

## Pitfalls in Financial Model Building: A Clarification

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In their well-known "Pitfalls" paper, William Brainard and James Tobin advocated "a 'general disequilibrium' framework for the dynamics of adjustment to a 'general equilibrium' system." The Pitfalls framework has subsequently been widely used in the construction of flow of funds models of financial markets. The Brainard-Tobin paper has also elicited notes from Mark Ladenson and Kevin Clinton in this *Review* which attempt to interpret and elaborate upon the Pitfalls model. However, these authors seriously misinterpret the model and seemingly obscure rather than extend the Pitfalls framework in a maze of unnecessary mathematical techniques and notation.

Ladenson and Clinton both attempt to work with linearly dependent explanatory variables whose coefficients are not identified and cannot be meaningfully interpreted. Clinton uses this indeterminacy to provide a superficial counterexample to a Brainard-Tobin argument. Ladenson, on the other hand, discusses two (of many possible) sets of "expedient" parameter restrictions which will allow him to assign values to all of the coefficients of the linearly dependent variables. He believes that estimation of the model necessitates substantive behavioral assumptions, whereas in fact the elimination of a redundant explanatory variable alters only the appearance of the model and the interpretation of individual coefficients.

In addition to discussing the errors of Clinton and Ladenson, this paper will demonstrate the simplicity with which adding up constraints can be derived, the relationship between the form of a model and the interpretation of its coefficients, and how the parameters might be estimated subject to adding up constraints. In Section I the Pitfalls model is compared with the Clinton-

Ladenson formulation; Sections II and III concern the respective details of the Clinton and Ladenson articles; and estimation procedures are discussed in Section IV.

### I

All of the authors consider a portfolio model in which desired asset shares depend linearly upon the competing rates of return and other explanatory variables:

$$(1) \quad \frac{a_i^*}{w} = b_{i0} + \sum_{j=1}^q b_{ij} r_j \quad i = 1, \dots, n$$

where  $a_i$  is the actual holding of the  $i$ th asset,  $a_i^*$  is the desired holding of the  $i$ th asset,  $r_j$  is the  $j$ th rate of return or other explanatory variable, and  $w$  is equal to the sum of the  $a_i$ .

If, as the authors assume, desired holdings sum to available funds ( $\sum_i a_i^* = w$ ) for all values of the explanatory variables, then

$$1 = \sum_{i=1}^n \frac{a_i^*}{w} = \sum_i b_{i0} + \sum_{j=1}^q \left( \sum_i b_{ij} \right) r_j$$

Clearly this equation can hold for all values of  $r_j$  if and only if

$$\sum_i b_{i0} = 1 \quad \sum_i b_{ij} = 0 \quad j \neq 0$$

That is, if all changes in asset value are included in  $w$ , then a change in any desired asset share must be compensated by offsetting changes in the remaining shares, and an increase in funds must be fully accompanied by an increase in desired holdings.

All of the authors allow actual holdings to differ from desired shares and assume that holdings will adjust in response to portfolio disequilibria. Brainard and Tobin pointed out that the common specification in which each asset adjusts solely to its own discrepancy between this period's desired and last period's actual holdings

$$(2a) \quad \Delta a_i = \epsilon_i [a_i^* - a_i(-1)]$$

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is likely to result in inconsistent or unintentionally implausible behavior since each of these assets is reacting to an incomplete description of the portfolio disequilibria.<sup>1</sup>

The danger of inconsistency arises if specification (2a) is adopted for all  $n$  assets. In this case, using  $\Delta w = \sum_i [a_i^* - a_i(-1)]$ , we have

$$\begin{aligned}\Delta w &= \sum_i \Delta a_i = \sum_i \epsilon_i [a_i^* - a_i(-1)] \\ &= \epsilon_k \Delta w + \sum_{i \neq k} (\epsilon_i - \epsilon_k) [a_i^* - a_i(-1)]\end{aligned}$$

This equation can hold for all potential disequilibria situations if and only if  $\epsilon_i = 1$   $i = 1, \dots, n$ . That is, consistency in a pure linear own-adjustment model requires that actual holdings adjust fully so as to always coincide with desired holdings. If some of the  $\epsilon_i$  are instead allowed to assume values other than 1, then either desired or actual holdings will be inconsistent with available funds for some values of the explanatory variables.

A more frequent practice is to specify own-adjustment equations (2a) for an incomplete list of assets, say for  $i = 1, \dots, q < n$ . In this case, the consistency requirement can be used to residually derive the implicit specification of the adjustment equation for the remaining assets as a group,

$$\begin{aligned}\Delta \left[ \sum_{q+1}^n a_i \right] &= \Delta w - \sum_1^q \Delta a_i \\ &= \left[ \sum_{q+1}^n a_i^* - \sum_{q+1}^n a_i(-1) \right] \\ &\quad + \sum_1^q (1 - \epsilon_i) [a_i^* - a_i(-1)]\end{aligned}$$

Thus the residual collection of assets implicitly adjusts quite differently from the explicitly specified assets. In particular, the residual collection fully adjusts to its own discrepancy and passively absorbs any funds which are not immediately shifted in accord with desired holdings of any of the assets with explicit adjustment equations. This

<sup>1</sup> Similar arguments apply when some desired asset shares are assumed to depend upon only a subset of the explanatory variables.

model could conceivably describe some group's actual behavior. In practice, however, assets are usually included in the residual category not for theoretical reasons, but rather because the model builder is not immediately interested in these items. As a consequence, the specification usually appears to be inadvertent and implausible due to the illiquid nature of the residual assets. One Pitfalls moral is thus that the residual specification should always be directly checked since any inadequacy in this equation must be due to inadequacies in the explicit equations.

To avoid these problems, Brainard and Tobin advocated having each asset adjustment depend upon a complete description of the disequilibria in the portfolio. Ladenson and Clinton instead attempt to work with explanatory variables which overdescribe the portfolio disequilibria:

$$(2b) \quad \Delta a_i = \sum_{j=1}^n \epsilon_{ij} [a_j^* - a_j(-1)] + f_i \Delta w$$

$i = 1, \dots, n$

The interpretation of the parameters in this specification is completely arbitrary since the explanatory variables are linearly dependent

$$(3) \quad \sum_j [a_j^* - a_j(-1)] = \Delta w$$

The coefficients of these variables consequently cannot be meaningfully interpreted nor empirically identified since no variable can change without the remaining variables also changing in any of an infinite number of ways to maintain the balance sheet identity. There is thus an infinite variety of values for the parameters of (2b) which are consistent with any observed behavior, and each individual parameter is meaningless in that any value is consistent with a particular unique behavioral specification.

A portfolio disequilibrium is fully described by any  $n$  of the  $n+1$  variables in (2b) and all information relevant to an adjustment model is contained in  $n$  such variables. The assignment of specific values to  $n$  associated parameters will consequently uniquely describe the behavioral response of a particular asset

holding in all possible portfolio situations. It does not require behavioral assumptions to select  $n$  variables that will describe the disequilibria, and the specific  $n$  variables that are used in the adjustment equations do not constrain behavior in any way. The choice of variables will of course define the interpretation of the individual associated coefficients, but any particular representation can be uniquely rewritten in terms of the coefficients of any other  $n$  selected variables.

Rather than simply choosing  $n$  variables to describe the disequilibria, a Pitfalls model could be formally constructed by substituting the identity (3) into the Clinton-Ladenson model (2b) in order to eliminate a redundant variable. Two of the more popular forms of the Pitfalls model will be illustrated here.

If  $\Delta w$  is eliminated from (2b), then the model can be written as

$$(3') \quad \Delta a_i = \sum_{j=1}^n (\epsilon_{ij} + f_i) [a_j^* - a_j(-1)] \\ = \sum_{j=1}^n \theta_{ij} [a_j^* - a_j(-1)]$$

The adding up restrictions can be directly derived from

$$\Delta w = \sum_i \Delta a_i = \sum_i \sum_j \theta_{ij} [a_j^* - a_j(-1)] \\ = \sum_i \theta_{ik} \Delta w + \sum_{j \neq k} \sum_i (\theta_{ij} - \theta_{ik}) \\ \cdot [a_j^* - a_j(-1)]$$

This equation can hold for all possible discrepancies if and only if  $\sum_i \theta_{ij} = 1, \forall j$ . A coefficient  $\theta_{ij}$  should be interpreted as the partial effect on holdings of the  $i$ th asset of a unit increase in desired holdings of the  $j$ th asset accompanied by an equal increase in  $\Delta w$  (since one discrepancy is increasing while the remaining discrepancies are held constant). The adding up restrictions state that such an increase in funds must be fully absorbed in the portfolio.

An alternative form of writing the Pitfalls model can be obtained by instead eliminating

the  $k$ th discrepancy from (2b):

$$(3'') \quad \Delta a_i = \sum_{j \neq k} (\epsilon_{ij} - \epsilon_{ik}) [a_j^* - a_j(-1)] \\ + (f_i + \epsilon_{ik}) \Delta w \\ = \sum_{j \neq k} \phi_{ij} [a_j^* - a_j(-1)] + h_i \Delta w$$

The adding up restrictions follow from

$$\Delta w = \sum_i \Delta a_i = \sum_{j \neq k} \left( \sum_i \phi_{ij} \right) \\ \cdot [a_j^* - a_j(-1)] + \left( \sum_i h_i \right) \Delta w$$

This can always hold if and only if

$$\sum_i \phi_{ij} = 0, \quad j \neq k \quad \sum_i h_i = 1$$

Here the explicit coefficients  $\phi_{ij}$  should be interpreted as the partial effect on holdings of the  $i$ th asset of desiring to hold one unit more of the  $j$ th asset and one unit less of the  $k$ th asset. Since  $\Delta w$  is held constant, the adding up restrictions require that any increases in asset holdings must be fully offset by decreases in other holdings. The  $h_i$  represents the partial effect of an increase in  $\Delta w$  which the sector desires to hold as the  $k$ th asset; the total portfolio will consequently expand by one unit.

The representations (3') and (3'') are equivalent characterizations, and unique parameter values could be assigned in either case which would identically describe behavior in all portfolio situations. Specifically, the variables in (3') could be rearranged as

$$\Delta a_i = \sum_{j=1}^n \theta_{ij} [a_j^* - a_j(-1)] \\ = \sum_{j \neq k} (\theta_{ij} - \theta_{ik}) [a_j^* - a_j(-1)] + \theta_{ik} \Delta w$$

Since this is the same form as (3''), it follows that

$$\phi_{ij} = \theta_{ij} - \theta_{ik}, \quad j \neq k \quad h_i = \theta_{ik}$$

is necessary and sufficient to insure identical behavior no matter what the values of the explanatory variables. That is, one does not need to make behavioral assumptions in

order to rearrange the explanatory variables. Note also that rearranging the variables and redefining the coefficients only rearranges the basic adding up restrictions.

II

After assuming that  $\sum_i \epsilon_{ij}$  is constant over  $j$ , Clinton proves that

$$\sum_i (\epsilon_{ij} + f_i) = \sum_i \theta_{ij} = 1, \quad \forall j$$

is sufficient to insure adding up in adjustment model (2b). Completely ignoring the linear dependency in this model, he then offers the case

$$\epsilon_{ij} = \begin{cases} 1 - \sum_i f_i, & i = j \\ 0, & i \neq j \end{cases} \quad i \neq k$$

as a counterexample to Brainard and Tobin's statement that "If no cross-effects were allowed in the explicit equations of adjustment . . . , then the counterparts of all the own-adjustments specified would be loaded into the implicit adjustment equation for [the omitted assets]" (p. 106).

This statement in the Pitfalls paper follows a summary argument against own-adjustment equations such as (2a), the details of which I've given above, and refers to the consequences of omitting cross-effects from the Pitfalls model when written in the form (3''). Clinton's example applies to a different set of parameters and, in light of the arbitrariness of the parameter values in his equations, is neither surprising nor interesting.

What Clinton's example does illustrate is that one must be careful when discussing coefficients (such as "cross-effects") to specify which variables are being held constant. To be meaningful, Clinton's example has to be rewritten in terms of identifiable coefficients.

If, for instance, his example is written in the form of (3'), then

$$\Delta a_i = \left(1 - \sum_j f_j + f_i\right) [a_i^* - a_i(-1)] + \sum_{j \neq i} f_j [a_j^* - a_j(-1)]$$

The cross-effects ( $f_i$ ) here specify that when one has a unit increase in available funds ( $w$ ) accompanied by a unit increase in desired holdings of the  $j$ th asset, how much of these funds will go into other asset positions. Clinton has not eliminated these cross-effects, and indeed he could not eliminate them without assuming complete adjustment,  $a_i = a_i^*$ .

If Clinton's example is instead rewritten in the form (3'') that is used throughout the Pitfalls paper,

$$\Delta a_i = \left(1 - \sum_i f_i\right) [a_i^* - a_i(-1)] + f_i \Delta w,$$

$$\Delta a_k = \sum_{j \neq k} - \left(1 - \sum_i f_i\right) [a_j^* - a_j(-1)]$$

$$+ \left(f_k + 1 - \sum_i f_i\right) \Delta w$$

The cross-effects now specify that when one wishes to switch 1 unit from asset  $k$  to asset  $j$ , how much will be put into asset  $i$  ( $i \neq j$ ). Instead of contradicting Brainard and Tobin, Clinton's example directly and completely confirms their statement. Precisely as Brainard and Tobin wrote, leaving cross-effects out of all but the  $k$ th equation necessitates cross-effects in this residual equation which fully offset the own-effects in the other equations. And cross-effects cannot be absent from all of the equations without eliminating the own-effects,  $\Delta a_i = f_i \Delta w$ .

In summary Brainard and Tobin are correct in their criticism of own-adjustment equations (2a) and in their discussion of the consequences of setting the cross-effects of model (3'') equal to zero. A similar argument has been made here against equating to zero the cross-effects of model (3'). None of this rules out someone arbitrarily calling some other parameters "cross-effects" and then assigning them a value of zero.

III

In contrast, Ladenson recognizes the linear dependency in the explanatory variables when he substitutes (1) into (2b) for

estimation purposes

$$(4) \Delta a_i = \left[ \sum_{j=1}^n \epsilon_{ij} b_{j0} \right] + w \sum_h \left[ \sum_{j=1}^n \epsilon_{ij} b_{jh} \right] r_h w - \sum_{j=1}^n \epsilon_{ij} a_j(-1) + f_i \Delta w$$

where  $w - \sum_{j=1}^n a_j(-1) = \Delta w$ .

Although the coefficients can consequently not be uniquely estimated, Ladenson is nevertheless determined to assign values to all of them so that he may calculate estimates for all of the parameters in (1) and (2b). He therefore views the necessary elimination of a linearly redundant explanatory variable from his estimation equation (4) as an "expedient" parameter restriction with behavioral implications. He thus asserts that the elimination of  $w$  from (4) "... involves the assumption that assets adjust instantaneously to a change in wealth," whereas if  $\Delta w$  is eliminated "... we assume that such adjustment occurs with a distributed lag" (1973, p. 1006).

It is of course always possible to impose parameter constraints which alter the model; Ladenson (1971) demonstrates this by resorting to the expedient of setting  $b_{j0} = 0$  for all  $j$ , even though this violated the consistency requirement that  $\sum_j b_{j0} = 1$ . However, it is certainly not necessary to impair the model in order to estimate it, and eliminating  $w$ ,  $\Delta w$ , or  $a_k(-1)$  from (4) in truth has no effect upon the model beyond a simple rearrangement of the identifiable parameters.

It was demonstrated above that there are a variety of equivalent and uniquely related ways in which the Pitfalls adjustment equations can be written; it will now be shown that there are similarly a variety of equivalent and uniquely related ways of writing the corresponding Pitfalls estimation equations. In both cases, the Ladenson alternative is to work with linearly dependent variables and unidentifiable parameters; and in both cases it is the elimination of a redundant variable rather than a behavioral assumption that is required to obtain identifiable parameters.

The substitution of (1) into form (3') of

the Pitfalls adjustment equations yields the estimation equation

$$(5a) \Delta a_i = \left[ \sum_j \theta_{ij} b_{j0} \right] w + \left[ \sum_h \left[ \sum_j \theta_{ij} b_{jh} \right] r_h \right] w - \sum_j \theta_{ij} a_j(-1)$$

This is indistinguishable from Ladenson's estimation equations (4) when  $\Delta w$  is eliminated. Behavioral restrictions were not needed to derive (3') from Ladenson's (2b), and are consequently not needed to eliminate  $\Delta w$  from Ladenson's (4).

There are clearly an infinite number of equivalent ways in which (5a) could be rewritten. For example, substituting for  $a_k(-1) = w - \sum_{j \neq k} a_j(-1) - \Delta w$  yields this arrangement of explanatory variables

$$(5b) \Delta a_i = \left( \sum_j \theta_{ij} b_{j0} - \theta_{ik} \right) w + \sum_h \left( \sum_j \theta_{ij} b_{jh} \right) r_h w - \sum_{j \neq k} (\theta_{ij} - \theta_{ik}) a_j(-1) + \theta_{ik} \Delta w$$

This is Ladenson's (4) with  $a_k(-1)$  eliminated, which could also be directly derived by substituting (1) into (3'') with the correspondence between the parameters again defined by

$$\phi_{ij} = \theta_{ij} - \theta_{ik}, \quad j \neq k \quad h_i = \theta_{ik}$$

Since (3'') is equivalent to (3'), it follows that deleting  $a_k(-1)$  from Ladenson's (4) is equivalent to deleting  $\Delta w$ .

Alternatively, substituting in either (5a) or (5b) for  $w = \Delta w + \sum a_j(-1)$  yields yet another equivalent rearrangement of the variables which this time is indistinguishable from Ladenson's (4) with  $w$  eliminated.

All of these cases (and all alternative linear rearrangements of the variables and parameters) are just different ways of writing the same model. If one is careful, the form in which the model is written has absolutely no behavioral implications. In particular, irrespective of whether  $w$ ,  $a_k(-1)$ , or  $\Delta w$  is eliminated from (4) and no matter how the

resultant linearly independent variables are juggled, the final equations will be equivalent to form (5a), and the coefficients of the rearranged variables will be uniquely related to the parameters of (5a). Indeed, it would be very strange if one were only happy with the model when the variables were arranged in a particular way; again we have the Pitfalls moral that explicit representations are no more important than implicit ones.

#### IV

Single equation ordinary least squares (*OLS*) parameter estimates will generally obey adding up restrictions if the regressors are the same in each equation and if some linear combination of the regressors is always equal to the sum of the regressands.<sup>2</sup> This can be seen directly by writing an estimation equation as

$$a_i = X \cdot \beta_i + \epsilon_i$$

$T \times 1 \quad T \times p \quad p \times 1 \quad T \times 1$

where we have (possibly) rearranged  $X$  so that the first regressor is

$$x_1 = \sum_{i=1}^n a_i$$

The sum across equations of the *OLS* parameter estimates are the elements of the column vector

$$\sum_{i=1}^n \hat{\beta}_i = \sum_i (X'X)^{-1} X' a_i = (X'X)^{-1} X' \sum_i a_i$$

which is what would result from a regression of  $\sum a_i$  on  $X$ . The squared residuals for that regression are minimized (are equal to zero) when the coefficient of  $x_1$  is one and all other coefficients are zero.

For *OLS* estimation of the model considered here, the Gauss-Markov Theorem implies that linearly rearranging the explanatory variables in (5a) will not affect the forecasts nor the implicit estimates of any estimatable parameters. In particular, the implicit estimates of the coefficients of (5a) do

<sup>2</sup> Ladenson incorrectly states that it is necessary and sufficient that all lagged endogenous variables appear in all equations.

not depend upon whether  $w$ ,  $\Delta w$ , or  $a_i(-1)$  is deleted from Ladenson's (4). Thus, Ladenson's alleged behavioral constraints are just awkward ways of eliminating a redundant variable and differ only in providing different arbitrary estimates of uninteresting parameters.

In considering *OLS* estimates of the identified structural parameters, it consequently suffices to consider the single case of using the estimation equations (5a) to estimate the parameters of the Pitfalls model (1) and (3'). Rewriting (5a) in obvious matrix notation,

$$a = \theta \cdot B_0 \cdot w + \theta \cdot B \cdot rw + (I_n - \theta)a(-1) + \epsilon$$

(where  $a$  is  $n \times 1$ ,  $\theta$  is  $n \times n$ ,  $B_0$  is  $n \times 1$ ,  $B$  is  $n \times q$ , and  $r$  is  $q \times 1$ ). The column elements of the estimates  $\hat{\theta} \hat{B}_0$  will sum to one and the elements of  $\hat{\theta} \hat{B}$  and  $(I - \hat{\theta})$  will sum to zero. The column elements of  $\hat{\theta} = (I - \hat{\theta}) + I$  will thus sum to one as desired. The elements of  $\hat{B}_0$  and  $\hat{B}$  will also sum correctly, since it can be easily shown that the column sums for any matrix  $Z = \theta^{-1}(\theta Z)$  will be the same as the column sums of  $\theta Z$  if the columns of  $\theta$  sum to one.

It is worth noting that hypothesis tests of any coefficients or combinations of coefficients are also invariant to the way in which the equations are written for estimation purposes. However, the usual computer output provides  $t$ -statistics for testing the null hypotheses that the explicitly estimated parameters are equal to zero. If one is in the (dubious) habit of imposing zero constraints whenever these are unrejected by the data, then the form in which the equations are written may affect the final estimates if one allows this arbitrary choice to define the subset of possible hypotheses that will be tested. This is of course not a limitation of the model but rather a well-known drawback to mechanically using hypothesis tests for decision-making purposes.

Turning to the question of more complex estimation methods, the automatic imposition of adding up constraints obviously extends to a variety of single equation methods, such as the use of instrumental variables and semi-first differences if and only if the instru-

ments used for each variable and the autoregressive parameters do not vary across equations. It will also remain true that the properly interpreted estimates are independent of the particular linearly dependent variable omitted from (4) and of any subsequent rearrangement of the variables.<sup>3</sup>

For the class of feasible Aitken generalized least squares system estimators, we will generally have to delete one equation (since the disturbances must sum to zero) and calculate the parameter estimates for this residual equation from the adding up constraints. Fortunately Alan Powell has shown that the estimated coefficients do not depend upon the equation chosen for deletion. In addition, it is well known that such system estimates will be identical to the single equation estimates on which they are based if the final step in the single equation method is the OLS regression of equations in which the right-hand side variables do not vary across equations.

The likelihood that the effective dimensionality of the data will be much less than the number of parameters is a persuasive argument for the imposition of a priori parameter restrictions. If one does specify the coefficients associated with a particular variable in every equation, then the single equation methods discussed above will continue to automatically impose the adding up constraints, and the estimated parameters

<sup>3</sup> With instrumental variables, this conclusion requires that all predetermined variables in the equation be included in the list of instruments. We can then view all of the explanatory variables as being regressed upon a common list of regressors. By the earlier adding up proof, forecasts of linear combinations of the explanatory variables will then equal linear combinations of the forecasts of the individual explanatory variables.

will be identical to the ones that would result from an Aitken procedure. If the parameters are not specified in every equation, then one can enforce the adding up constraints with a stacking procedure or with a Lagrangian modification of the Aitken method (see the discussion by Gordon Sparks for example). If one has useful information about the reliability of the parameter restrictions then a Bayesian or mixed estimation method might be of interest (see Smith and Brainard for example). Unfortunately, all of the non-single equation methods typically involve formidable computational difficulties and the possibility of serious rounding errors.

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