2}\left(\frac{1+2\varrho}{1+\varrho}\right),\ \frac{A}{2},\ 0,\ 0\right]$$

and for the banking system

$$\left[0,\ \frac{A}{2}\left(\frac{\varrho}{1+2\varrho}\right),\ 0,\ 0\right],$$

and

$$p_1 = \frac{1+\varrho}{1+2\varrho},\quad p_2 = 1$$

are the prices.

Noncooperative Equilibrium with "Bertrand" Bankers

Suppose we assume that $\partial \Psi / \partial q^i_2 > 0$, i. e. that the bankers each have a positive marginal utility for the means of payment (this is a statement about $m+1$, *not* $m+2$ or $m+3$). The interest they earn by definition has to be paid in this commodity in the sense that the wiping out of all the credit they advanced retires all of $m+2$ without covering interest, which must be paid in $m+1$ or $m+2$.

If there are two bankers with unlimited credit lines each naming on interest rate, and if borrowers always select the bank with the lowest rate and split on ties then the "duopoly game" of the bankers is as follows

$$\begin{aligned} \Pi_B{}^1 &= 0 && \text{if } \varrho_1 > \varrho_2 \\ &= \Psi\left[0, \frac{A}{4}\left(\frac{\varrho_1}{1+2\varrho_1}\right)\right] && \text{if } \varrho_1 = \varrho_2 \\ &= \Psi\left[0, \frac{A}{2}\left(\frac{\varrho_1}{1+2\varrho_1}\right)\right] && \text{if } 0 \leqq \varrho_1 < \varrho_2. \end{aligned} \tag{19}$$

This is the classical Bertrand duopoly. As long as $\varrho_1, \varrho_2 > 0$ there is a discontinuity in the payoffs associated with just undercutting the competitor. At $\varrho_1 = \varrho_2 = 0$ there is an equilibrium*.

An immediate inspection of final holdings and prices in the noncooperative market with a credit cost of $1+\varrho$ shows that for $\varrho = 0$ we obtain the same distributions and prices as in the competitive market, except that here the bankers are explicit players supplying credit at no gain to themselves.

Noncooperative Equilibrium with "Cournot" Bankers

If we assume that the two bankers face an ocean of traders then at equilibrium if the credit line of banker i is x_i we have the condition derived from (9) and (18)

$$1+\varrho = \frac{\frac{A}{2}\left(\frac{1}{1+2\varrho}\right)}{x_1+x_2} \tag{20}$$

From which we obtain:

$$\varrho = \frac{-3+\sqrt{5+\frac{2A}{x_1+x_2}}}{4}.$$

* There are two fine points not covered. We need to specify a condition that for $\varrho_1 < 0$ the payoff is less than Ψ (0, 0). Furthermore for this and the next model it might be more congenial to imagine the traders to have been fractionated rather than replicated. Otherwise in (19) a factor of n is missing in the interest payments, but for $n \rightarrow \infty$ this is unbounded.

The noncooperative game the two banks are in amounts to:

$$\begin{aligned} &\underset{x_1}{\text{maximize}}\ \Pi_B{}^1 = x_1\,\varrho = x_1 \left(\frac{-3 + \sqrt{5 + \dfrac{2A}{x_1 + x_2}}}{4} \right) \\ &\underset{x_2}{\text{maximize}}\ \Pi_B{}^2 = x_2\,\varrho = x_2 \left(\frac{-3 + \sqrt{5 + \dfrac{2A}{x_1 + x_2}}}{4} \right) \end{aligned} \tag{21}$$

Solving these we obtain:

$$x_1 = x_2 \cong 0.134\,A$$

and

$$\varrho \cong 0.131.$$

As in the simple example given by Cournot where the duopolists can offer quantities of a costless product, but perceive their control over supply it pays them to restrict the quantity of credit offered. The final distribution is as already given for the noncooperative equilibrium with a credit cost of $1+\varrho$ and with $\varrho \cong .131$.

It is easy to see that by increasing the number of bankers the oligopoly effect attenuates.

References

L. S. Shapley and M. Shubik: Some Strategic Models Related to General Equilibrium, mimeographed, February 1973.

L. S. Shapley and M. Shubik: Trade Using One Commodity as a Money, mimeographed, March 1974.

Address of author: Prof. Dr. Martin Shubik, Department of Economics, Cowles Foundation for Research in Economics, Box 2125, Yale Station; Yale University, New Haven, CT 06520, U. S. A.