Additively Consistent Relationships for Personal Savings and the Categories of Consumption Expenditures, U.S.A., 1949–1963

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1. Introduction

There are two approaches, broadly speaking, to the estimation of empirical demand relationships for the categories of consumption expenditures. On the one side, there are studies like those of A. P. Barten ([3] and [4]), Richard W. Parks [16], Pollak and Wales [17], Alan Powell ([18] and [19]), Somermeyer, Hilhorst, and Wit [24], and Stone, Brown, and Rowe [27]. These writers employ a neo-classical model, in which the demand for a consumption category is related to relative prices, total expenditure on all the categories under consideration, and (in the empirical applications) some non-structural variable, such as a time trend. In general, the estimation of the parameters of these models is more

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‡ Barten’s model ([3], [4], and the relevant sections of [16]) is estimated without the explicit use of an exogenous variable such as a time trend. However, as Barten and Parks make clear, the first differencing procedure entailed in estimating Barten’s model is, in a sense, equivalent to the introduction of a time trend.

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complicated than the computation of the coefficients of a least squares regression; often non-linear estimation problems are encountered. Nevertheless, the model has been successfully estimated in all of the cited studies, and parameter estimates which are broadly consistent with the underlying theoretical framework have been obtained. Despite the fact that the theoretical model refers to individual demand relationships obtained from a utility maximization procedure and the fitted relationships refer to market demand functions estimated from time series data, most of the above studies do not concern themselves with the problem of aggregating from the individual to the market relationships.\(^2\)

A second approach, in which the underlying theoretical framework is much further in the background, is that of the econometric model builders. In several more detailed econometric models of the United States economy, total consumption expenditures are disaggregated into several categories. (See [9], [14], [15], [20], [29], and [30].) The model-builder then appears to concentrate on finding the “best” (according to his criterion or criteria) empirical relationship for each category of consumption expenditure, generally utilizing a somewhat different list of explanatory variables for each consumption category.

Both of these approaches have been reasonably successful in their applications. The neo-classical models have yielded fairly close fits, significant parameter estimates, and (typically) results that broadly agree with economists’ general presumptions. The relationships in larger econometric models have also been reasonably successful, as judged by statistical criteria or by conformity with the prior expectations of the model-builder. Moreover, the consumption category relationships of the econometric models have functioned reasonably well, in the larger systems in which they are embedded; they do not appear to be the weak links in applications of the system.

Nevertheless, both kinds of approaches can be criticized. We have already mentioned the problem of aggregating from the individual to the market demand relationships, in connection with the neo-classical models. The (implicit or explicit) time trends in the neo-classical models suggest that they are incomplete; presumably time is serving as a proxy for some slowly changing structural variables, possibly in connection with aggregation. A more serious short-coming, as Powell [18] acknowledges, is the failure to explain the allocation of disposable income between personal savings and total consumption. Instead of being explained, total consumption is taken as a datum.\(^3\) This approach could be taken on

\(^2\)A notable exception is Somermeyer, Hilhorst, and Wit [24] in which the aggregation problem is explicitly considered and it is conceded that the use of market demand functions blurs considerably the features of the household utility maximization model, unless some strong simplifications are introduced. (Indeed, one interpretation of these authors’ time trend is as an aggregation effect.) Stone, Brown, and Rowe [27] also consider the agreement of their time series results for the market demand relationship for food with cross-section findings for individual households at four widely separated dates in their long (1900-1960) sample period. (In general, the agreement of the two sets of elasticities of food expenditure with respect to total expenditure is quite striking.)

\(^3\)As exceptions to this assertion, we have the studies of Wit [33] and Somermeyer, Hilhorst, and Wit [24]. In the latter study, the authors used alternative treatments, making their total financial resources variable either total expenditures or total disposable income and thus excluding or including (respectively) total savings with the category of durable goods expenditures. (No separate price of personal savings was utilized; instead, its implicit price was the average price of the three subcategories of durable goods with which it was combined.) Somermeyer et al. concluded that the results supported the total expenditure model (with its implicit view of multi-stage utility maximization), but this is questionable, in my view. Although the
the view that households carry out the maximization of their utility in two (or more) stages: first, total financial resources are allocated between total consumption and additions to wealth (savings, in the absence of capital gains or losses), usually on the basis of fairly long run considerations (as in [1], [10], [23], and [25]), and then total consumption is parcelled out among categories, presumably with a much shorter term outlook. 4

One might criticize the approach of most econometric model-builders on somewhat different grounds. While personal savings are generally determined residually in these models, the consumption category relationships appear to be fitted with little concern for the implied behaviour of total consumption and savings. Several writers do not even bother to state what is the implicit total marginal propensity to consume out of income (either in the short run or long run). 5

Thus one has the distinct feeling that the model-builder would have obtained different estimates if personal savings had been the subject of an explicit relationship with some category of expenditures (say durable goods expenditures or non-housing services) obtained residually. 6

The approach of this paper attempts to capture the strengths of the two differing approaches outlined above. 7 Unlike most of

consistent in the following paragraph. Thus McCarthy says:

"Casual consideration of the theory of consumer behavior suggests, however, that if such a relative price enters the equation for one consumption item, some relative price must enter at least one other consumption demand curve." ([13], p. 37.)
The insight is, however, not applied to some of the other explanatory variables, such as a time trend, certain dummies, or a credit rationing variable. Furthermore, the implicit savings function is not considered in this light, either.

6 Of course, if the relationships for personal savings and all of the categories of consumption are obtained by an explicit fitting procedure, with each regression considered in isolation and no limitations on the explanatory variables to be included in each regression, then there is no guarantee that the accounting identity will always hold. In general (i.e., for most values of disposable income and the set of the other explanatory variables), it will not hold.

We may note, in passing, that similar considerations were pointed out by Brainerd and Tobin [5], in connection with a discussion of econometric models of the financial sector. Indeed, it was the desire to respond to the problems posed by Brainerd and Tobin that led Sparks [26] to develop the estimating technique that has been utilized in Appendix 2 of this paper.

7 Neither of these two broad approaches neatly categorizes the recent study [13] by H. S. Houthakker and Lester D. Taylor. In this book, the authors utilize an ingenious generalization of a stock concept to nondurable goods and to services (they term this generalization of a stock concept a "state" variable), in order to develop short and long run marginal propensities to consume particular categories of expenditure, short run and long run price elasticities, and the like. Their approach, which is very neat, has been applied with great success to data for the U.S.A. and for other countries. (In particular, their model has been utilized, by Schweitzer and Siedule [22], in the CANDIDE model of the postwar Canadian economy.) However, a purist could still quibble about the fact that, in the basic
the neo-classical models in the articles cited above, in this paper we shall be very much concerned with the allocation of disposable income between personal savings and total consumption. We shall also be concerned that the relationships for all uses of disposable income display the property of additive consistency: for any configuration of disposable income and the other explanatory variables, the sum of the predicted values of the categories of consumption plus the predicted value of personal savings should identically equal the generating value of disposable income. In terms of variables employed, the paper blends the two approaches just described. It is neo-classical in spirit in that relative price effects are introduced, while the use of variables measuring household net worth and household stocks of consumer durables suggests the wealth and stock adjustment effects so strongly emphasized by the econometric model-builders.

In outline, then, the remainder of this paper is as follows. In the following sec-

Houthakker-Taylor model, there is no logical reason for the sum of the marginal propensities to consume the various categories (out of total consumption expenditure) to add up to unity, even though in practice the discrepancies tend to be small.

This theoretical deficiency can be overcome in (at least) two ways. In the CANDIDE model, a mechanism forcing additive consistency under all circumstances has been added. (See [22], especially pp. 24-28.) Houthakker and Taylor have developed, in Chapter 5 of [13], an extension of their basic model which forces the property of additive consistency on their demand functions. (They term the resulting demand functions, which can include or exclude personal savings as a commodity, "additive"). It is worth noting that the inclusion of the property of additive consistency does appear to improve some forecasts with their models for the U.S.A. and Canada for the year 1965, as they themselves note ([13], p. 231.) Moreover, despite the fact that they like their results better with personal savings excluded from the model, they do note a rather marked interdependence between personal savings and expenditures on automobiles ([13], p. 233).

tions, the principal estimating technique employed (inclusion of the complete set of explanatory variables in every regression) is discussed, and some rationalizations for the choice of the explanatory variables are given. The actual results are presented and discussed in Section 3, with a comparison to the estimates from the Brookings Model given in Section 4. The principal conclusions and caveats appear in Section 5. Some supplementary material appears in the appendices: Appendix 1 presents the data employed (and some notes on their derivation), while alternative estimation techniques are the subject of Appendixes 2 and 3. Finally, Appendix 4 deals with the derivation of long term marginal propensities to save (and to consume particular expenditure categories), when one attempts to incorporate fully the stock-flow effects implicit in the underlying model.

2. The Techniques, the Variables, and the Model Employed

The model to be estimated is the following system of M equations:

$$C_{mt} = \sum_{s=1}^{K} \beta_{ms} X_{st} + u_{mt},$$

$$m = 1, 2, \ldots, M; t = 1, 2, \ldots, T;$$

8 The estimates of Appendix 2 are based on the technique developed by Gordon Sparks [26], which permits one to specify in advance that some of the coefficients in the set of regressions for the various categories are zero (i.e., that not every variable appears in every category regression). Sparks's technique is basically a generalization of Zellner's "Seemingly Unrelated Regressions" method [35], to allow for equality constraints on the coefficients being estimated. (A good discussion of estimation with equality constraints, in the case of restricted least squares regression of the parameters of a single equation, may be found in Goldberger's celebrated text [12], pp. 255-258.)

The estimates of Appendix 3 are based on the two stage least squares technique, which has been employed in an attempt to circumvent simultaneous equation biases.
where: $C_{mt}$ is expenditure on the $m^{th}$ category at time $t$; $X_{Kt}$ is total income at time $t$; $X_{Kt} = \sum_{m=1}^{M} C_{mt}$, for all $t$; $X_{1t}, X_{2t}, \ldots, X_{Kt}$ are other variables influencing (contemporaneously) expenditure on the $m^{th}$ expenditure category; $u_{mt}$ is a random disturbance in the $m^{th}$ relationship; and $\beta_{ms}, m = 1, 2, \ldots, M,$ $s = 1, 2, \ldots, K$, are the parameters to be estimated.\(^9\)

Since total income is included among the explanatory variables, constraints must be imposed on the estimated coefficients to preserve the identity relating income to the sum of the expenditures on the categories and personal savings. For example, a wealth effect may stimulate the demand for some categories of expenditure, but this added demand must come out of personal savings and/or some other categories. Similarly, a change in total disposable income must be distributed over all categories, including personal savings. Hence we require:

\[
\begin{align*}
(2a) \quad \sum_{m=1}^{M} \beta_{mK} = 1 & , \text{ and} \\
(2b) \quad \sum_{m=1}^{M} \beta_{ms} = 0, s = 1, 2, 3, \ldots, K = 1.^{10}\end{align*}
\]

\(^9\)If we do not wish every variable to appear all $M$ relationships, this might be accomplished by setting the associated $\beta_{ms}$ equal to zero.

\(^{10}\)It may be remarked that equations (2a) and (2b), together with the accounting equality of $\sum_{m=1}^{M} C_{mt}$ to $X_{Kt}$, imply that the disturbance terms are not independent but rather, for any given point in time, must cancel with addition across categories. In symbols, we must have:

\[
(2c) \quad \sum_{m=1}^{M} u_{mt} = 0 , \text{ all } t.
\]

Finally, we should acknowledge explicitly that the linearity of the category expenditure relationships (1) may of course be regarded as an approximation to a "true" relationship that may well be nonlinear. Such a form might be selected for the ease of modeling the constraints implying additive consistency.

The easiest way to accomplish our objectives is to insist that each explanatory variable appear in every regression, i.e. that none of the $\beta_{ms}$ parameters are equal to zero. Then, as is well known,\(^11\) ordinary least squares regression applied to each relationship individually will ensure that the set of category relationships are additively consistent. In other words, equations (2a) and (2b) will be satisfied for the least squares estimates of the parameters.\(^12\) Accordingly, the principal technique to be utilized in estimating these category rela-

\(^{11}\)This result has been known for some time, although it is difficult to find an early published statement of it. One of the earliest statements of this result is that of Worswick and Champernowne [34]. Although the discussion in the text of this article runs in terms of nonlinear functions of the income variable (such as powers, logarithms, exponential functions), the argument is more general than the presentation suggests. So long as a rank condition on the moment matrix is satisfied, Worswick's argument (in terms of the solution of a system of linear equations) will continue to hold, yielding a zero sum of the coefficients on the variables other than income itself and a sum of coefficients for the income variable equal to unity.

\(^{12}\)A sketch of a proof of this result is as follows. Because of the linearity (in the dependent variable) of the least squares estimators of the regression coefficients, the sum (across categories) of the vectors of coefficients for the category regressions is the coefficient vector for the regression of the sum of the expenditures on all categories, on the set of explanatory variables. But the regression of total income on a set of variables which includes total income must be such as to have a coefficient of unity on total income and zero on all other explanatory variables, in the absence of perfect multicollinearity (in the sample) among the set of explanatory variables. This must be the case, in order to minimize the criterion sum of squared deviations (at a value of zero, in this particular case).
tionships will be ordinary least squares regressions, with the same set of explanatory variables for each regression. Table I below is based on this estimating technique.\textsuperscript{13}

We may now briefly to the more general case in which not every variable appears in every regression. First, it should be observed that ordinary least squares regressions will not, in general, guarantee that the criteria for additive consistency, equations (2a) and (2b), will continue to hold. For example, if an explanatory variable is included in only one category regression (e.g., the stock of automobiles in the automobile expenditure relationship), there is no way that condition (2b) can hold for this variable if it has a non-zero coefficient in that one relationship.\textsuperscript{14} However, the technique developed by Sparks [26], on which the results of Appendix 2 are based, mitigates this difficulty by imposing conditions (2a) and (2b) as constraints on the estimation process.

We now turn to the variables employed in this study; the actual data pertain to the U.S. economy for the 15 years between 1949 and 1963. Let $Y_d$ be disposable (personal) income as measured in the national income accounts. In this study, six uses of disposable income are distinguished: $C_d$, consumption expenditures on durable commodities; $C_n$, expenditures on non-durables; $C_h$, consumer expenditures on housing services (rent and imputed rent, but not including the expenses of household operation); $C_{os}$, consumer expenditures on all services other than housing services; $C_i$, interest payments my households; and, $S$, personal savings.\textsuperscript{15}

Thus the accounting identity $X_{kt} = \sum C_m$ above becomes:

$$M = \sum_{m=1}^{3} \left( C_d + C_n + C_h + C_{os} + C_i + S \right).$$

All of these flow variables are in current dollars until explicitly deflated. For each of the consumption categories (except consumer interest), the national income accounts yield a corresponding implicit deflator which will be denoted in a symmetric manner, e.g., $P_{d}$ is the implicit deflator for expenditures on consumer durables, $P_{n}$ is the implicit deflator of nondurable consumption expenditures, and so forth. $P$ (without a subscript) represents the implicit deflator of consumption as a whole. Most studies of the determinants of consumption expenditures work in terms of "real" (i.e., price-deflated) magnitudes; hence one might wish to explain the values of the first four categories on the right-hand side of equation (3) divided by their own implicit deflators, deflating nominal consumer interest payments and nominal savings by the implicit deflator of all consumption. However, such a procedure would disturb the equality described in that equation; the accounting identity need not necessarily hold (and, in general, will not hold) for this manner of deflating the nominal magnitudes of this relationship, regard-

\textsuperscript{13} The estimates of Appendix 3, which are based on the method of two stage least squares, are also ones in which each explanatory variable appears in every regression. The property of additive consistency also characterizes the two stage least squares estimators in this particular case. Two stage least squares estimators are utilized in an attempt to obviate possible single equation biases, which may arise when one equation from a complete system is estimated by ordinary least squares techniques.

\textsuperscript{14} Of course, in a practical econometric model, the unique variable (say automobile stocks in the automobiles expenditures equation) would also appear (by implication) in the residual relationship as well, and so logical consistency would be satisfied. But such a specification is open to question, as Brainard and Tobin have pointed out in another context [5].

\textsuperscript{15} In the accounting framework of the U.S. Department of Commerce, another use of disposable income, personal transfers to foreigners, is explicitly delineated. This category of expenditures has been thrown into consumption of other services, as it did not seem of paramount interest and as it appeared more closely related to non-housing services than to any other alternatives. In any case, the amounts involved are comparatively small.
less of which of the deflators is applied to disposable income, \( Y_d \). If the accounting identity is ruptured, the theory underlying the estimation procedure no longer applies and in particular there is no longer any reason to expect the fitted category relationships to possess the additivity properties discussed above. Consequently, we select \( P \), the implicit deflator of total consumption expenditures, as the deflator of the variables on both sides of equation (3). In addition, one may modify this identity still further by expressing all of the variables in per capita terms. This is done by dividing all of the "real" variables by \( N \), the population of the United States as of July 1; this variable includes the residents of Alaska and Hawaii and the members of the armed services stationed abroad.\(^{16}\) Thus population is utilized as an additional deflator of expenditures, savings, and the income flow variables, rather than as an additional explanatory variable. There is some theoretical justification for this procedure, which is also helpful in saving on precious degrees of freedom.

The basic accounting identity, in the form in which it is employed, becomes:

\[
\frac{Y_d}{PN} = \frac{C_i}{PN} + \frac{C_a}{PN} + \frac{C_b}{PN} + \frac{C_{or}}{PN} + \frac{S}{PN},
\]

Thus the dependent variables are the six right-hand side components of equation (4); these variables correspond to the \( C_m \)'s of equation (1) above, with \( M = 6 \).

Next, the rationale for the explanatory variables (the \( X_i \)'s discussed above) may be briefly indicated. Rearranging the order of the appearance of the variables, we take the first two explanatory variables for the category regressions to be a dummy which is always equal to unity (thus generating the estimated constant terms) and real, per capita disposable income. Following received theory, we expect all categories of expenditure (including personal savings) to be positively related to the level of real income; the categories appear to be broad enough to rule out inferior goods, with the possible exception of consumer interest payments.\(^{17}\)

We seek to modify the bold absolute income hypothesis presented so far by two wealth variables. We may denote, by the symbol \((WE)_{-1}\), the net worth of the personal sector, in current dollars, as of December 31 of the preceding calendar year; this variable refers to both households and private non-profit institutions, except that the value of the physical assets of these private non-profit institutions has been excluded.\(^{18}\) The third explanatory variable for our category regressions is \((WE)_{-1}/PN\), the level of "real" per capita wealth of the personal sector. The inclusion of this variable can be rationalized with a permanent income hypothesis [10] or a life cycle hypothesis [1] of the household's consumption behaviour. However, one can equally regard this variable as being a relevant one under a modified absolute income hypothesis, in which the household's wealth or net worth is predetermined in the short run.\(^{19}\)

\(^{16}\) The data, along with their sources and, in some cases a further description, are presented in Appendix 1.

\(^{17}\) Actually, the suspicion does not appear to be confirmed in the results of Section 3 below, although this may reflect a peculiarity of this historical period.

\(^{18}\) In particular, the net worth of the personal sector does include the net stock of consumer durable owned by households (the \( DUB_N \) \(-\) 1 variable, defined below).

\(^{19}\) The modified absolute income hypothesis is often presented with income and liquid assets as the principal arguments explaining personal savings (or its complement, total consumption expenditures). Some years ago [31], James Tobin argued that, in normal times, adjustment of the liquid portion of a household's portfolio can be made fairly rapidly, and so a deeper determinant of the level of personal
ing standard views, we should expect the net worth of the personal sector to be positively related to the expenditures that can be regarded as approximating true consumption activity, namely nondurable expenditures and the two types of services expenditures. This variable should be negatively related to savings (accumulation of wealth) and to expenditures on durable goods (if the wealth accumulation aspects of the purchase of durables outweighs the consumption aspects, which seems probable in view of the length of life of most consumer durables). Finally, the effect of personal net worth on consumer interest is an open issue. The presence of financial assets (or a low level of debts) should reduce consumer interest, while the presence of consumer durables or physical capital for savings (given income) is the value of household net worth, which can in general be changed only slowly. More recently, theoretical work by Tobin [32] and by Dolde and Tobin [6] has suggested a distinction between liquidity-constrained and wealth-constrained households. To the extent that liquidity-constrained consumption behaviour is widespread, a liquid assets variable might come back into its own. Nevertheless, in [32], Tobin argues that wealth-constrained behaviour predominates (as, for example, in the 1968 temporary tax surcharge in the U.S.), thus returning essentially to his earlier hypothesis.

Some evidence supporting Tobin’s view appears in a study by Arena, who found that liquid assets were a significant explanatory variable for personal savings over a long-term period when net worth was not included as an explanatory variable in the regression but that liquid assets lost statistical significance when the household net worth variable was included. (However, the evidence for the postwar period was less clear.) On the other hand, Evans [8] interpreted his study to show that “...the time-series evidence gives very little reason to believe that wealth should be included in the consumption function, either implicitly or explicitly,” (p. 349) However, in my view, his regressions don’t warrant such a strong conclusion, and it is interesting to note in the acknowledgments at the beginning of this piece that Ando remains unconvinced.

The argument that the appearance of the savings (or consumption function) is similar under these two differing theoretical approaches is developed at greater length in [21].

the production of income (including residential housing) should raise such payments.20

The fourth explanatory variable in the category regressions is \((\text{DUR}_n)_t / \text{PN}\), the level of “real”, per capita stocks of households’ durables.21 The inclusion of this variable can be justified in terms of attempting to measure stock-adjustment effects. Given the view that stock adjustment does not occur instantaneously, one would expect the initial stock of consumer durables to exert a negative influence on expenditures on new consumer durables. In addition to any influences present from the basic accounting identity, one might expect the stock of consumer durables to be positively associated with consumer interest (on the assumption that some debt was incurred to finance the acquisition of this stock) and also possibly to have positive influence on personal savings (which is principally financial savings and which excludes acquisition of durables), through attempts to adjust portfolios when the consumer durables portion becomes larger than “desired”.

Finally, two relative price variables have been employed as explanatory variables in

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20 These considerations led us to constrain this parameter to be equal to zero, in applying Spark’s method to obtain the results presented in Appendix 2.

21 For this explanatory variable, there is no need to deflate the current value of the stocks of consumer durables by the implicit deflator for all consumption; one could equally well deflate by \(P_n\), the implicit deflator for durable goods expenditures, in an attempt to get closer to the “real” value of the undepreciated portion of the stocks of consumer durables owned by households. (This is true as the only restriction that was placed on the explanatory variables of the category regressions was the fairly weak one regarding the rank of the matrix of the explanatory variables.) In point of fact, both variants of “real”, per capita stocks of consumer durables were tried in preliminary experiments. While the results were very similar in both cases, there appeared to be a modest superiority for the real stock variant based on the implicit deflator for all consumption. Consequently, this variant was selected.
the category regressions. These relative price variables, \( P_a/P \) and \( P_a/P \), are formed by taking the ratio of an implicit deflator of a particular category of expenditures (durable goods and non-durables, in the regressions of this study) to the implicit deflator of consumption as a whole.\(^{22}\) In this way, one can attempt to capture the relative price effects suggested by the traditional theory of utility maximization; in addition, it is interesting to observe whether these relative price variables play any significant role in the fitted relationship for personal savings. As Suits [28] has pointed out, it is often assumed that the structure of prices has no appreciable effect on the level of personal savings, although this proposition has rarely been tested.

This section might be concluded with a word of explanation for the absence of an interest rate variable. Presumably if one is testing for the presence of the effects of relative prices on the category of personal savings, among others, one might wish to test for the influence of an interest rate variable on personal savings. This is not done in the present paper for three reasons. First, in the framework of an analysis which is somewhat neo-classical in spirit, one should distinguish between nominal and "real" interest rates. However, attempting to correct nominal interest rates for expectations of future inflation held by "representative" opinion is a thorny issue into which I did not wish to venture. Secondly, a realistic analysis of this issue should probably distinguish between lending and borrowing rates of interest, if one considers the imperfections in capital markets, particularly with regard to the household sector. (In this connection, see [6] and [32].) Finally, it seems plausible to believe that one will be successful in capturing the elusive effects of interest rates only if their influence on personal savings is modelled quite precisely, and this has not been the focus of this particular paper.\(^{23}\)

3. Results of the Application of the Categories Model

The results obtained in applying the model to U.S. data for 1949–1963 appear in Table 1, which is based on ordinary least squares. Each of the six fitted relationships is represented by a row in the table, the dependent variable in all cases being shown by the entry in the first column. In the second column, the constant terms of the regressions appear without their associated t ratios in Table 1, as these latter statistics were not computed in this case. For all of the independent variables, which are represented by columns (3) through (7), there are three entries: the uppermost entry in a cell is the computed regression coefficient; below it, in parentheses, is its simple t ratio (ratio of the regression coefficient to its associated error); the lowermost entry in the cell, which is enclosed in braces, is the estimate quantity elasticity of the consumption.

\(^{22}\) One cannot include all four possible relative price explanatory variables because there is a perfect mathematical relationship among them and hence there exists a high degree of multicollinearity in the observations on these variables. Given that one must drop one or more of these relative price explanatory variables, one may then engage in preliminary experimentation. Such an examination of experimental regressions (which unkind critics might call "data mining") suggested the choice of the two relative price explanatory variables named in the text.

\(^{23}\) In this connection, it is interesting to note that Somermeyer and van de Boe [25] formulate a model in which the influence of the interest rate variable enters multiplicatively with disposable income (after suitable simplifications). Applying this model to Dutch data for 1949-1966, they find the expected positive influence of the government bond rate on personal savings, although the regression coefficient of their "best" relationship has only marginal statistical significance, by the usual criteria.
Examine the final three columns of Table I, we may observe that the fits are fairly tight and the corresponding estimated standard deviations of residuals are reasonably small. The Durbin-Watson test statistics are interesting; although this formal test has very little power for 15 observations and 3-6 estimated parameters (and may be beside the point in the presence of simultaneous equation bias), there is some suggestion of positively autocorrelated residuals in the housing services equation (third row) and of negatively autocorrelated residuals in the consumer interest regressions (fifth row). In evaluating the $t$ ratios in the cells of Table I, it might be noted that the 10 per

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant Term</th>
<th>$Y_{it}$</th>
<th>$W_{it}$</th>
<th>$D_{it}$</th>
<th>$P_{it}$</th>
<th>$P_{at}$</th>
<th>$R^2$</th>
<th>$S_u$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{it}$</td>
<td>(1) $494.0$</td>
<td>$0.3477$</td>
<td>$-0.02468$</td>
<td>$-0.2273$</td>
<td>$153.8$</td>
<td>$-625.4$</td>
<td>$0.8152$</td>
<td>$10.35$</td>
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</tr>
<tr>
<td>$P_{it}$</td>
<td>(2)</td>
<td>$0.1592$</td>
<td>$0.01580$</td>
<td>$0.0072$</td>
<td>$5.6$</td>
<td>$673.2$</td>
<td>$0.9855$</td>
<td>$2.679$</td>
<td>$2.40$</td>
</tr>
<tr>
<td>$C_{it}$</td>
<td>(3) $329.8$</td>
<td>$0.0968$</td>
<td>$0.00503$</td>
<td>$0.0797$</td>
<td>$-305.8$</td>
<td>$-70.0$</td>
<td>$0.9968$</td>
<td>$2.561$</td>
<td>$1.43$</td>
</tr>
<tr>
<td>$S_{it}$</td>
<td>(4) $597.4$</td>
<td>$0.1376$</td>
<td>$0.03329$</td>
<td>$-0.0883$</td>
<td>$-225.4$</td>
<td>$-409.1$</td>
<td>$0.9822$</td>
<td>$9.298$</td>
<td>$1.88$</td>
</tr>
<tr>
<td>$S_{it}$</td>
<td>(5) $81.0$</td>
<td>$0.0318$</td>
<td>$0.00037$</td>
<td>$0.0112$</td>
<td>$-39.3$</td>
<td>$-80.2$</td>
<td>$0.9961$</td>
<td>$0.7065$</td>
<td>$2.54$</td>
</tr>
<tr>
<td>$S_{it}$</td>
<td>(6) $-1201.6$</td>
<td>$0.2369$</td>
<td>$-0.02981$</td>
<td>$0.2174$</td>
<td>$411.1$</td>
<td>$511.5$</td>
<td>$0.7764$</td>
<td>$8.917$</td>
<td>$2.09$</td>
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<tr>
<td>SUM (7)</td>
<td></td>
<td>$0.10000$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

As is well known (and not difficult to demonstrate algebraically), the quantity elasticities are identical to the expenditure elasticities, for all of the explanatory variables except own (relative) price. For own relative price, it can be shown that the elasticity of quantity with respect to this variable is always equal algebraically to the corresponding expenditure elasticity minus unity. (Thus, for example, an expenditure elasticity with respect to own price of +0.5 corresponds to a quantity elasticity of -0.5.)
cent, 5 per cent, 2 per cent, and 1 per cent points of the $t$ distribution with 9 ($= 15 - 6$) degrees of freedom are 1.83, 2.26, 2.82 and 3.25, respectively.

The conditions for retaining a variable as an explanatory variable if it is to appear in all regressions (the situation in Table I) are relatively undefined in circumstances like this. If we insist on statistical significance for an explanatory variable in all the regressions, we shall probably be left with only the income variable. On the other hand, the simultaneity aspect of the problem would seem to require that an explanatory variable be at least mildly statistically significant in at least one equation other than the one in which it has a strongly significant influence. Thus, we might set up some criterion such as the following: an explanatory variable must be significant at the 2 or 1 per cent level in one or more of the regressions and significant at the 10 or 5 per cent level in at least one other equation (in the case that the first condition only holds for one regression). It should be noted that the category regressions of Table I fulfill this requirement, as did several other sets of category regressions with all variables in all regressions which were also computed.25

25 As is often the case, the author computed a number of intermediate sets of category regressions, which were generally based on data available before the Fall, 1965, revisions of the national income data by the Department of Commerce. These computations suggested that the years 1946-1948 could usefully be excluded from the sample, as durable goods expenditures appeared to be abnormally low in relation to the stock of durables, presumably due to supply shortages in the immediate postwar period. Also, unsuccessful attempts were made to allow the functional or personal distribution of income to influence consumption. In the first case, estimated disposable labor income, disposable income of the owners of unincorporated enterprises, and disposable pure property income were introduced, as separate explanatory variables in the set of category relationships. The results were wild and unbelievable; presumably multicollinearity was the villain. In the second case, the share of after-tax income going to

Of course, one can make his results look better by deleting insignificant variables and rerunning the regressions by the method developed of Appendix 2. But it should be noted that there are still statistically insignificant variables there (by conventional criteria) which appear to play an important structural role in that set of estimated relationships.

Turning to the structure of the fitted relationships, we may observe that the effects on the categories may be classified into three types: income effects, stock effects, and relative price effects. The income effects are all significant at the 5 per cent level or better, except for expenditures on other services. As we suspected, the categories are broad enough that inferior categories of expenditure do not appear. These values do not look unreasonable as intermediate term "impact" effects on consumption; they cannot, however, be identified as the long-term marginal propensities because a change in income, through its effects on personal savings and on expenditures on consumer durables, will of course influence the stock variables, namely households' net worth and their stock of consumer durables.26 Thus, the "true" long run marginal propensity to consume these durables would appear to be much lower than 0.35, while the "true" long run marginal propensities to consume nondurables and services

the families constituting the top 5 per cent of this distribution was introduced as an explanatory variable in the set of category relationships. This variable had a theoretically inappropriate sign in the personal savings regression, although it was never statistically significant in any of the category regressions. Finally it was interesting to note that, with the revision of the national income data, the estimated structure of the category relationships was not modified to any large extent, although the individual relative price effects did jump around somewhat with regard to magnitude and statistical significance.

26 The method of computing these long run marginal propensities is explained in Appendix 4.
would appear to be higher than their column (3) values. Also worthy of comment is the intermediate term marginal propensity to save, at a value of 0.24, as well as the coefficient of real per capita income in the consumer interest equation. Taken at face value, this coefficient suggests that roughly $0.03 of every dollar of an increase in disposable income finds its way into consumer interest payments, at least for this historical period. Moreover, when the effect of the stock of consumer durables is taken into account, the long run coefficient of income on consumer interest payments would appear to be even somewhat higher.

The stock variables are both statistically significant at the 5 per cent level in three out of the six category relationships in Table I. Personal net worth is positively related to expenditures on non-durables, housing services, and other services, although the effect on housing services is questionable. It is negatively related to personal savings (largely financial) and (insignificantly) to durable goods outlays. The overall marginal propensity to consume (including expenditure on consumer durables) out of wealth is estimated to be 0.03, somewhat lower than the central tendency of the values of this coefficient estimated by Ando and Modigliani [1] and also by Arena [2], although it is within the range (or close to it) of the estimates from these studies. The durables stock variable has the expected significantly negative effect on durable expenditures, and it appears to be strongly and positively related to housing expenditures, which appears sensible. The positive coefficient on personal savings (statistically significant, by usual tests) might reflect portfolio adjustment among the financial and physical assets available to households. Finally, the positive coefficient of the durables stock variable (which is almost statistically significant) on consumer interest payments could be rationalized in terms of a larger stock of consumer durables being accompanied by higher level of debt, which entails correspondingly larger interest payments by households.

The relative price explanatory variables play a role which is by no means trivial. (Preliminary experimentation suggests that the results are decidedly inferior when the relative price variables are omitted.) The effect of a rise in the implicit deflator on consumer durables, relative to the overall deflator, appears to be to raise personal savings (this effect being significant only at the 10 per cent level) and possibly durable goods expenditures, at the expense of expenditures on housing services and consumer interest payments. Thus there appears to be a sort of gross complementarity between consumer durables and expenditures for housing services, and also between consumer durables and consumer interest payments. Both of these effects might be expected on the basis of intuition, reinforced by the sign of the stock of consumer durables as an explanatory variable in these regressions. Similarly, the almost significant positive coefficient of relative price of consumer durables in the personal savings equations suggests a gross substitutability between consumer durables and personal savings. This might be expected on the basis of portfolio adjustment considerations, which are corroborated by the sign of the durable stock variable’s coefficient in this regression. The point estimate (−0.34) suggests that the price elasticity (with respect to quantity) of the demand for durable goods is unitary, to a crude first approximation, and this suspicion is corroborated by the insignificant effect of the relative price of durables on durable goods expenditure. The price elasticity of the demand for nondurable goods is both low, at −0.15, and significantly less than unitary.
The only other significant coefficient for the relative price of nondurable goods as an explanatory variable is the negative one in the consumer interest equation. However, the point estimates of the relevant coefficients in Table I, which are generally fairly large despite their lack of statistical significance, suggest that a rise in the relative price of nondurable goods raises personal savings (as well as nondurable expenditures themselves) at the expense of durable goods expenditures, outlays on non-housing services, and, of course, consumer interest payments.\textsuperscript{27}

4. A Comparison with some Estimates from the Brookings Model

In Table II, a comparison of the category marginal propensities to consume (MPCs) implied by the relationships of this study with those of an easily accessible set of estimates based on the Brookings econometric model of the U.S. economy is presented.\textsuperscript{28}

This table was formed by aggregating the two Brookings categories of durable goods (automobiles and others), the two Brookings categories of nondurables (food and beverages, and others) and the two services categories of Sections 2 and 3 of this paper. The first set of MPC's from this study are taken directly (aside from the addition of the two categories mentioned directly above) from Table I; as argued in the preceding section, I am interpreting them as "intermediate term" MPC's, in which stock adjustment effects are ignored. The second set of estimates from this study, which appear in the third column of Table II, are interpreted as "long term" MPC's, in which stock adjustment effects are taken into account. The methodology underlying these estimated long term MPC's is explained in Appendix 4; basically, we assume a dynamic equilibrium with constant rates of growth of stock variables and

<table>
<thead>
<tr>
<th>Category</th>
<th>Brookings Estimate</th>
<th>Intermediate Estimate from this Study</th>
<th>Long Term Estimate from this Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nondurables</td>
<td>0.302</td>
<td>0.159</td>
<td>0.242</td>
</tr>
<tr>
<td>Durable</td>
<td>0.265</td>
<td>0.034</td>
<td>0.096</td>
</tr>
<tr>
<td>Services</td>
<td>0.224</td>
<td>0.022</td>
<td>0.042</td>
</tr>
<tr>
<td>Consumer Interest</td>
<td>--</td>
<td>0.002</td>
<td>0.040</td>
</tr>
<tr>
<td>Personal Savings</td>
<td>0.059*</td>
<td>0.237</td>
<td>0.212</td>
</tr>
<tr>
<td>Sum</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

\textsuperscript{*} Calculated as the residual from unity.

Sources: See the text.

\textsuperscript{27} Hence, as Suits [28] has suggested, it may well be true that total consumption outlay is governed to some extent by the structure of relative prices. However, it would appear that this cannot be a definitive conclusion of this study, in view of the fact that the estimated coefficients of the relative price variables that are presented in Appendices 2 and 3 do not agree closely with those of Table I. As is so often the case, further research on this matter would appear to be in order.

\textsuperscript{28} The sources of the equations from which the "Brookings Estimate" column has been constructed is not the original book [7] but instead the more recent book [11]. The sample period underlying the Brookings estimates is the first quarter of 1948 through the fourth quarter of 1960, which is nearly the same as the present study. The method of parameter estimation appears (the discussion on p. 17 of [11] is not too explicit) to have been ordinary least squares regressions.
constant rates of depreciation. (The figures in the third column of Table II are based on the second column of Table D-1.) The Brookings MPC's are adjusted for the existence of a lagged dependent variable in the ease of the services and the nondurables categories, but for nothing else. Thus no account has been taken of the fact that the non-automobile durables equation of the Brookings model contains a stock adjustment term, nor that in the Brookings model expenditures on automobiles also depend on the rate of unemployment in addition to real disposable income; obviously, the unemployment rate should fall when real disposable income rises (rapidly enough), in a short-term context.

The agreement between the results of the two studies is not particularly close, although the disagreement is not extreme, either. Comparing the first two columns, we may note that the closest agreement is between the two estimates of the intermediate term marginal propensity to consume durables. It is interesting to observe that the forms of the fitted equations in the two models are most similar in this case. The Brookings estimates yield higher estimates of the marginal propensities to consume nondurables and services, in the intermediate term, than do the present results. This is not altogether surprising, as the forms of the equations fitted in the two models are quite different for these cases; in particular, the Brookings MPC's for these categories might well be regarded as long term in nature. In any case, comparing the first and third columns, we see that the agreement is quite close for services, moderately close for nondurables, and quite far apart for durables. Finally, the largest discrepancy (whether one interprets the Brookings estimates as "intermediate" or "long term") is found in the case of the estimated marginal propensities to save. As noted in Table II, the Brookings estimate is the difference between unity and the category MPC's, as no formal treatment of personal savings is presented in the Brookings model and hence this phenomenon is treated (implicitly) residually, by identity. It is interesting to observe that the greatest difference (either relatively or absolutely) in the estimates from the two studies occurs in the category which is treated asymmetrically in the Brookings model; some possible limitations of this approach were pointed up in Section 1. Moreover, one might claim a superiority for the estimates of the marginal propensities to save of the present study, on the grounds that they have been obtained from an estimation procedure which, as outlined in Section 2, preserves additive consistency among income, saving, and expenditures on the various categories of consumption. However, it must be admitted that, judged by average savings rates, the estimated marginal propensities to save of the present study are probably too high as long term savings propensities.

5. Concluding Remarks

In this paper, the issue of estimating the parameters of a set of relationships explaining the various categories of consumption expenditures and personal savings has been raised. It is argued that many previous studies have been deficient in that all the categories (including personal savings) have
not been treated symmetrically; it is quite possible that different parameter estimates and even different structural forms would have been obtained if a different category had been taken as residually determined (in a loose sense) by the accounting identity connecting disposable income, personal savings, and the categories of consumption. One can attempt to circumvent these difficulties by employing estimation techniques in which this accounting identity is preserved for the predicted values of the category relationships. Such a set of category relationships may be term "additively consistent", as has been done in this paper. As outlined in Section 2, one method of obtaining additive category relationships is to estimate each category relationship by the method of ordinary least squares, provided we employ a common set of explanatory variables for all of the category regressions. As an intermediate result, we would obtain the property that, under these conditions, the sum of the regression coefficients on disposable income (the estimated marginal propensities) adds up to unity, while the sum of the regression coefficients on all the other explanatory variables (including the fitted constant terms) adds up to zero. It is worth noting that these properties of additive consistency also apply to category relationships fitted by the method of Appendix 2 (Sparks's generalization [26] of the Zellner technique of "seemingly unrelated regressions" [35]) or by the method of Appendix 3 (two stage least squares).

These estimating procedures were then applied to U.S. data over the period 1949-1963 and relationships explaining expenditures on several broad categories of personal consumption (including consumer interest payments and personal savings) were fitted. The explanatory variables are the conventional income flow and relative price variables, in addition to two stock variables (real, per capita household net worth and real, per capita stocks of consumer durable goods).

The parameter estimates of these two sets of relationships, which are shown in Table I above, appear to be reasonably satisfactory, in terms of tightness of the fits, significance of the individual regression coefficients, and agreement of the fitted parameters with received theory and/or appropriate values, according to intuition, for the magnitudes in question. These parameter estimates are hardly ideal: two difficulties in particular (in addition to possible errors of observation in the explanatory variables, and repeated experimentation tending to blur the statistical significance of conventional tests of statistical significance) may be pointed up. There is some evidence of positively autocorrelated residuals for one regression and negatively autocorrelated residuals for three of the six consumption categories equations that were fitted. The existence of this phenomenon, if it is in fact present, would tend to vitiate the conventional tests of statistical significance because of a tendency to underestimate the "true" values of the standard errors. In this connection, however, it should be observed that the magnitude of the autocorrelation of the sample residuals is not very great. More serious is the presence of an unknown amount of simultaneous equations bias in the parameter estimates of Table I. A second attempt was made to circumvent this difficulty, through the use of the two stage least squares estimating technique. (See Appendix 3.) This attempt was less than a smashing success; in particular, the standard errors associated with the two stage least squares parameter estimates were so large as to render these estimates consistent with virtually any null hypothesis. However, it is true that the second set of two stage least squares parameter estimates (the only
ones presented in Appendix 3) showed broad agreement, for the point estimates, with the results of Table I.

Because of these qualifications, the substantive results of this paper should be regarded as suggestive (and also as indicative of the methodology of the approach), rather than as definitive. The factors tending to limit the accuracy of the results of Table I are important qualifications, but, in view of the author, they are not so overwhelming that these results should be discarded. In particular, it is encouraging to find that one can obtain reasonable results from a fitting procedure that places great limitations on the ingenuity of the investigator, in order that the basic criterion of additivity in the fitted relationships may be satisfied. In principle, these relationships could be employed to generate predictions of the level and structure of consumption expenditures beyond the sample period.\(^{30}\) An advantage of forecasting with a set of category relationships like those of the present paper is that consistency of the forecasts, in the additive sense, is insured. Moreover, the fact that in the least squares approach to parameter estimation, all the information has been treated symmetrically would appear to make it more likely that the structure of the forecasted values of the categories will remain in an appropriate relationship to each other.\(^{31}\)

Finally, some supplementary materials are given in the four appendixes. In Appendix 1, the data are presented, along with some surrounding discussion. The results of two supplementary estimation techniques have been relegated to Appendixes 2 and 3. Finally, the theoretical underpinning of the long-term marginal propensities to consume given in Table II is presented in Appendix 4, along with some intermediate results.

\(^{30}\) Some preliminary experimentation five years ago, in 1968, with the category relationships of Table I suggested that, if disposable income were to grow along a full employment path, savings rates in the early 1970's would be higher than those experienced over the sample period. In addition, both the structure and the level of consumption expenditures appeared to be quite sensitive to the assumptions regarding the development of relative prices. (In a more complete model, the time path of relative prices would be one of the determinates of the system.)

\(^{31}\) It may be well to note that the Sparks variant of the method of seemingly unrelated regressions does treat one relationship asymmetrically, which we have taken to be the savings relationship. Although one is very careful about the variables that are to appear in this residual relationship, there is some evidence that this relationship suffers somewhat from its treatment as the set of residual parameter estimates. If this is true with the careful treatment of the Sparks method, it would appear to be true a fortiori with the usual treatment of a residual relationship in which the implications of accounting identities may not even be examined explicitly.
The Data

In this appendix, the sources of the data employed in the consumption categories regressions are discussed, and the data are presented. In the case of data which were directly affected by the extensive revisions of the national income statistics in 1965, only data after these revisions are presented.

Disposable income, the five categories of consumption, and personal saving may be found in the August, 1965, and November, 1965, issues of the Survey of Current Business. The implicit price deflators, for the various categories of consumption and also for total consumption, appear in the same sources. Finally, with the data prior to this set of revisions, a decomposition of personal income into various functional shares was presented on p. 10 of the July, 1964, issue of the Survey of Current Business; from this information, along with data on tax payments, one can make rough estimates of the shares in disposable income accruing to the various functional groups. Since these income shares are pre-revisions estimates, they are not presented here. As noted in Footnote 25 of the text, this approach was considered so unpromising that it was not pursued with the post revisions data.

The symbol N denotes the U.S. population on July 1 of the current year; as mentioned in the text above, it includes Alaska and Hawaii and members of the armed forces stationed abroad. The source of these figures is the U.S. Bureau of the Census’s publication, Statistical Abstract of the United States, 1965, p. 5. The share of after-tax income received by the highest 5% percent of this distribution of U.S. households (denoted by the symbol \( \chi \)) is taken from several sources: for 1946–1950, Arena ([2], p. 295); for 1951–1954, the April, 1958, issue of the Survey of Current Business, p. 17; for 1955–1962, Survey of Current Business, April, 1964, p. 8.

Personal contributions for social insurance are explicitly subdivided into employee contributions and contributions of the self-employed, back to 1952. Before that, no break-down is given; however, since the ratio of employee contributions to those of the self-employed was reasonably stable at 0.94, this ratio was employed in projecting the subdivision back to 1949.
The stock of consumer durables is the net stock, in current dollars on December 31 of the preceding year. Straight line depreciation was employed in estimating this stock. This variable was taken directly from Raymond W. Goldsmith's book, *The National Wealth of the United States in the Postwar Period*, \(^8\) for the years 1946–1958. The figures for the years 1959–1963 are based on some unpublished estimates of the components of the stock of consumer durables, prepared by Helen Stone Tice for her doctoral dissertation at Yale University. From Mrs. Tice's work, a series for the net stock of consumer durables, in current dollars, for the period 1952–1963, was obtained by summation. The two series were then linked by a regression based on the overlapping years, for which, incidentally, the fit was very tight. The Tice series was then used as an extrapolator for the Goldsmith series.

The personal net worth series is also the stock as of December 31 of the preceding year. For 1946 through 1959, this magnitude is the sum of the net worth of non-farm households (including the equity in non-corporate, non-farm businesses) and the net worth of agriculture, \(^6\) minus the tangible assets of non-profit institutions. \(^7\) This series was extrapolated to 1963 in the following manner. A crude net worth series, running from 1951 to 1963, was prepared, based on Federal Reserve Board data on financial assets less financial liabilities of the consumer sector, the agricultural sector, and the non-farm, non-corporate business sector \(^8\) and on tangible asset data. The tangible asset data, which cover only non-farm households and the agricultural sector, were pieced together from a variety of sources—principally U.S. Department of Agriculture publications and Mrs. Tice's unpublished series. As in the case of the stock of consumer durables, the technique of regression (using the years for which both series contain observations) was used to link the two net worth series, and, as before, the fit was very satisfactory. The Goldsmith series was then extended to 1963 by using the cruder net worth series as a linear predictor. \(^9\)

Finally, three series were useful in constructing additional predetermined variables for the two stage least squares estimates of Appendix 3: data on real government expenditures (\(G^e\)), constant-dollar exports of goods and services (\(E^e\)), and an index of the unit value of imported crude materials (\(P_m\)). The source of the first two series was the August, 1965, issue of the *Survey of Current Business* \(^10\); the Bureaus of International Programs and of International Commerce were the sources of the unit value index for crude materials imported into the United States. \(^11\)

\(^9\) As is evident from the text discussion, the kind cooperation of Mrs. Tice was invaluable in reconstructing the series for consumer durables and personal net worth.
TABLE A-1
POPULATION (N), PERSONAL NET WORTH \((\text{We}_{-1})\), STOCK OF CONSUMER DURABLES \((\text{DUR}_{-1})\), AND PERCENTAGE OF AFTER-TAX INCOME RECEIVED BY THE TOP 5 PER CENT OF THIS DISTRIBUTION OF HOUSEHOLDS \((\chi)\), U.S.A., 1949–1963

<table>
<thead>
<tr>
<th>Year</th>
<th>(N) ((\times) 1000)</th>
<th>(\text{We}_{-1}) ($ bill.)</th>
<th>(\text{DUR}_{-1}) ($ bill.)</th>
<th>(\chi) (percentage points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949</td>
<td>149,767</td>
<td>859.9</td>
<td>85.31</td>
<td>18.9</td>
</tr>
<tr>
<td>1950</td>
<td>152,271</td>
<td>972.0</td>
<td>91.20</td>
<td>19.2</td>
</tr>
<tr>
<td>1951</td>
<td>154,878</td>
<td>974.2</td>
<td>111.32</td>
<td>18.4</td>
</tr>
<tr>
<td>1952</td>
<td>157,553</td>
<td>1052.4</td>
<td>122.45</td>
<td>18.2</td>
</tr>
<tr>
<td>1953</td>
<td>160,184</td>
<td>1094.9</td>
<td>127.63</td>
<td>17.7</td>
</tr>
<tr>
<td>1954</td>
<td>163,026</td>
<td>1119.8</td>
<td>134.77</td>
<td>18.3</td>
</tr>
<tr>
<td>1955</td>
<td>165,931</td>
<td>1211.3</td>
<td>138.72</td>
<td>18.2</td>
</tr>
<tr>
<td>1956</td>
<td>168,903</td>
<td>1314.2</td>
<td>150.77</td>
<td>18.1</td>
</tr>
<tr>
<td>1957</td>
<td>171,984</td>
<td>1393.5</td>
<td>163.38</td>
<td>18.1</td>
</tr>
<tr>
<td>1958</td>
<td>174,882</td>
<td>1426.8</td>
<td>173.61</td>
<td>18.1</td>
</tr>
<tr>
<td>1959</td>
<td>177,830</td>
<td>1576.7</td>
<td>180.03</td>
<td>18.0</td>
</tr>
<tr>
<td>1960</td>
<td>180,684</td>
<td>1661.9</td>
<td>187.33</td>
<td>17.7</td>
</tr>
<tr>
<td>1961</td>
<td>183,756</td>
<td>1690.9</td>
<td>192.55</td>
<td>17.8</td>
</tr>
<tr>
<td>1962</td>
<td>186,656</td>
<td>1844.8</td>
<td>196.95</td>
<td>17.7</td>
</tr>
<tr>
<td>1963</td>
<td>189,375</td>
<td>1829.9</td>
<td>203.65</td>
<td>17.7</td>
</tr>
</tbody>
</table>

* Not available.
Sources: See the discussion above.

TABLE A-2
DISPOSABLE INCOME \(\left( Y_d \right)\), CONSUMPTION EXPENDITURES ON DURABLE GOODS \(\left( C_d \right)\), NON-DURABLE CONSUMPTION EXPENDITURES \(\left( C_{nd} \right)\), EXPENDITURES ON HOUSING SERVICES \(\left( C_h \right)\), EXPENDITURES ON NON-HOUSING CONSUMER SERVICES \(\left( C_{nh} \right)\), CONSUMER INTEREST PAYMENTS \(\left( C_i \right)\), AND PERSONAL SAVINGS \(\left( S \right)\), IN CURRENT DOLLARS, U.S.A., 1948 (OR 1949) THROUGH 1963

<table>
<thead>
<tr>
<th>Year</th>
<th>(Y_d) ($ bill.)</th>
<th>(C_d) ($ bill.)</th>
<th>(C_{nd}) ($ bill.)</th>
<th>(C_h) ($ bill.)</th>
<th>(C_{nh}) ($ bill.)</th>
<th>(C_i) ($ bill.)</th>
<th>(S) ($ bill.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1949</td>
<td>188.585</td>
<td>24.628</td>
<td>94.545</td>
<td>19.252</td>
<td>38.899</td>
<td>1.860</td>
<td>9.401</td>
</tr>
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<td>1951</td>
<td>226.583</td>
<td>29.648</td>
<td>108.753</td>
<td>23.823</td>
<td>44.398</td>
<td>2.656</td>
<td>17.275</td>
</tr>
<tr>
<td>1952</td>
<td>238.312</td>
<td>29.334</td>
<td>113.950</td>
<td>26.476</td>
<td>47.336</td>
<td>3.041</td>
<td>18.175</td>
</tr>
<tr>
<td>1953</td>
<td>252.564</td>
<td>33.248</td>
<td>116.792</td>
<td>29.315</td>
<td>51.090</td>
<td>3.790</td>
<td>18.329</td>
</tr>
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<tr>
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<td>40.795</td>
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<td>67.085</td>
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<td>37.881</td>
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<td>8.995</td>
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</table>

Sources: See the Discussion above.
**TABLE A-3**

Implicit Deflator of Consumer Durable Goods Expenditures ($P_d$), Implicit Deflator of Non-Durable Consumption Expenditures ($P_n$), Implicit Deflator of Housing Services Expenditures ($P_h$), Implicit Deflator of Total Consumption Expenditures ($P$), Unit Value Index of Imported Crude Materials ($P_m$), Real Government Expenditures on Goods and Services ($G^*$), and Constant-Dollar Exports of Goods and Services ($E^*$), U.S.A., 1948–1963

<table>
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<td>100.0</td>
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<td>94.2</td>
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<td>101.9</td>
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<td>94.7</td>
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<td>101.1</td>
<td>103.1</td>
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<tr>
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<td>107.5</td>
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<td>1963</td>
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<td>106.9</td>
<td>106.1</td>
<td>95</td>
<td>109.8</td>
<td>32.2</td>
</tr>
</tbody>
</table>

Sources: See the Discussion above.
APPENDIX 2

Results of a Constrained Estimation Procedure

In this appendix, the results of estimating the model of the text by an alternative procedure are presented. This alternative procedure, developed by Gordon R. Sparks in [26], is a variant of Zellner's technique of "Seemingly Unrelated Regressions" [35]. To avoid singularity in the matrix of disturbance terms (and hence in the corresponding matrix for the sample residuals), the technique requires us to treat one of the category relationships as residually determined. We have elected to take the relationship for personal savings as determined residually, thus following the treatment of several econometric models.

The important gain from the use of this technique is that one is no longer required to have every variable appear in every regression. The omitted variables (those with zero coefficients in Table B-1 below) were chosen on the basis of theoretical considerations (such as those pointed up in Footnote 20 of the text) and partly, it must be admitted, after a peek at which regression coefficients were quite insignificant in Table I of the text. (It must be admitted that Brainard and Tobin [5] have adduced good reasons why this is a methodologically suspect procedure.)

The results of this constrained estimation procedure appear to agree, in broad outline, with the results of Table I of the text. Interestingly, the category relationship that appears to suffer most is the one that is treated residually (personal savings) rather than being the outcome of an explicit fitting procedure. Despite the fact that all the variables in the relationship for personal savings were put there explicitly rather than "absent-mindedly", this relationship definitely is weaker because of its asymmetrical treatment, in terms of the tightness of the fit and the significance of the individual coefficients. Also, in the case of the relative price of nondurables as an explanatory variable, the measured effect appears to be the resultant of being the residual estimate into which any errors emanating from three zero restrictions are dumped, rather being believable in its own right. Finally, it may be observed that, if the residual category relationship appears to suffer even with the careful treatment by the Sparks method, this would appear to be true a fortiori with the usual treatment of residual category relationships in various econometric models, in which the implications of the relevant accounting identities may not even be examined explicitly.

The results of this estimation technique
are presented in Table B-1 below. The interpretation of the figures appearing in this table is identical to that of Table I of the text, except that those cells that have simply a zero coefficient, with no t ratio nor computed value for the quantity elasticity, represent explanatory variables that have been dropped from the category regression under consideration.

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>Constant Term</th>
<th>Y_d</th>
<th>(Wc)_{-1}</th>
<th>(Dut)_{-1}</th>
<th>P_d</th>
<th>P_s</th>
<th>R^2</th>
<th>Su</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_d PN (1)</td>
<td>121.4 (0.38)</td>
<td>0.3253</td>
<td>-0.01441</td>
<td>-0.2270</td>
<td>-145.0</td>
<td>0</td>
<td>0.7686</td>
<td>10.99</td>
<td>2.05</td>
</tr>
<tr>
<td>C_o PN (2)</td>
<td>-260.7 (2.19)</td>
<td>0.1594</td>
<td>0.01558</td>
<td>0</td>
<td>0</td>
<td>647.3</td>
<td>0.9853</td>
<td>2.444</td>
<td>2.32</td>
</tr>
<tr>
<td>C_k PN (3)</td>
<td>200.5 (3.31)</td>
<td>0.0929</td>
<td>0.00600</td>
<td>0.0921</td>
<td>-277.4</td>
<td>0</td>
<td>0.9962</td>
<td>2.637</td>
<td>1.52</td>
</tr>
<tr>
<td>C_g PN (4)</td>
<td>-244.0 (5.10)</td>
<td>0.1859</td>
<td>0.03754</td>
<td>0</td>
<td>0</td>
<td>0.9720</td>
<td>10.01</td>
<td>1.74</td>
<td></td>
</tr>
<tr>
<td>C_l PN (5)</td>
<td>77.1 (2.59)</td>
<td>0.0340</td>
<td>0.0126</td>
<td>-3.10</td>
<td>-86.7</td>
<td>0.9960</td>
<td>0.6196</td>
<td>2.49</td>
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<tr>
<td>S PN (6)</td>
<td>105.7 (0.31)</td>
<td>0.2025</td>
<td>-0.04471</td>
<td>0.1223</td>
<td>453.4</td>
<td>-560.7</td>
<td>0.5442</td>
<td>13.95</td>
<td>1.78</td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
As a check on the seriousness of possible single equation biases vitiating the estimates of the parameters, the model was estimated by the technique of two stage least squares. The reader will note that the estimates based on this technique that are presented in this appendix preserve additive consistency in the estimated category relationships. This result has general validity; as long as every variable is included in all the category relationships (and as long as the same set of predetermined variables are used in all first stage regressions), the resulting set of category relationships the parameters of which are estimated by the method of two stage least squares will be additively consistent.

For this study, two sets of two stage least squares estimates of the parameters were carried out, although only the second set of estimates is presented here. (The first set of two stage least squares estimates were quite wild and unbelievable.) For the first set of two stage least squares estimates, the predetermined variables of the first stage regressions were real government expenditures per capita, real exports of goods and services per capita, the ratio of a unit value index of crude materials imported into the U.S.A. to the implicit deflator of total consumption expenditures, and the two stock variables which appear in the structural relationships (the second stage regressions). For the second set of two stage least squares regressions, the set of predetermined variables included all five of the variables delineated above and also the lagged values of the other three variables (the income flow and the two relative price variables) appearing in the second stage regressions. The results of this second set of two stage least squares regressions appear in Table C-1 (the coefficients of multiple determination for the first stage regressions) and in Table C-2 (the estimates of the parameters of the model by this technique). The interpretation of the figures appearing in Table C-2 is the same as that of Table I of the text, although no estimates of the relevant quantity elasticities have been calculated.

Several comments on Table C-2 may be offered. The two stage least squares estimates of this table are in broad agreement with those of Table I of the text, although their precision in a statistical sense is woefully small. None of the coefficients of real, per capita disposable income is statistically significant, although there is at least one coefficient which is significant by conventional criteria for all of the other explana-
variables of the structural relationships of the model. In general, where there is a divergence between the point estimates of the ordinary least squares estimates of Table I and the two stage least squares results of Table C-2, the former appear, in my judgment, to be more reasonable figures. Thus, the two stage estimate of the short-term marginal propensity to consume durables (0.48) is almost certainly too high; on the other hand, the estimated marginal propensity to consume non-durables (0.06) and the estimate short-term marginal propensity to save (0.11) are very likely too small. In addition, the high positive coefficient of own relative price in the regression for non-durables expenditures implies an algebraically positive price elasticity of demand with respect to quantity, which is far less plausible than the inelastic point estimate of this concept (−0.15) generated by the corresponding ordinary least squares regression of Table I. Parenthetically, it may be remarked that it is hardly surprising that the two stage estimates of Table C-2 are fairly close to the ordinary least squares estimates of Table I, in view of the very high coefficients of multiple determination shown in Table C-1 for the first stage regressions. (It may also be mentioned that all of the predetermined variables for this second set of computations were significant in at least one first stage regression.) What does appear to be somewhat puzzling is the absence of more statistically significant regression coefficients, for the second stage regressions.

**TABLE C-1**

**VALUES OF THE COEFFICIENTS OF MULTIPLE DETERMINATION (R^2) IN THE FIRST STAGE OF A TWO STAGE LEAST SQUARES ESTIMATION PROCEDURE, SECOND SET OF PREDETERMINED VARIABLES, 1949–1963**

<table>
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<tr>
<th>Endogenous &quot;Explanatory&quot; Variable</th>
<th>Value of R^2</th>
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<td>Y_d</td>
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</tr>
<tr>
<td>PN</td>
<td></td>
</tr>
<tr>
<td>P_d</td>
<td>0.9820</td>
</tr>
<tr>
<td>P</td>
<td></td>
</tr>
<tr>
<td>P_n</td>
<td>0.9942</td>
</tr>
<tr>
<td>P</td>
<td></td>
</tr>
</tbody>
</table>
**TABLE C-2**  

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Constant Term</th>
<th>( Y_d ) PN</th>
<th>( (\text{We})_{-1} ) PN</th>
<th>( (\text{DUR})_{-1} ) PN</th>
<th>( P_d ) PN</th>
<th>( P_r ) PN</th>
<th>( R^2 )</th>
<th>( S_n )</th>
<th>( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_d ) PN</td>
<td>305.5</td>
<td>0.4766</td>
<td>-0.0347</td>
<td>-0.2635</td>
<td>468.7</td>
<td>869.1</td>
<td>0.6703</td>
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<td>2.48</td>
</tr>
<tr>
<td>( C_n ) PN</td>
<td>-237.5</td>
<td>0.0589</td>
<td>0.0252</td>
<td>0.0408</td>
<td>-215.5</td>
<td>903.7</td>
<td>0.9622</td>
<td>4.33</td>
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<tr>
<td>( C_{sp} ) PN</td>
<td>387.4</td>
<td>0.0599</td>
<td>0.0069</td>
<td>-0.0869</td>
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<td>-52.9</td>
<td>0.9943</td>
<td>3.43</td>
<td>1.79</td>
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<tr>
<td>( C_{pl} ) PN</td>
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<td>0.0218</td>
<td>-0.1289</td>
<td>89.5</td>
<td>-719.4</td>
<td>0.9802</td>
<td>9.80</td>
<td>1.95</td>
</tr>
<tr>
<td>( S ) PN</td>
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<td>0.0001</td>
<td>0.0101</td>
<td>-29.7</td>
<td>-74.9</td>
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<td>0</td>
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APPENDIX 4

Estimates of Long Term Marginal Propensities to Consume Particular Categories, Incorporating Effects of Stocks or Flows

We may begin by restating our model, after which the basic problem can be stated and two tentative solutions suggested. Let \( C_0 \) be (real, per capita) expenditures on consumer durable goods, and let \( D \) be the corresponding initial stock. Let \( Y \) be (real, per capita) disposable income, \( S \) (real, per capita) personal savings, and \( W \) (real, per capita) net worth of the personal sector, at the beginning of the period. Finally, let \( C_j \), \( j = 1, 2, 3, \) and \( 4 \), be (real, per capita) expenditures on the other four consumption categories delineated in the text. If we denote parameters by the subscripted Greek letters and let the effects of relative prices be impounded in the constant term (the Greek letters with the “0” subscripts), the model of the text can be written:

\[
\begin{align*}
(\text{I}) & \quad C_D = a_0 + a_1 Y + a_2 W + a_3 D; \\
(\text{II}) & \quad S = \beta_0 + \beta_1 Y + \beta_2 W + \beta_3 D; \text{ and} \\
(\text{III}) & \quad C_j = \gamma_{0j} + \gamma_{1j} Y + \gamma_{2j} W + \gamma_{3j} D, \\
& \quad j = 1, 2, 3, 4.
\end{align*}
\]

At this point, we are ready to state our problem. Over time, the stocks \( W \) and \( D \) depend on the flows \( S \) and \( C_D \), and hence long run estimates of the relevant marginal propensities to consume should presumably take these dependencies into account. Moreover, any calculated long run marginal propensities to consume should add up to unity across categories in order to preserve additive consistency, as the accounting identities don’t cease to hold in the long run. It will be recalled that, in the short run, the property of additive consistency implies (and is implied by) the following conditions:

\[
\begin{align*}
(\text{IV}) & \quad a_0 + \beta_0 + \sum_{j=1}^{4} \gamma_{0j}^j = 0; \\
(\text{V}) & \quad a_1 + \beta_1 + \sum_{j=1}^{4} \gamma_{1j}^j = 1; \\
(\text{VI}) & \quad a_2 + \beta_2 + \sum_{j=1}^{4} \gamma_{2j}^j = 0; \text{ and} \\
(\text{VII}) & \quad a_3 + \beta_3 + \sum_{j=1}^{4} \gamma_{3j}^j = 0.
\end{align*}
\]

The first approach to this problem is to assume equilibrium growth rates. Let \( g_1 \) and \( g_e \) be the assumed rates of growth, in dynamic equilibrium, of the stocks \( D \) and
W, respectively. Then, by definition, we shall have:

(IX) \[ \Delta D/D = g_D \text{ or } \Delta D = g_D D; \] and

(X) \[ \Delta W/W = g_w \text{ or } \Delta W = g_w W, \]

where the \( \Delta \) symbol refers to the change in the stock from the preceding to the current calendar year. Let us assume that the physical disappearance of a portion of the stock of consumer durables is proportional to the initial stock, with the factor of proportionality being \( \delta_1 \); and, similarly, the parameter \( \delta_w \) may be defined as the rate of physical disappearance of the portion of the stock of wealth of the personal sector that is not comprised of consumer durables. (While net financial wealth does not depreciate, physical assets, such as the stock of housing and agricultural capital, do, and so \( \delta_w \) can be regarded as an appropriately defined weighted average.) With these assumptions, we may write down the relationships between the gross flows (\( C_1 \) and \( S \)), the initial stocks, and the net changes as follows:

(XI) \[ \Delta D = C_D - \delta_1 D; \] and

(XII) \[ \Delta W = C_D + S - \delta_1 D - \delta_w (W \cdot D). \]

(Recall that the stock of consumer durables is included in our measure of the net worth of the personal sector. Also, it should be clear that we are assuming an absence of capital gains or losses on financial assets and liabilities held in the portfolios of households and non-profit institutions.)

This is all the basic machinery that we need to give one solution to our problem. First, substitute Equations (I) and (IX) into Equation (XI), obtaining:

(XIII) \[ g_D D = a_0 + a_1 Y + a_2 W + (a_3 - \delta_1) D. \]

Next, substitute Equations (I), (II), and (X) into Equations (XII); this yields:

(XIV) \[ g_w W = (a_0 + \beta_0) + (a_1 + \beta_1) Y + (a_2 + \beta_2 - \delta_w) W + (a_3 + \beta_3 - \delta_1 + \delta_w) D. \]

In turn, we may rewrite Equations (XIII) and (XIV) as a system of two linear equations in the two dependent variables, \( D \) and \( W \):

(XV) \[ (a_3 - \delta_1 - g_D) D + a_2 W = -a_0 - a_1 Y; \]

(XVI) \[ (a_3 + \beta_3 - \delta_1 + \delta_w) D + (a_2 + \beta_2 - \delta_w - g_w) W = -(a_0 + \beta_0) - (a_1 + \beta_1) Y. \]

This system has the following solution:

(XVII) \[ D = k^D_0 + k^D_1 Y, \] where

\[ k^D_1 = \frac{\beta_1 \alpha_2 - \alpha_1 (\beta_2 - \delta_w - g_w)}{(a_3 - \delta_1 - g_D) (\beta_2 - \delta_w - g_w) - a_2 (\beta_3 + \delta_w + g_D)}. \]

(XVIII) \[ W = k^W_0 + k^W_1 Y, \] where

\[ k^W_1 = \frac{a_1 (\beta_3 + g_3 + \delta_w) - \beta_1 (a_3 - \delta_1 - g_D)}{(a_3 - \delta_1 - g_D) (\beta_2 - \delta_w - g_w) - a_2 (\beta_3 + \delta_w + g_D)}. \]
With this solution, we are ready to calculate the relevant long term marginal propensities. First, we may rewrite Equation (XI) above, substituting the dependence of the stock variable $D$ on $Y$ in the long term, as shown in Equation (XVII):

\[(XIX) \quad C_D = \Delta D + \delta_D D = (g_d + \delta_d) \left( y^D + k^D_1 Y \right).\]

Accordingly, the long term marginal propensity to consume durable goods is, according to this approach, given by the following expression:

\[(XX) \quad \left( \frac{\partial C_D}{\partial Y} \right)_{LT} = (g_d + \delta_d) k^D_1,\]

where the parameter $k^D_1$ is given in Equation (XVII) above. Similarly, we may rewrite Equation (XII) as:

\[(XXI) \quad S = \delta w - C_D + (\delta_D - \delta_D) D + \delta_w W = g_s W - (g_d + \delta_d) D + (\delta_D - \delta_D) D + \delta_w W = (g_s + \delta_w) W - (g_d + \delta_D) D.\]

Accordingly, making the appropriate substitutions for the stock variables $W$ and $D$, we may calculate the long term marginal propensity to save as:

\[(XXII) \quad \left( \frac{\partial S}{\partial Y} \right)_{LT} = (g_s + \delta_w) k^w_1 - (g_d + \delta_D) k^D_1,\]

where the parameters $k^w_1$ and $k^D_1$ are given by Equations (XVIII) and (XVII), respectively. Finally, from an appropriate substitution into Equation (III), it is easily verified that, according to this approach, the long run marginal propensities to consume the various other expenditure categories are given by:

\[(XXIII) \quad \left( \frac{\partial C_j}{\partial Y} \right)_{LT} = \gamma_j + \gamma_2 k^w_1 + \gamma_3 k^D_1, \quad j = 1, 2, 3, 4.\]

It is worth remarking that the sum of these long run marginal propensities, across the six uses of disposable income distinguished here, is indeed identically equal to unity. While the proof of this proposition involves a lot of tedious algebra, it involves no deep principles of mathematical analysis, and so (to avoid making a long article still longer) it will not be reproduced here. (The only slightly tricky intermediate step is to observe that one could have equally calculated the long run marginal propensity to consume durable goods from Equation (I) as $(a_1 + a_2 k^w_1 + a_3 k^D_1)$ and the long run marginal propensity to save from Equation (II) as $(\beta_1 + \beta_2 k^w_1 + \beta_3 k^D_1)$; this is where the tedious algebraic manipulations come into play.)

The second approach, which is different conceptually but which comes down to much the same figures in practice, is to assume equilibrium stock-flow ratios. Specifically, assume that, in the long run, the ratio of net worth to disposable income is constant at the equilibrium ratio $(W/Y)^e$ and assume as well that, in the long run, the ratio of the stock of consumer durables also has a constant equilibrium value equal to $(D/Y)^e$. (These conditions might come about, for instance, if personal disposable income, the stock of consumer durables, and consumer durables all grew at the same rates in a dynamic equilibrium.) Then, in the long run, we should have:

\[(XXIV) \quad \frac{W}{Y} = (W/Y)^e \quad \text{or} \quad W = (W/Y)^e Y \quad \text{and} \]

(XXV) \[ \frac{D}{Y} = (\frac{D}{Y})^e \]
\[ \text{or} \quad D = (\frac{D}{Y})^e Y \]

Substituting Equations (XXIV) and (XXV) into Equation (I), we obtain:

(XXVI) \[ C_D = a_0 + \left[ a_1 + a_2 \left( \frac{W}{Y} \right)^e + a_3 \left( \frac{D}{Y} \right)^e \right] Y \]

Accordingly, the estimate of the long run marginal propensity to consume durable goods is, according to this approach:

(XXVII) \[ \frac{\partial C_D}{\partial Y}_{\text{LT}} = a_1 + a_2 \left( \frac{W}{Y} \right)^e + a_3 \left( \frac{D}{Y} \right)^e \]

Similarly, the other long run marginal propensities are, according to this approach, equal to the following expressions:

(XXVIII) \[ \frac{\partial S}{\partial Y}_{\text{LT}} = \beta_1 + \beta_2 \left( \frac{W}{Y} \right)^e + \beta_3 \left( \frac{D}{Y} \right)^e \] and

(XXIX) \[ \frac{\partial C_j}{\partial Y}_{\text{LT}} = \gamma_j^1 + \gamma_j^2 \left( \frac{W}{Y} \right)^e + \gamma_j \left( \frac{D}{Y} \right)^e, \quad J = 1, 2, 3, 4. \]

Again, it is easily verified that the property of additive consistency holds for the long term marginal propensities calculated according to this approach.

We may illustrate these two approaches on the basis of some estimates of the relevant parameters. These estimates, which are based on the worksheets underlying the data of Appendix 1 above, are fairly rough, and so none of the issues underlying their construction will be discussed. Nevertheless, they are intended to be realistic, at least to a first approximation. Since we have estimated our model with annual data, all flows are at annual rates. From the worksheets mentioned above, I have estimated the parameters in question as:

\[ \delta_d = 0.145; \]
\[ \delta_w = 0.032; \]
\[ g_w = 1.74\%; \]
\[ g_d = 2.52\%; \]
\[ \left( \frac{W}{Y} \right)^e = 4.5; \quad \text{and} \]
\[ \left( \frac{D}{Y} \right)^e = 0.5. \]

The estimated long term marginal propensities appear in Table D.1 below. The picture in the long term is considerably different from that presented by the interim marginal propensities, especially for consumer durables and the two categories of services. However, the two sets of estimates of the long term marginal propensities do not differ markedly from each other. This is hardly surprising, in view of the fact that we have, as intermediate estimates:

\[ k_1^b = 0.567 \quad \text{and} \]
\[ k_1^w = 4.96. \]
### TABLE D-1
ESTIMATES OF CATEGORY MARGINAL PROPENSITIES, INTERMEDIATE AND LONG TERM, U.S.A., 1949-1963

<table>
<thead>
<tr>
<th>Category</th>
<th>Intermediate Marginal Propensities From Table I Above</th>
<th>Long Term Marginal Propensities According to First Approach (Constant Growth Rates)</th>
<th>Long Term Marginal Propensities According to Second Approach (Equilibrium Stock-Flow Ratios)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Durables</td>
<td>0.3477</td>
<td>0.0905</td>
<td>0.1230</td>
</tr>
<tr>
<td>Nondurables</td>
<td>0.1592</td>
<td>0.2416</td>
<td>0.2339</td>
</tr>
<tr>
<td>Housing Services</td>
<td>0.0868</td>
<td>0.1569</td>
<td>0.1493</td>
</tr>
<tr>
<td>Services other than Housing</td>
<td>0.1376</td>
<td>0.2525</td>
<td>0.2433</td>
</tr>
<tr>
<td>Consumer Interest</td>
<td>0.0318</td>
<td>0.0400</td>
<td>0.0391</td>
</tr>
<tr>
<td>Personal Saving</td>
<td>0.2369</td>
<td>0.2124</td>
<td>0.2115</td>
</tr>
<tr>
<td>Check Sum</td>
<td>1.0000</td>
<td>0.9999</td>
<td>1.0001</td>
</tr>
</tbody>
</table>
REFERENCES


[30] ———, and Sparks, Gordon R. “Consumption Regressions with Quarterly Data”, Chapter 7 of [7].


