

TAXATION, CORPORATE FINANCIAL POLICY, AND THE COST OF CAPITAL¹

Joseph E. STIGLITZ

Yale University, New Haven, Conn., U.S.A.

First version received April 1972, revised version received September 1972

1. Introduction

Students of taxation are used to deriving qualitative propositions about the effect of taxation on one aspect or another of economic activity, and then, in attempting to get quantitative estimates of the magnitudes of the effect, finding that it is insignificant. Empirical studies of the effects of taxation on corporate financial structure suggest that taxation has not had a very significant effect on corporate financial structure, let alone the dramatic change that one might have anticipated given the very large increases in the corporate tax rates in the last fifty years. This poses a far more difficult problem for the theory of taxation than similar empirical findings in other areas; for here the theory has made strong quantitative as well as qualitative predictions, and these appear contradicted by the facts.

1.1. The tax structure

The following are the features of the U.S. tax code which are relevant for our purpose:

(1) Dividends (apart from some minor exception) are taxed at the rate of ordinary income.

¹ The author is indebted to A.K. Klevorick, R. Merton, and P. Miezkowski for conversations over the years on the questions discussed in this paper and to A.B. Atkinson and J. Whalley for their comments on an earlier draft. This paper was originally presented as a lecture at a conference in Hakone, Japan, June 25–26, 1970. I am very much indebted to H. Uzawa, who organized the conference, and to the other participants, for their helpful comments and discussion. The research described here was supported by the Guggenheim Foundation, the National Science Foundation, and the Ford Foundation.

(2) Capital gains are taxed at $\frac{1}{2}$ that rate;² they are taxed only at realization, and escape taxation completely if the individual dies before realization.

(3) Bond interest payments (coupons) are taxed at the rate of ordinary income.

(4) Interest payments are exempt from corporate profits taxes.

There are several other features of any tax code which are important for an understanding of the effect of taxation on corporate behavior: (a) Treatment of losses: (i) Can capital losses be offset against capital gains? The U.S. code provides for complete loss offset within any year, and limited provisions for carrying losses in excess of income over or back in time. (ii) Can capital losses be offset against ordinary income? When losses exceed capital gains in any year, then the U.S. tax code allows the excess to be deducted against ordinary income. The absence of complete loss offset provisions and the ability to offset capital losses against ordinary income both introduce a non-linearity in the tax structure, i.e. they make the tax structure non-proportional. Consider an individual with income from capital gains. His initial wealth is W_0 . He receives a wage w . His before tax income depends on the size of his capital gains. His after tax income and his tax payments as a function of the size of capital gains are plotted for four different types of loss offset³ provisions. (b) Progressivity of the tax structure: again, this has the effect of taxing losses and gains at different rates; provisions for averaging may mitigate the effects of progressivity. (c) Depreciation allowances, accelerated depreciation and investment credits. The effects of these provisions on the durability of capital and the capital intensity of production are extensively discussed in the literature. The tax savings which result from allowing depreciation to be taken against ordinary income at a rate faster than true economic depreciation, and taxing the capital gain (over the tax-depreciated value of the asset) at capital gains rates have been noted both by real estate operators and the Internal Revenue Service.

Although, these provisions are clearly important, their introduction complicates the analysis and obfuscates the role played by what we consider to be the four essential provisions listed above. Hence in this analysis below we shall assume proportional taxation, with full loss off-

² Formerly, there was a maximum rate of 25%.

³ In 'limited' loss offset, losses can only be offset against present income.

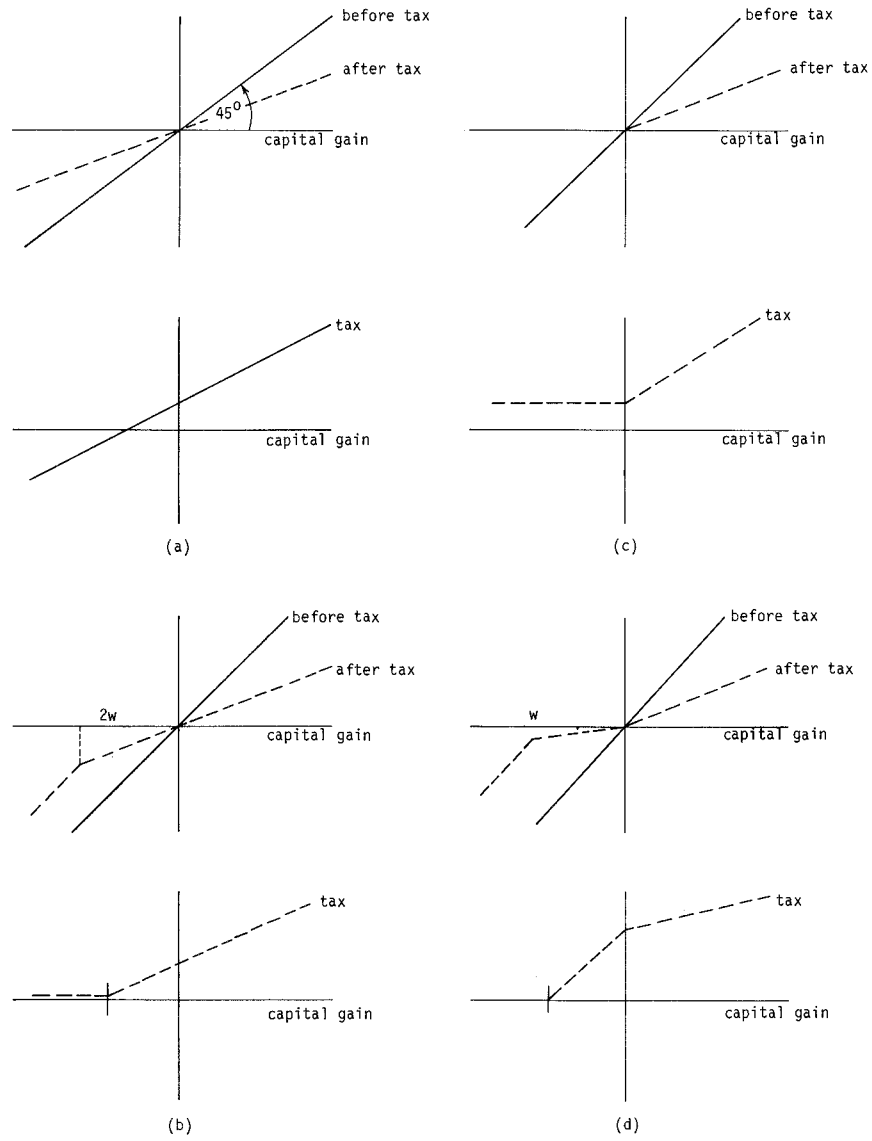


Fig. 1a. Full loss offset. 1b. Loss offset against current tax liabilities at capital gains rates. 1c. No loss offset. 1d. Loss offset against current income at ordinary rates.

set provisions (at the same rate at which gains are taxed) with true economic depreciation.⁴

In addition to these, there are a number of other provisions, pertaining to recapitalizations of firms, taxes of accumulated earnings, personal holding companies, tax treatment of preferred stock convertible bonds and other securities, and the determination of when a security is a bond which are important particularly for understanding closely held firms and about which we shall have a little to say later; unfortunately not only do these complicate the analysis but many of these issues are a matter of case law, and a full exploration would take us beyond the scope of this paper.

1.2. The effects of taxes on financial structure: a review

Three related questions have been posed:

- (1) What would be the effect of an income tax on the financial structure of the firm, in the absence of these special provisions?
- (2) What is the effect of the special provisions on the debt-equity ratio?
- (3) What is the effect of the special provisions on the dividend retention ratio?

To understand the problem and the position we shall take, it is useful to summarize briefly the history of its analysis.

Early students of corporate finance observed that the variance of the return per dollar invested in a share of the common stock of a company increased as the debt equity ratio increased. Since a corporate income tax reduces both the mean and standard deviation of the return, by the same reasoning that Domar and Musgrave (1944) used to argue that a proportional income tax (with full loss offset provisions) increases risk taking, since the government acts as an equal partner in absorbing risk,⁵ it can be argued here that the corporate profits tax increases the debt equity ratio. The deductibility of interest payments for the corporate income tax leads to an even further increase in the debt-equity ratio.⁶

Following the work of Markowitz (1959) it became clear that one

⁴ For a discussion of the meaning and implications of true economic depreciation, see Samuelson (1964).

⁵ Mossin (1968) and Stiglitz (1969a) have recently shown that the conclusions of the Domar–Musgrave analysis are valid only under a more restrictive set of assumptions.

⁶ The favorable treatment of capital gains was largely ignored in early studies – see later comments.

could not judge the riskiness of an asset in isolation; Modigliani and Miller (1958) showed in fact that, if firms could be grouped into risk classes, then any change in the debt equity ratio of one firm could be completely offset by changes in the portfolio composition of the individual. Stiglitz (1969b) and (1972b) extended the argument to show that the financial policy of the firm – its debt equity ratio, the maturity structure of its debt, its dividend retention ratio – was of no consequence under a much weaker set of assumptions. A *uniform* income tax – with no special provisions for capital gains and interest deductibility – would leave this result unaffected; thus the imposition of the tax would have no implications for the firm's financial policy; any changes that one observed in the financial structure could not be attributed to the imposition of the tax. The consequence of this result was to focus the analysis on the effects of the 'special provisions'. Modigliani and Miller ignored the special provisions for capital gains, and focusing on the interest deductibility provisions observed that tax payments could be reduced by increasing the debt equity ratio. This implied that firms should have as high a debt equity ratio 'as possible'; that is, they should at least increase their debt equity ratio to the point where there is a positive probability of bankruptcy. This was more than just a qualitative prediction that the tax should increase the debt equity ratio; this was a quite strong quantitative prediction. It implied moreover that firms that had essentially the same pattern of returns across the states of nature (i.e. belonged to the same risk class) should all have the same debt equity ratio. The very statistics that Modigliani and Miller presented to support their original analysis, in which taxes were not properly treated, provided the evidence against this result. Clearly, something was missing from the analysis.⁷

One thing at least was missing; the implications of the special treatment of capital gains, which encouraged equity financing.⁸ But when examined in the context of a two period model this yielded an equally strong quantitative prediction: firms should either have a very high debt equity ratio or should have no debt. This too seems inconsistent with the evidence.

⁷ Baumol and Malkiel (1967) implicitly recognize this problem but to resolve it, they revert to the pre-Modigliani–Miller analysis of looking at securities in isolation.

⁸ This was pointed out by Farrar and Selwyn (1967) and by Stiglitz (1972e).

1.3. The cost of capital

The effect of the tax structure of the financial structure is of some interest in its own right. But our primary interest should be in the effect of the tax structures on resource allocation; if in the absence of taxation financial policy were irrelevant for the real investment and consumption decisions of the economy – as in the Modigliani–Miller and Stiglitz analysis – it is clearly conceivable that taxed induced changes in the financial structure have no real effects on the investment decision of the firm. This we shall show is in fact the case: the optimal investment conditions for the firm are unchanged by the imposition of the tax – e.g. for a safe industry the rate of return should still be equal to the before tax rate of interest.⁹ The taxes do, of course, affect the savings investment decision, and accordingly may affect the rate of interest which will prevail in the economy.¹⁰

2. The multi-period model

2.1. Introduction

It is our contention that what the earlier studies referred to in section 1 lacked was a complete analysis of the interrelations among the different financial policies which become apparent in a multi-period model. We consider the effects of changes in financial policy for a given ‘real plan’, i.e. a given investment plan, specifying what the firm will do in each state of nature at each date, corresponding to which there is a given level of profit in each state of nature at each date. If for instance the firm decides to increase its dividend–retention ratio, to obtain the requisite funds for financing its investment, it must either issue new bonds or issue new shares. If it issues new bonds, it will mean that at future dates, more of the gross profits will be distributed to bond holders, less will be available to be distributed to shareholders (either in the form of capital gains or dividends). Not only must we be careful

⁹ This result needs to be qualified in the presence of uncertainty, if there is a finite probability of bankruptcy. See below, section 3.1.

¹⁰ We assume that depreciation allowances are equal to true economic depreciation; obviously, accelerated depreciation, investment tax credits, etc. affect optimal investment rules.

to trace out the full repercussions of any change in financial policy today, we must be careful in evaluating the effects of such a change: a given change may save us taxes today, but only at the expense of increasing them tomorrow. Whether the change is desirable thus may depend on our marginal rate of substitution. We shall show that, for most tax brackets, the policy that seems to be pursued by most firms – financing most of their new investment by retained earnings, raising any additional capital required by issuing bonds – is in fact optimal.

Before proceeding with the formal analysis, it is perhaps worthwhile to provide an intuitive interpretation of this result. There are basically three reasons that bond financing is not as attractive as the earlier literature would have suggested:

(1) A larger debt means less if the returns to capital can be taken in the form of capital gains, which are taxed at lower rates than interest income.

(2) Capital gains are taxed only upon realization rather than upon accrual; even if capital gains and ordinary income were taxed at the same rate there would be an advantage to the use of equity.

(3) Personal borrowing is a substitute for corporate borrowing, and interest payments on personal account are also tax deductible. Thus the return to a firm borrowing – as opposed to an individual borrowing on his own account – is not the savings in the corporate profits tax, but only the difference between this and the savings which would have accrued to the individual if he had borrowed.

This explains why an all debt policy is not as attractive as appears at first sight; but it does not explain why all debt or all equity policies are not optimal: if equity is better than debt, shouldn't one still have an all equity policy, and conversely if debt is better than equity? The explanation of why firms do not pursue an all debt or all equity policy lies in a basic asymmetry (which arises even in our idealized tax structure) between payments to shareholders and receipts from them. Payments to shareholders are taxed, so reductions in dividends or in shares purchased back from shareholders reduce the taxes paid, but receipts from shareholders are not taxed. Accordingly, if the firm is not paying out any dividends, using all its retained earnings for investment, and financing the excess of investment over retained earnings by debt, an attempt to increase the equity by reducing the new debt issue and increasing the new equity issue will have disadvantageous tax effects; there will be no reduction in taxation on 'equity account' this period but an increase in corporate profits taxes paid in future periods because of the reduction in interest payments.

2.2. The model

The best way to proceed with this problem at least initially is to ignore uncertainty¹¹ and individual differences. We consider a firm owned by a single individual (under these assumptions the single individual is equivalent to the whole household sector). The individual is rather clever; he knows that in the absence of taxation, and without uncertainty, debt and equity are equivalent. He also knows that for some mysterious reason, the tax authorities do not perceive them to be equivalent and hence there are special tax laws pertaining to each. He thus must choose a financial structure with a single objective in mind – to maximize his lifetime utility.¹²

The individual lives for T periods; in the T th period, he sells his firm and ‘consumes’ the proceeds. The T th period consumption may be viewed as a bequest. The individual wishes to maximize his lifetime utility

$$\max U(c_1, c_2, \dots, c_t, \dots, c_T), \quad (1)$$

where U is assumed to be a concave function monotone increasing in each of its arguments, and where c_t = consumption in the t th period. Consumption is equal to his net receipts from owning shares (dividends minus purchases of new shares), plus his net borrowing (i.e. gross borrowing minus repayment of outstanding debt), minus interest payments on outstanding debt and minus taxes (paid on his ‘personal’ account). We introduce the following notation

$$\begin{aligned} b_t &= \text{indebtedness on personal account of the individual the } t\text{th} \\ &\quad \text{period,} \\ d_t &= \text{flow of funds to or from shareholders,} \end{aligned}$$

¹¹ And hence bankruptcy will play no role in our discussion. The importance of bankruptcy for financial policy in the absence of taxation is discussed in Stiglitz (1972c).

¹² One might have thought of the individual as minimizing the present discounted value of taxes he pays to the tax authorities. This way of viewing the problem is not completely fanciful; the Internal Revenue Service appears aware of the fact that many firms, particularly closely held ones, view the problem of their financial structure in exactly this way. Some of the ways the IRS attempts to counter actions taken by firms to reduce their tax liability are discussed below.

The difficulty with this approach arises in determining, at what interest rate should we discount taxes in different periods: the before tax interest rate, the after tax interest rate using the personal tax rate or the capital gains tax rate, etc.? There is no obvious answer to this question; to proceed we must write down our formal model.

r_t = interest rate on one period bonds issued at time t (paid at time $t + 1$),
 $\tau_d(t)$ = tax rate applicable to receipts of equity owners at time t ,
 $\tau_p(t)$ = personal tax rate (tax rate applicable for interest income) at time t .

In the absence of taxation, we would have

$$c_t = d_t + b_t - b_{t-1}(1+r_t).$$

The taxation of distributions to shareholders is somewhat complicated. Clearly, if the individual is just receiving back what he put into the firm, there is no tax imposed. Let D_t be the total amount he has received from the firm from time 0 to time t . Then

$$D_t - D_{t-1} = d_t.$$

Thus, if $d_t > 0$, but $D_t < 0$, the individual is just receiving back what he is investing and there is no tax imposed.¹³ If we define

$$\tau_d^*(t) \equiv \begin{cases} \tau_d & \text{if } D_t > 0 \text{ and } d_t > 0 \\ 0 & \text{if } D_t \leq 0 \text{ or } d_t < 0 \end{cases},$$

then we have

$$c_t = d_t(1 - \tau_d^*(t)) + b_t - b_{t-1}(1 + r(1 - \tau_p)). \quad (2)$$

A few remarks are in order concerning eq. (2):

(1) For simplicity, we assume all bonds are one period bonds. Thus, b_t is the number of bonds issued at time t . If $b_t > b_{t-1}$ the individual is borrowing the t th period, if $b_t < b_{t-1}$, he is paying back some of his indebtedness.

(2) If $d_t > 0$, the firm is distributing profits to shareholders. This may take one of several forms; the two most obvious are paying dividends and buying back shares. Dividends are taxed at the personal tax rate,

¹³ Clearly, it is in the interests of the taxpayer to postpone taxes as long as possible. Hence the convention we describe here, of calling the initial distributions to 'shareholders' repayment of principal, rather than dividend, is optimal.

while income from repurchased shares is taxed at the capital gains rate. Since, except for tax purposes, the two are exactly equivalent it is obviously in the interests of shareholders to have the firm repurchase shares rather than issue dividends directly. The Internal Revenue Service is also aware of this, and there are a number of regulations which limit the extent to which firms can distribute profits at rates subject to capital gains taxation. These regulations seem to have had the effect that few firms actually repurchase shares, but there are other methods by which income subject to ordinary tax rates can be converted into capital gains, e.g. the firm can grow or acquire other firms; the individual can then sell off the incremental value in the 'shares' of his firm. The net effect of all these regulations¹⁴ is that the average rate on ' d ' is between the capital gains rate and the ordinary income rate. In addition the marginal rate may well differ from the average rate. Since we are considering the effects of a uniform proportion tax structure, we shall ignore this possibility and assume τ_d is constant.

(3) If $d_t < 0$, the firm is raising capital from shareholders (issuing new

¹⁴ Thus Holzmann writes, 'If a corporation has earnings and profits, any redemption of shares of a stockholder (particularly a major stockholder) probably will be regarded by the Internal Revenue Service as a dividend, unless the corporation is contracting its business and thus returning capital. I.R.C., section 346'. Other provisions of the U.S. tax code which limit the extent to which the firm, by retaining earnings, can 'convert' income into capital gains are the following: (1) A firm which retains a large proportion of its profits may be classified as a personal holding company (if (a) it is closely held and (b) 60% of its income comes from dividends, interest income, rents, and sources other than enterprises directly managed by the firm). Undistributed personal holding company income (i.e. income after paying corporate profits taxes with certain other deductions allowed) is taxed at a rate of 70%. (2) A firm may be labeled a collapsible corporation and its income taxed at ordinary income rather than capital gains rates if it is principally engaged in construction and if the shareholders receive their income (e.g. through selling stock) before a substantial part of the income is derived from the property. (3) Perhaps most important is the tax on accumulated earnings. The tax is imposed on retained earnings in excess of that retained 'for the reasonable needs of the business' (with a number of other adjustments), at a rate of 27½% for the first \$ 100,000 and 38½% on the excess over \$ 100,000. Clearly, the critical question is, what is considered to be the reasonable needs of the business. Excess liquidity, and loans to stockholders are taken as evidence of a surplus of funds beyond that required for the business; and investments unrelated directly or indirectly to the business are not acceptable as grounds for retaining earnings (including entering a new line of business); more striking, some investments in the business may not be acceptable; for instance if the object of the retentions was 'to make the stock more attractive to the public', and hence 'benefit the stockholders' (Holzman (1965), p. 158, citing *Trico products corporation v. commissioner*, 137 F.2d 424 (2d Cir., 1943)) or if the firm customarily leased a machine, retaining earnings to purchase the machine might not be acceptable. The economic logic of the court decisions interpreting reasonable needs of the corporation is not always apparent. Nonetheless, it is clear that this tax may act as a strong deterrent from not issuing dividends.

shares). Note that receipts from the firm are taxed, but payments to the firm are not ‘tax deductible’.

(4) For the purposes of this section, where we focus on the financial policy of the firm, we shall assume the tax rates are constant. We shall also for convenience assume the interest rate is constant, so that in the subsequent discussion, there will be no subscripts on r .

We have thus far related the consumption of the individual to his receipts as an ‘equity owner’ in the firm and as a lender (or borrower). We must now relate the receipts as an equity owner to the financial policy of the firm. The set of feasible financial policies corresponding to any given real plan of the firm is described by the two basic financial identities (Stiglitz, 1972b): (i) Investment is financed by retained earnings, issuing new bonds, and issuing new shares; (ii) gross profits after taxes are either retained, paid out as dividends, or paid out to bond holders as interest payments. Introducing the notation

$\pi(t)$ profits of the firm,
 $I(t)$ investment of the firm,
 $B(t)$ bonds outstanding at end of period t ,
 T_c corporate profits taxes,

we can solve out these two identities for retained earnings, obtaining the simple relationship

$$d_t = \pi_t - I_t - T_c - rB_{t-1} - B_{t-1} + B_t. \quad (3)$$

Payments to shareholders (receipts from them, if $d_t < 0$) are equal to profits minus investment, minus corporate profits taxes, minus interest payments on outstanding debt minus repayment of outstanding debt plus new debt issued. Corporate profits taxes are given by

$$T_c(t) = (\pi_t - rB_{t-1}) \tau_c, \quad (4)$$

where τ_c = tax rate on corporate profits.¹⁵

2.3. *The conditions for an optimal financial policy*

The individual has three sets of controls:

¹⁵ As elsewhere in this paper, we assume a proportional tax with full loss offset provisions.

(1) He chooses a ‘real investment plan’, i.e. a sequence of $\{I_t\}$ and the associated sequence of $\{\pi\}$. This choice will be the focus of section 3.

(2) He chooses a financial policy for the firm. Given a ‘real investment plan’ the financial policy for the firm can be completely characterized by the sequence $\{B_t\}$.

(3) He chooses a financial policy for his personal account, which is characterized by $\{b_t\}$.

These three decisions together determine his consumption sequence $\{c_t\}$ from eqs. (2)–(4). He wishes to make these decisions to maximize $U(c_1, \dots, c_T)$, where

$$c_t = \{\pi(t)(1-\tau_c) - I_t - (1+r(1-\tau_c))B_{t-1} + B_t\}(1-\tau_d^*(t)) \\ + b_t - (1+r(1-\tau_p))b_{t-1}. \quad (5)$$

In this section we focus on the financial decisions of the firm. We substitute (5) into (1) and take the derivative with respect to B_t and b_t ; but in doing this we must be careful: $\tau_d^*(t)$ is a discontinuous function of D_t and d_t , and hence of B_t . Ignoring the points of discontinuity, we obtain

$$\frac{\partial U}{\partial b_t} = U_t - U_{t+1}(1+r(1-\tau_p)) \quad (6a)$$

$$\frac{\partial U}{\partial B_t} = (1-\tau_d^*(t))U_t - (1-\tau_d^*(t+1))U_{t+1}(1+r(1-\tau_c)). \quad (6b)$$

The condition

$$\frac{\partial U}{\partial b_t} = 0 \quad (7a)$$

immediately yields the result

$$-\left(\frac{\partial C_{t+1}}{\partial C_t}\right)_{\bar{U}} = \frac{U_t}{U_{t+1}} = (1+r(1-\tau_p)). \quad (8)$$

The marginal rate of substitution is equal to the after tax rate of interest, using the personal tax rate.

Since at the points of discontinuity in τ_d^* , the left- and right-handed

derivatives of U with respect to B_t are unequal, we write our optimality condition for B_t in the form

$$\frac{\partial U}{\partial B_t^+} \leq 0 \leq \frac{\partial U}{\partial B_t^-}. \quad (7b)$$

It is immediately clear that except in the special case where $\tau_c = \tau_p$, $(\partial U/\partial b_t)$ and $(\partial U/\partial B_t)$ cannot both equal zero simultaneously. Accordingly, either D_t, D_{t+1}, d_t or d_{t+1} must be equal to zero for all t .

2.4. Interpretation of optimality conditions

Consider what happens to the vector of consumption when we change B_t , keeping everything else constant. From eq. (5), it is clear that only c_t and c_{t+1} are affected.

$$\frac{\partial c_t}{\partial B_t} = (1 - \tau_d^*(t))$$

$$\frac{\partial c_{t+1}}{\partial B_t} = -(1+r(1-\tau_c))(1-\tau_d^*(t+1)).$$

When we increase B_t , keeping everything else constant, we increase dividends paid out at time t (reduce the revenue raised at time t by issuing new shares). If we are paying out funds to shareholders, consumption goes up by $1 - \tau_d$ times the increase in bonds. If we are raising new capital, consumption goes up by exactly the amount of the additional bonds. Next period, when we repay the bonds, dividends must go down. We must pay back $1 + r$ for every additional dollar borrowed, but there is a tax savings of $r\tau_c$. Hence dividends are reduced only by $1 + r(1 - \tau_c)$. If the firm is issuing dividends, the reduction in dividends results in a reduction in consumption of only $1 - \tau_d$ as much. Hence, increasing B_t (keeping everything else constant) increases consumption at time t but decreases it at $t + 1$.

The rate of return is dependent on $\tau_d^*(t)$ and $\tau_d^*(t+1)$, i.e. on just the sign of d_t, d_{t+1}, D_t and D_{t+1} . Very large values of B_t necessitate our raising new equity at $t + 1$ to pay back the debt and interest due, but allow us to pay out dividends at time t . Hence $d_{t+1} < 0, d_t > 0$. Conversely, very small (possibly negative) values of B_t imply that $d_{t+1} > 0$ and $d_t < 0$.

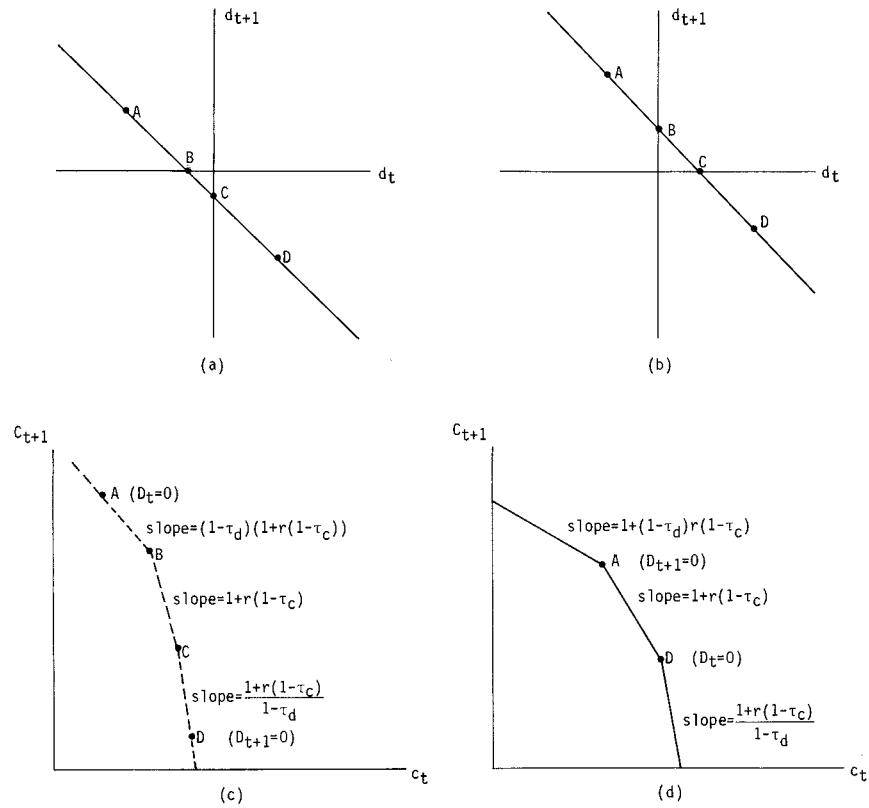


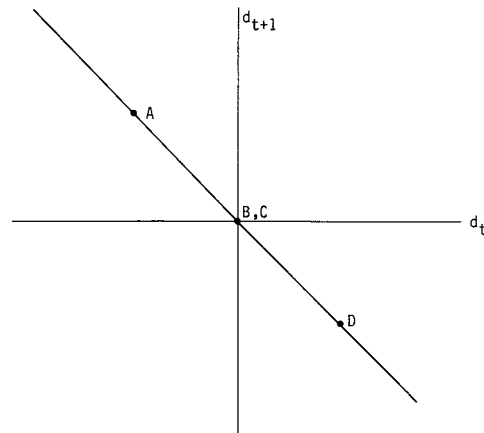
Fig. 2a. 2b. 2c. $D_t, D_{t+1} \geq 0$ at $d_t = 0$. 2d. $D_t, D_{t+1} < 0$ at $d_t = 0$ and $d_{t+1} = 0$.

More formally, from (3), it is clear that if all other aspects of the financial plan and the real plan of the firm are constant, d_{t+1} is a linear negatively sloped function of d_t . There are three possibilities, as depicted in figs. 2a, 2b and 3a, in which the curve passes below, above, and through the origin, respectively. Corresponding to each of these we can derive the consumption possibilities schedule, as depicted in figs. 2c and 3b, for the case of $D_{t+1} \geq 0$ when $d_t = 0$; and in figs. 2d and 3c for $D_t < 0$ when $d_t = 0$.¹⁶ To find the optimal point, we must compare the

¹⁶ Since

$$\frac{\partial d_{t+1}}{\partial d_t} = -\frac{1}{1+r(1-\tau_c)},$$

if $D_{t+1} \geq 0$ when $d_t = 0$, $D_{t+1} \geq 0$ for $d_t < 0$.



(a)

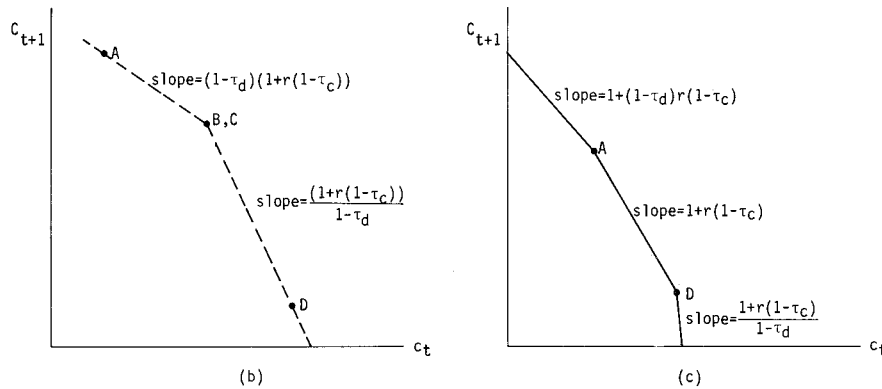


Fig. 3a. 3b. $D_t, D_{t+1} > 0$ at $d_t = 0$. 3c. $D_t, D_{t+1} \times 0$ at $d_t = 0$.

marginal rate of substitution (which by eq. (6a) is equal to $(1+r(1-\tau_p))$) with the slope of the consumption possibilities schedule which is denoted by ρ .

There are three important cases¹⁷ which we need to distinguish.

¹⁷ One further case is illustrated in figs. 2d and 3c; to the left of A, $d_t < 0$, $d_{t+1} > 0$, $D_t < 0$, $D_{t+1} > 0$. The slope is $1 + (1-\tau_d)r(1-\tau_c)$ (since the tax is paid only on the amount by which D_{t+1} exceeds zero; a decrease in d_t decreases D_{t+1}), and this is clearly greater than, equal to, or less than the MRS as

$$(1-\tau_d)(1-\tau_c) \begin{matrix} \geq \\ \leq \end{matrix} (1-\tau_p).$$

- (a) $\tau_d^*(t) = \tau_d^*(t+1)$.^{18,19} Then $MRS \geq \rho$ as $\tau_c \geq \tau_p$.
 (b) $\tau_d^*(t) = 0$, $\tau_d^*(t+1) = \tau_d$.²⁰ Then $MRS \geq \rho$ as $\tau_d \geq r[(1-\tau_d)(1-\tau_c) - (1-\tau_p)]$.
 (c) $\tau_d^*(t) = \tau_d$, $\tau_d^*(t+1) = 0$.²¹ Then $MRS \geq \rho$ as $-\tau_d \geq r[(1-\tau_c) - (1-\tau_p)(1-\tau_d)]$.

It is clear from eq. (6) that $\partial U/\partial B_t \geq 0$ as $MRS \geq \rho$. We would argue that *normally*, in case (b) $MRS > \rho$ and in case (c) $MRS < \rho$. The former be true for any r if

$$(1-\tau_d)(1-\tau_c) < (1-\tau_p), \quad (9a)$$

or equivalently, if

$$\tau_d + \tau_c - \tau_d\tau_c < \tau_p,$$

e.g. (a) if $\tau_c \geq \tau_p$; (b) if $\tau_c = 0.5$, and $\tau_d = 0.5\tau_p$ and $\tau_p \leq 0.67$; (c) if $\tau_c = 0.5$ and $\tau_d = \min[\frac{1}{2}\tau_p, 0.25]$, and $\tau_p \leq 0.625$. If $r \leq 20\%$, and $\tau_c = 0.5$, then the slope of AB will be less than the MRS for any τ_p if $\tau_d \geq \frac{1}{11}$, or if $\tau_d = \frac{1}{2}\tau_p$, or if $\tau_d = \min(\frac{1}{2}\tau_p, 0.25)$. Similarly, in case (c), the MRS $< \rho$ for any r if

$$(1-\tau_c) > (1-\tau_d)(1-\tau_p), \quad (9b)$$

or equivalently if

$$\tau_d + \tau_c - \tau_d\tau_c > \tau_p,$$

e.g. if $\tau_c \leq \tau_p$ but if $r \leq 20\%$, and $\tau_c = \frac{1}{2}$, and $\tau_d = \frac{1}{2}\tau_p$, this will be true for all values of τ_p .

The implications of this should be clear: for instance, from fig. 2c, the individual chooses point B if $\tau_d + \tau_c - \tau_d\tau_c > \tau_p > \tau_c$, point C if

¹⁸ There are two conditions under which (a) will be true:

- (i) $\tau_d^*(t) = \tau_d^*(t+1) = 0$, i.e. D_t and $D_{t+1} < 0$ and/or d_t and $d_{t+1} < 0$;
 (ii) $\tau_d^*(t) = \tau_d^*(t+1) = \tau_d$, i.e. $D_t, D_{t+1}, d_t, d_{t+1} > 0$.

¹⁹ This corresponds to the segment BC in fig. 2c.

²⁰ This implies (i) either d_t or $D_t < 0$ and (ii) d_{t+1} and $D_{t+1} > 0$. This corresponds to the segment AB in fig. 2c or 3b.

²¹ This implies (i) d_t and $D_t > 0$ and (ii) $d_{t+1} < 0$ (since $d_{t+1} > 0, D_{t+1} < 0$ is not possible). This corresponds to the segment CD in fig. 2c or 3b. It is easy to check that the cases delineated in this and the previous footnotes are exhaustive.

$\tau_c - \tau_d + \tau_d \tau_p < \tau_p < \tau_c$, and is indifferent to any point on the line BC if $\tau_c = \tau_p$.²² The implications of this for the optimal financial policy in the multiperiod firm are set forth in the next subsection.

2.5. Characterization of optimal financial policy

In the previous subsections, we derived and interpreted the first order conditions for the optimal financial policy of the individual and the firm which he ‘controls’. In this subsection we shall show what these conditions imply for the optimal debt equity ratio. We consider the following case history. An individual has an ‘idea’; to implement it requires capital. He is sufficiently wealthy that he could provide all the capital himself. Alternatively, he could ‘borrow the funds’; it should be clear that from an economic point of view, since there is no uncertainty in this model, this is perfectly equivalent to his ‘labeling’ some of his ownership claims as bonds, some as equity. In the initial years of the firm, investment exceeds profits, while in the final stages of the ‘life cycle’ of the firm, profits exceed investment.

In the previous subsection we characterized ‘three normal’ cases (and two ‘unusual cases’); the optimal financial policy depends to some extent on which of these situations we are in.

(a) If

$$\tau_d + \tau_c - \tau_d \tau_c > \tau_p > \tau_c$$

then the optimal financial policy is given by

$$d_1 = d_2 = d_3 = \dots d_{T-1} = 0, d_T > 0.$$

Since $D_t = 0$ for all $t < T$, fig. 3c is the appropriate diagram for periods 1 to $T-1$. The point BC is the optimal point in those diagrams, i.e. $d_t = d_{t+1} = 0$. Between period $T-1$ and T , the appropriate diagram is 2c, where the optimal point is B , implying $d_{T-1} = 0$ and $d_T > 0$.²³

It is thus clear that the policy we have described satisfies the necessary conditions. Is it the only such policy? Assume at some $t \neq T$,

²² The ‘abnormal’ cases where AB is flatter than the MRS or CD steeper than the MRS are discussed in note 26, below).

²³ Note that in this path, $D_t = 0$ for $t < T$.

$d_t \neq 0$. Let i be the first such date. Assume $d_i > 0$. Again, the appropriate diagram for the period $(i, i+1)$ is fig. 2c; the firm operates at point B , which implies either $d_i < 0$ (fig. 2a) or $d_i = 0$ (fig. 2b). In either case, $d_i > 0$ is not consistent with the necessary conditions. Assume $d_i < 0$, $i \neq 1$. Again, for the period $(i-1, i)$, since $D_{i-1} = 0$ when $d_{i-1} = 0$, the appropriate diagram is 2c, implying either $d_i > 0$ (fig. 2a) or $d_i = 0$ (fig. 2b). The one remaining case is $d_1 < 0$. To see that this cannot be optimal, we consider the consequences of changing d_1 and the corresponding changes in d_T , keeping $d_t = 0$, $t \neq 1, T$. We obtain diagrams identical to figs. 2b and 2c, with the rate of interest now interpreted to be the T period rate of interest. From this, it is clear that we operate at B , i.e. $d_1 = 0$.

In short, the individual receives the capitalization of his 'idea' in the form of the equity of the firm: all the capital required to finance the implementation of the ideas are raised by debt. The individual does not receive anything as an equity owner until time T (the dissolution of the firm).

The debt equity ratio depends simply on the life history of the firm; the initial ratio of debt to equity is equal to the ratio of initial capital requirements to the present discounted value of the 'idea', i.e. the surplus which the 'idea' can earn over the market rate of interest. As the investment continues in excess of profits earned, the debt rises and since the date at which the returns to the equity will be realized approaches, the value of equity also rises. Initially, the presumption is that the debt rises faster than the equity, but then it rises more slowly. Eventually, the firm's profits exceed investment, so it starts repaying debt, the debt equity ratio falls, and finally approaches zero at time T .

To put it another way, all the investment requirements in excess of retained earnings are financed by issuing bonds. The advantages of retaining earnings are sufficiently strong that they mitigate against an all debt policy.²⁴ The advantages of debt are sufficiently strong that they mitigate against an all equity policy.

$$(b) \quad \tau_c = \tau_p .$$

²⁴ One might have wondered why in the above discussion I said the present value of equity rises as the date at which the returns will be realized approaches, but did not mention the effect of retained earnings. The reason is that the value of the equity is predicated on the firm following an optimal policy, i.e. retaining earnings, so the value of these retained earnings has already been capitalized. If the firm contemplated some other financial (or real) policy it would obviously have an effect on the value of the equity of the firm.

This is the one case where there are a wide variety of financial policies consistent with optimality. The general class of optimal policies may be characterized by

$$d_1, \dots, d_T \geq 0 .$$

It is obvious that any such policy satisfies the necessary conditions. To see that only such policies are optimal, recall from fig. 2c that we are indifferent to any point along BC . This implies that for adjacent periods, d_t and d_{t+1} must be of the same sign. But if there were any period in which $d_j < 0$, since there clearly must be some period in which $d_t > 0$, by exactly the same kind of argument we used in the previous case, we could issue a bond (without coupon) for $t - j$ periods (if $t > j$) and make ourselves better off.

Among all policies which have positive or zero returns to equity owners in all periods the individual is indifferent, since borrowing on the corporate account (within these bounds) is a perfect substitute for individual borrowing. There is no optimal debt equity ratio. The debt equity ratio must be greater than that described in the previous case but less than that described in the next case.

$$(c) \quad \tau_c - \tau_d - \tau_d \tau_p < \tau_p < \tau_c .$$

This is the one case where an all debt policy is optimal:

$$d_1 > 0, \quad d_2 = \dots = d_T = 0 ,$$

the individual capitalizes the value of his idea at time 0 and receives for it debt in the firm (rather than equity).²⁵ The argument follows in exactly the same manner as the two previous cases.²⁶

It should be noted that the character of the optimal financial policy

²⁵ The provisions discussed earlier in connection with regulations pertaining to the payment of dividends are relevant here: such a firm would be considered undercapitalized and the bond payments might well be taxed as if they were dividends.

²⁶ The 'abnormal cases' where the slope of AB is greater than the marginal rate of substitution or the slope of CD is less than the MRS, lead to the firm attempting an infinitely negative debt equity ratio (i.e. the firm raises equity in order to lend funds) or an infinitely positive debt equity ratio (i.e. no equity at all). The argument follows along the lines of the discussion above; the explanation for why these extreme policies are not pursued arises from the regulations cited earlier (note 14) and from problems associated with uncertainty, discussed in section 2.8.

derived in the previous subsection required a constant value of τ_d , but the analysis did not depend on the rate at which capital gains were taxed. Capital gains could be taxed at half the rate of personal income – or at the full rate – and the optimal financial policy would be unchanged.^{27,28,29}

2.6. Uncertainty

The modifications that uncertainty requires are straightforward when $\tau_c \geq \tau_p$. We argued that if he knew what the sequence of states of nature, $\theta(t)$ would be, and hence $\pi_t(\theta(t), t)$ for all t , he would pursue under case (a) (with $\tau_c > \tau_p$) a policy of issuing no new shares to raise equity and making no payments to shareholders (except in the last period); since he can pursue exactly the same policy without foreknowledge of the

²⁷ In the 'normal cases'; whether we were in the 'normal cases' will depend on the rate at which capital gains is taxed.

²⁸ The special provision which exempts capital gains from taxation at death means that τ_d is not constant. Consider an idealized case where $\tau_d = \bar{\tau}_d$ for all $t < T$, and $\tau_d = 0$ at $t = T$. The implications that this has for the analysis may best be seen by redrawing figs. 2c and 3b for the periods $T-1$ and T (see figs. 4a and 4b). Notice that the individual can borrow and receive a tax deduction worth $\tau_p r$. The firm can lend paying a tax of $\tau_c r$ on the interest. Thus if $\tau_p > \tau_c$ the net tax savings are positive. To see this more clearly, assume the individual borrows, buys equities in the firm, which lends the money (the firm is acting partially as a finance company) perhaps even back to the shareholder, and then upon the dissolution of the firm at time T , receives back $(1+r(1-\tau_c))$ for each dollar invested in the firm (since capital gains are not taxed, the return over investment, $r(1-\tau_c)$ is not taxed), with which he is able to pay back his debt and interest, which after the tax savings, amounts to $1+r(1-\tau_p)$. Thus for each dollar borrowed, the individual makes a profit of $r(\tau_c - \tau_p)$: if there were unlimited provisions for interest deductibility, clearly, not only would the individual have an all equity position in the firm, but the firm would have an infinite negative debt; even if it had limited deductibility provisions, the individual would borrow until his interest payments equalled his taxable income against which he could offset the interest payments. For closely held firms, it is clear that the regulations and tax provisions which have the effect of limiting the income from non-operating sources are important here as is the tax on undistributed profits for both closely and non closely held firms; whether such regulations can be completely effective is another question.

It is important to observe that this special treatment does not change the behavior of the individual with $\tau_p < \tau_c$. He continues to pursue the same financial policy as before.

²⁹ The modifications of the analysis to make it applicable to an ongoing firm are simple and straightforward. Considering case (a) ($\tau_c < \tau_p$) we noted earlier that if the individual could borrow on his own account, it made no difference whether he lent to the firm or lent his funds on the general bond market, so long, of course, as his firm pursued the same borrowing-lending policy. The firm should not buy back shares or issue dividends (until its termination date). The individual who is more short lived than the firm can realize his capital gains whenever he wishes: it is as if the firm stood willing to buy back his shares at any date he wished, but then immediately issued the shares to new individuals, so that the only taxes that are paid are the capital gains taxes paid by the individual upon realization of his gain.

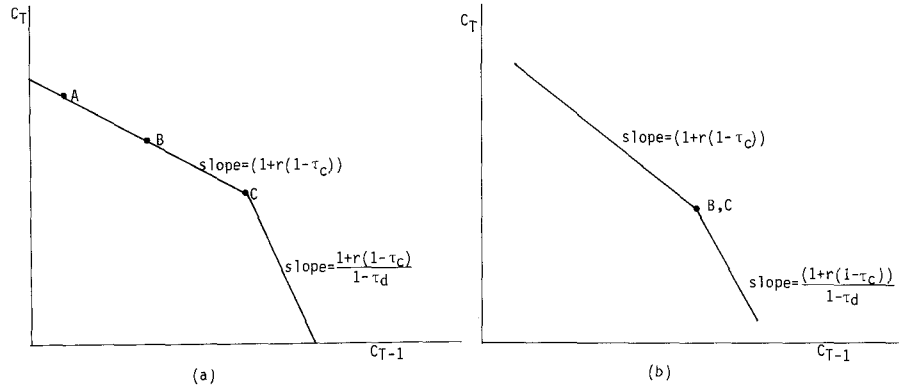


Fig. 4a. 4b.

states of nature, it is clear that uncertainty need not affect his optimal financial behavior, if he decides to retain sole ownership of the equity, and if he can raise the requisite bond finance at the safe rate of interest.

For this firm, the debt equity ratio is a function simply of the past profit and investment history of the firm. If the firm has been very successful, it may have a very low debt equity ratio, financing most of its investment out of retained earnings. If there have been a number of years when profits are low, and yet future investment opportunities still look good, it may have a very high debt equity ratio.

The case of $\tau_p < \tau_c$ is somewhat more difficult. An all debt policy in the presence of uncertainty necessarily involves the possibility of bankruptcy, and hence the firm cannot be borrowing at the safe rate of interest. Indeed, an all debt policy would imply that the firm went bankrupt in essentially all states of nature (otherwise, there would be some return to equity owners, and hence there would be a positive value of equity). It is clear that under these circumstances, the Internal Revenue Service would stipulate that the firm was undercapitalized and would treat the bonds as equity.³⁰

Let us assume that the Internal Revenue Service imposes a regulation that to obtain treatment as a bond, there must be a zero probability of default.³¹ Thus firms will increase their debt equity ratio to the maxi-

³⁰ See note 14 above and Holzman (1965).

³¹ Obviously, whether there is a positive probability of default depends on the subjective probability distribution of returns. We shall assume for simplicity that everyone agrees and knows what the probability distribution of returns is.

mum level. In the standard two period model, the implications of this are straightforward:

$$(1+r)B \leq \min_{\theta} \pi(\theta) .$$

In the multiperiod model, profits can obviously fall short of interest payments – if the firm can borrow more to repay debt falling due. Let θ represent a whole sequence of states of nature from say t to T . If $d_t = 0$, $0 \leq t < T$ we can calculate $B_{T-1}(\theta)$ given B_0 by solving

$$0 = \pi_t(\theta) - I_t(\theta) + B_t(\theta) - (1+rB_{t-1}) + (1-\tau_c)(\pi_t - rB_{t-1}) .$$

Then, at the termination of the firm at time T

$$d_T(\theta) = (\pi_T(\theta) - rB_{T-1}(\theta)) (1 - \tau_c) - B_{T-1} .$$

We require for all θ , $d_T(\theta) \geq 0$. The maximum value of B_0 satisfying this constraint is the level of debt chosen. At time 1, we can solve the problem over again. The solution will clearly depend on what state of nature occurred the previous period. Let $\hat{\theta}_0$ be the state of nature at time 0 on the ‘worst possible’ sequence of states of nature, i.e. the sequence for which $d_t = 0$ all t . Then if $\hat{\theta}_0$ occurs, the firm issues no dividends (buys back no shares); it may borrow or repay debt. But if any other state of nature occurs, the firm’s equity owners will immediately capitalize the ‘good news’; if dividends are taxed at a higher rate than capital gains, they will obviously prefer the firm to buy back shares. If the firm is simultaneously borrowing, they may be subject to a special tax which will more than offset the tax advantages of capital gains: ‘If bonds are issued in a recapitalization to stockholders or bondholders, the receipt of any greater principal amount of bonds then is surrendered (which may be zero) will be taxed as “other income” or boot’ I.R.C., section 355(a)(3)(a). (Holzmann, 1965, p. 45.)

For this firm the debt equity ratio is a function not of the past history of investment and profits, but of the probability distribution of future prospects. If there were no risk, there would be no equity; but if there is a high variance to the returns, there may be a very low debt equity ratio.

Note that like the previous case, changes in the rate of taxation – except as it results in the individual moving from case (a) to case (b) or case (c) – do not affect the optimal financial policy.

In the preceding discussion, we have explicitly assumed that there was no bankruptcy, because the tax advantages of debt disappeared if bonds became risky. In fact, the greater the probability of bankruptcy, the greater the chance that the bonds would be treated as equity but there is no rigid rule. Although this may be a major deterrent on the use of a high debt policy by closely held firms, in the more general case other considerations are probably dominant. In particular, we have shown elsewhere (Stiglitz, 1972c) that even in the absence of taxation but in the presence of uncertainty, except under very special conditions, there will be an optimal debt equity ratio of the firm, e.g. if individuals have different expectations, and there are not a full set of markets for contingent claims. Thus, for the usual reasons for portfolio diversification, the individual wants to have some of the capital for the given firm to be raised by outsiders; they may view the bonds of the firm as risky – riskier than he views them – and insist that he pay a higher nominal rate of interest. It would appear as if (for the case $\tau_c > \tau_p$) the tax advantages of debt would increase the debt equity ratio from what it would have been otherwise, but would not result in the firm going to an ‘all debt’ position.

2.7. *Widely held firms*

The optimal policy of the firm when all individuals are in tax brackets for which the same optimal financial policy obtains is clear. The policy we have described for the closely held firm is the same policy which all shareholders would wish the firm to pursue. What happens, however, when some of the individuals are in a low tax bracket, some in a high? As the firm issues more bonds and the individuals reallocate their portfolio, individuals in the high tax brackets may be worse off, those in the low tax brackets better off. There are two possible results: (1) If there are two or more essentially identical firms (i.e. perfectly correlated returns) then one will have a ‘rich clientele’ and pursue the appropriate policy for that group, the other will have a ‘poorer’ clientele, and have a maximum debt equity ratio policy. (2) If in fact all firms are slightly different from one another, then there will exist an intermediate optimal debt equity ratio. To see this, consider a two period model; observe that we can write the demand for shares in the given firm by high and low tax bracket individuals as follows: the *percentage*, s , of the firm demanded will be (in general) a decreasing func-

tion of the value of the firm ³² and, for the upper brackets, a decreasing function of the debt equity ratio, z , and for the lower brackets, an increasing function.

$$s_i^r = f_i^r(z_i, V_i)$$

$$s_i^p = f_i^p(z_i, V_i) .$$

Equilibrium requires

$$s_i^p + s_i^r = 1$$

yielding an implicit equation for V_i as a function of z_i . The value of the firm is maximized when

$$\frac{\partial f_i^p}{\partial z_i} + \frac{\partial f_i^r}{\partial z_i} = 0 .$$

3. The cost of capital

In the previous section, we focused on the optimal financial policy, assuming that the ‘real plan’ of the firm – its investment-profit sequence $\{I_t, \pi_t\}$ – are already determined.

We now must determine the optimal investment policy. We let π_t be a function of the capital stock K_t , and

$$I_t = K_{t+1} - K_t . \tag{10}$$

We shall show that (in the absence of uncertainty) optimal investment requires

$$\frac{\partial \pi_t}{\partial K_t} = r , \tag{11}$$

i.e. the cost of capital is just the before tax rate of interest.

³² Consider, for instance, two firms whose profits, when scaled by their value are identically distributed but not perfectly correlated. Then, because of risk diversification, the individual will want to buy the same number of shares of each, i.e. a smaller percentage of the larger firm (see Stiglitz, 1972a).

This result is in marked contrast to previous studies, which have erroneously introduced tax terms into the cost of capital; Jorgenson (1965) for instance uses as his cost of capital, ν , where (assuming no depreciation and no changes in the relative price of capital goods)

$$\nu = r \frac{(1-t_c w)}{1-t_c}, \quad (12)$$

where r is the rate of interest, t_c the corporate profits tax rate, and w the proportion of the costs of capital which may be deducted for tax purposes, i.e.

$$w = \frac{rD}{\nu V}, \quad (13)$$

where D is the value of debt, V the total value of the firm. Where Jorgenson and others have erred is confusing average costs of capital with marginal costs of capital, and it is clearly the latter which is relevant for purposes of resource allocation.

The argument that optimality requires $r = \partial \pi_t / \partial K_t$ is straightforward. Assume we have an optimal policy. If it is optimal, there can exist no perturbation of the policy which makes us better off. Consider the effect of an increase in the capital stock at time t , keeping the size of the capital stock at all other dates unchanged. This necessitates increasing I_{t-1} by a unit and reducing I_t by a unit. To do this, assume the firm increases its borrowing at time $t-1$ by a unit. The incremental amount then that the firm can distribute to its shareholders (at time t) after paying interest on the loan and after taxes is ³³

$$\left(\frac{\partial \pi_t}{\partial K_t} - r \right) (1 - \tau_c). \quad (14)$$

³³ B_{t-1} is increased by a unit. If indebtedness at times subsequent is unchanged, in particular, if B_t is unchanged, since

$$d_t = \pi_t - I_t - B_{t-1}(1+r) + B_t - \tau_c(\pi_t - rB_{t-1})$$

and since

$$\Delta I_t = -\Delta B_{t-1},$$

(14) follows immediately.

No matter how the returns are distributed to shareholders — as dividends or capital gains — it is clearly profitable to undertake the investment if

$$\frac{\partial \pi_t}{\partial K_t} \geq r. \quad (15)$$

Similarly, consider the effect of reducing I_t by a unit,³⁴ i.e. reducing I_{t-1} by a unit and increasing I_t by a unit. By the same kind of argument as before, this reduces both profits and bond payments at time t ; the net amount which can be distributed to shareholders is

$$\left(r - \frac{\partial \pi_t}{\partial K_t} \right) (1 - \tau_c). \quad (14')$$

Again, this implies that

$$\frac{\partial \pi_t}{\partial K_t} \leq r. \quad (15')$$

(15) and (15') together imply that

$$\frac{\partial \pi_t}{\partial K_t} = r$$

as was to be established.

The same results may be obtained by substituting (10) into (5) and maximizing (1) with respect to K_t . (Again, we must be careful to take into account the non-differentiability of (5).)

Note that nowhere in the analysis did we need to know what tax bracket the individual was in. The analysis is equally applicable to the high equity as to the low equity firm.

In the absence of uncertainty, the corporate profits tax with the interest deductibility provision is completely non-distortionary. It does not shift resources (at the margin) from the corporate to the non-corporate sector. It is an infra-marginal tax on the return to capital (or pure profits) in the corporate sector.

³⁴ Even if B_{t-1} is zero, as in an all equity firm, we can still reduce B_{t-1} by lending. To put it another way, financial investment is an alternative to real investment, and clearly, we will not undertake the latter if its return is lower than the former.

3.1. Uncertainty

If we can ignore bankruptcy, the same result, that the corporate profits tax is non-distortionary, obtains in the presence of uncertainty. We may not, however, be able to ignore bankruptcy. Consider first the conditions for optimal investment in the absence of taxation. We shall assume the individual has an additive utility function, and that he is the sole owner of the equity in the firm (or all owners are identical). He does not plan to sell shares in the firm until time T .³⁵

Let π_t be a function of K_t and θ , the state of nature. For simplicity, let it take on the special form of

$$\pi(K_t, \theta) = \pi(K_t)g(\theta), \quad (16)$$

i.e. there is ‘multiplicative’ uncertainty. Without loss of generality, we let $Eg = 1$. We shall establish that optimal investment requires

$$\frac{\partial \pi_t}{\partial K_t} = r \frac{EU'_T}{EU'_T g} \equiv \gamma. \quad (17)$$

The right-hand side of (17) is the appropriate ‘cost of capital’ (cf. Diamond, 1967; Modigliani and Miller, 1958) for a risky firm. In the special case of $E(g-1)^2 = 0$, γ reduces to r , the cost of capital for a safe firm. In general, provided the firm’s return is positively correlated with all other sources of income, $\gamma > r$.

To establish (17), simply consider the effect of a perturbation on the investment path of the sort described in the previous section. The firm increases K_t keeping K at all other dates constant, i.e. it increases I_{t-1} and reduces I_t by a unit. To finance this it increases B_{t-1} by a unit. In the absence of taxation, if the net profits are invested in (or net losses financed by) a $T-t$ period bond, c_T changes by (in state θ)

$$\left[\frac{\partial \pi}{\partial K_t} g(\theta) - r \right] (1+r)^{T-t} \quad (18)$$

³⁵ The reason that these assumptions are made is that in the absence of a full set of markets for contingent commodities (i.e. a complete set of Arrow–Debreu securities) there may be disagreements over the desired objective of the firm; firm value maximization today may lead to a different policy than firm value maximization in the future; and if the individual plans to sell shares in the firm, he will have to take into account the expectations of returns to the given project by other investors, which may differ from his own expectations. See Stiglitz (1972c, d).

or expected utility increases by

$$(1+r)^{T-t} EU'_T \left\{ \frac{\partial \pi}{\partial K} g-r \right\} . \quad (19)$$

If (19) is positive, it is clearly profitable to expand I_t , and if it is negative, to contract I_t ; optimality requires

$$EU'_T \left\{ \frac{\partial \pi}{\partial K} g-r \right\} = 0$$

which is equivalent to (17).

With taxation, after tax profits at time t change by (in state θ)

$$(1-\tau_c) \left(\frac{\partial \pi}{\partial K} g-r \right) .$$

Clearly, we do not need to distribute the 'profits' at time t when they are earned; as the analysis of section 2 established; we could, for instance, capitalize them at time 0 or postpone the realization of these returns until time T , borrowing on the value of the unrealized capital gain. Of course, since at the optimal point, a small perturbation results in a negligible change in profits, for characterizing the optimal investment policy, we need not worry about when 'profits' are 'realized' by shareholders when there is no uncertainty. With uncertainty, however, in some states of nature there may be a loss, in others a profit. If before the perturbation $d_t = 0$, since the tax treatment of $d_t > 0$ is different that $d_t < 0$, we must be more careful.

Consider first the case of the high equity firm; its optimal policy, we described in section 2 required $d_t = 0$ for all t except $t = T$. Thus the increases in profits at time t are in effect invested in a $T-t$ period bond and losses are financed by a $T-t$ period bond.³⁶ At date T , the bonds are cashed (the debts are paid) and the returns distributed. Then d_T changes by

$$(1-\tau_c) \left(\frac{\partial \pi}{\partial K} g-r \right) (1+r)^{T-t}$$

³⁶ This is equivalent to a particular sequence of changes in B_t for $t \leq T$.

and since we have ruled out the possibility of bankruptcy, this must be positive in all states of nature; hence c_T changes by

$$(1-\tau_c)(1-\tau_d) \left(\frac{\partial \pi}{\partial K} g-r \right) (1+r)^{T-t}.$$

We thus obtain exactly the same optimality conditions as without taxes. (Obviously, there are 'income effects' from the taxes, i.e. U' in different states is changed, and hence γ is changed.)

The analysis of the high debt firm is somewhat more difficult; for we argued earlier that in the case, we could not ignore the problem of bankruptcy. Let us consider the special case discussed at some length in section 2.6 of a firm which wishes at each state to maximize its debt subject to the no bankruptcy condition constraint. In that case, the shadow price of investing retained earnings is different from the cost of raising outside capital.

Assume the state of nature turns out other than the 'worst' state; it will then be possible, as we argued earlier, for the firm to distribute some returns to its shareholders. Assume that it reduced the distribution by a dollar and invested the dollar in a safe investment, yielding a return $1+\hat{r}$ before tax, $1+\hat{r}(1-\tau_c)$ after the corporation profit tax, and $(1-\tau_d)(1+\hat{r}(1-\tau_c))$ after personal taxes. If instead, the funds had been distributed the first period, and invested by the individual, he would have at the end of the period

$$(1-\tau_d)(1+r(1-\tau_p)).$$

Thus equilibrium requires

$$\hat{r} = r \left(\frac{1-\tau_p}{1-\tau_c} \right). \quad (20)$$

This is not a very large distortion: the cost of capital is greater than the rate of interest, but not, as in some of the earlier analyses, by a factor of $1/1-\tau_c$ but by $1-\tau_p/1-\tau_c$. Again, the error in the conventional analysis follows from a failure to integrate the corporate tax structure with the personal tax structure.

The distortion is even smaller if when the firm distributed the returns to the shareholders the first period, the shareholders would have had to have paid ordinary income rates on it, but by investing it, and selling

the incremental value of the firm to others, they manage in effect to get capital gains treatment of the returns. Let τ_g be the tax rate on capital gains. Assume the firm invested the dollar in an infinitely lived asset that yielded an annual return of \hat{r} . Thus, the present discounted value of the after tax return, using the marginal rate of substitution, $r(1-\tau_p)$, is

$$\frac{\hat{r}(1-\tau_c)}{r(1-\tau_p)}.$$

After paying capital gains tax, the individual would receive

$$\frac{\hat{r}(1-\tau_c)(1-\tau_g)}{r(1-\tau_p)};$$

the investment should be undertaken if this is greater than what he would receive if the firm did not undertake the investment, which is just $1-\tau_p$. The cost of capital, in other words, is just

$$r \frac{(1-\tau_p)^2}{(1-\tau_c)(1-\tau_g)}. \quad (21)$$

Assume $\tau_c = 0.5$, $\tau_p = 0.4$ and $\tau_g = 0.2$. Thus, while the conventional rule (eq. (12)) says the cost of capital is $2r$,³⁷ and while (eq. (20)) where the favorable treatment of capital gains are not fully taken into account, says that the cost of capital is $1.2r$, (21) says that the cost of capital is $1.125r$, a relatively small distortion. Indeed, if $\tau_g = 0.5$, τ_p and $\tau_c = 0.5$, then the cost of capital equals the rate of interest if $\tau_p = 0.36$, and if τ_p is less than 0.36, the cost of capital is less than the rate of interest.

On the other hand, if the firm is raising new capital, and must have a marginal debt–equity ratio of $\alpha/1-\alpha$, the net return to investing one dollar after tax is

$$\alpha[r+(\hat{r}-r)(1-\tau_c)](1-\tau_p) + (1-\alpha)\hat{r}(1-\tau_d)(1-\tau_c).$$

³⁷ Since none of the incremental investment is financed by debt.

Again, if this equals $r(1-\tau_p)$,

$$\hat{r} = \frac{r(1-\tau_p)(1-\alpha\tau_c)}{\alpha(1-\tau_p) + (1-\alpha)(1-\tau_d)(1-\tau_c)}. \quad (22)$$

To return to our numerical example, with $\tau_p = 0.4$, $\tau_d = 0.2$ and $\tau_c = 0.5$, the cost of capital is just equal to the rate of interest at a marginal debt–equity ratio of 2/3, i.e. 40% of the marginal capital is raised by debt. For higher debt equity ratios, the cost of capital is even less than the rate of interest.

In the case of both (21) and (22) we noted the possibility that the cost of capital could be less than the rate of interest, because of the favorable treatment of capital gains. One should be careful in interpreting what this implies for the investment policy of the firm. Since the firm has the option of investing in financial assets rather than real assets, the firm would never undertake an investment at a return less than r . It does imply, however, say in the case of retained earnings, that if there are not profitable real investment opportunities, the firm will hold financial assets. The regulations and tax provisions discussed earlier pertaining to undistributed profits, particularly with respect to closely held firms, may however make it impossible for it to invest in financial assets, in which case the required rate of return on real investment is less than r .

What are the implications of these results for the marginal rate of return in the corporate sector as a whole? If everyone were in the same tax bracket, then we have delineated three possibilities: if $\tau_c < \tau_p$, so the firm has a low debt equity ratio and bankruptcy is not a constraint, the cost of capital is just equal to the before tax cost: there is no distortion. If the bankruptcy constraint is binding as it will normally be if $\tau_c > \tau_p$ and firms can invest in financial assets as well as real assets, then the cost of capital for retained earnings will in general be different from the cost of capital for new funds; if firms can invest in financial assets as well as real assets, then the marginal return to capital will be greater than or equal to the rate of interest, but for tax rates of many wealth-holders, the distortion is very small. If firms cannot invest in financial assets, it is even possible for the marginal return in the corporate sector to be less than the rate of interest.

A full general equilibrium analysis with many individuals in different tax brackets would take us beyond the scope of this paper. We merely suggest here that the marginal return for the corporate sector as a whole

will be equal to the lowest return demanded by any group. In that case, so long as the government allows closely held firms to invest their retained earnings in financial assets, and so long as there are firms pursuing a low debt equity policy then the marginal return in the corporate sector will be just equal to the rate of interest: the corporate profits tax is non-distortionary, with respect to its impact on the allocation investment. The consequence of the provisions not allowing firms to invest their returns in financial assets is to create a distortion in which the return in the corporate sector may be *lower* (not higher) than in the unincorporated sector.

4. Concluding comments

To analyze the effect of the corporate profits tax on the financial structure and the cost of capital of firms, one must include a full analysis of all the relevant provisions of the personal as well as the corporate tax code. The optimal financial policy which emerges from such an analysis is basically in accord with what one observes. Since the tax advantages of debt depend on the relative tax savings on personal borrowing versus corporate borrowing, the desirability of a high debt policy depends simply on whether the personal tax rate is greater or less than the corporate tax rate. When the personal rate is higher than the corporate rate, it pays to finance as much investment as possible through retained earnings, but the excess of investment over retained earnings is financed by debt. Although the tax savings on personal borrowing are greater than on corporate borrowing, it does not pay the individual to borrow to finance the investment through equities: for he must then pay not only the corporate profits tax on the total return, but he must also pay a capital gains tax when the profits are distributed, and the tax savings from personal borrowing is in general less than the increased corporate profits and capital gains tax. The actual debt equity ratio is the fortuitious outcome of the profit and investment history of the firm. Moreover, changes in the tax rates – so long as the appropriate inequalities (e.g. $\tau_p > \tau_c$) continue to hold – have no effect on the optimal financial policy; this, of course, is consistent with what many observers have claimed to be the case.

Although the tax structure does have an effect on the financial structure, since in the absence of taxes, the financial structure is of no relevance, there is not necessarily any economic significance to this change

in financial structure. In the absence of bankruptcy the optimal investment decision of the firm – whether for safe or risky assets – remains unaffected by the tax structure, and there is no inter-sector inefficiency resulting from the imposition of the corporate profits tax with the interest deductibility provision. Nor is there any misallocation between safe and risky industries. From an efficiency point of view, the whole corporate profits tax structure is just like a lump sum tax on corporations.³⁸ Those studies such as that of Harberger (1962), which treat the corporate profits tax as if it were a differential tax on capital on the corporate sector, and thus argue that it introduces an inefficiency, make the same error that Jorgenson and the other investment studies make: they confuse the average with marginal cost of capital.

Whether our tax code was deliberately designed not to interfere with economic efficiency is a moot question. The tax structure probably has as much to do with equity considerations as efficiency considerations. A reexamination of our argument will show, moreover, that for the case of $\tau_p > \tau_c$ this efficiency result does not depend on the particular rates being levied, in particular, on the favorable rates on capital gains. It does depend critically on the interest deductibility provisions and on the fact that capital gains are taxed only upon realization. Thus, what our analysis does make clear is that the lower rates paid on capital gains cannot be justified on efficiency grounds: if it is to be justified, it must be purely on grounds of equity.

Although our analysis has resolved the problem of the optimal debt equity ratios, there is another aspect of firm financial policy which is not adequately treated: what determines the amount firms pay out in dividends? As we have noted, it is obviously advantageous to retain earnings for investment or to buy back shares. We have noted a number of provisions of the tax code which limit the extent to which firms can avoid taxes by these means. We remain unconvinced that these provisions can explain the size of the dividends or the variations across firms in the amount they distribute by dividends. A full explanation may require modifying some of the assumptions underlying the conventional

³⁸ This assumes that the choice of whether the firm is to be 'incorporated' or not is not affected by these tax considerations or, if it is, that there are not important resource allocation implications of whether the firm is or is not incorporated. The former clearly is not a reasonable assumption; the latter is, since firms in both the corporate and noncorporate sector follow the same investment rule. Incorporation clearly has other effects, particularly when bankruptcy occurs. The full implications of incorporation and the economic consequences resulting from the discouragement of incorporation by the tax treatment of corporation requires further investigation.

model of the firm: transaction costs may make it less expensive to pay dividends than to buy back shares. Or changes in financial policy may affect individual expectations about the prospects of the firm; low dividend payments may, for instance, be a signal to the prospects of the firms are not very good.³⁹ These are still unresolved questions requiring further study.

References

- Baumol, W. and B. Malkiel, 1967, The firm's optimal debt-equity combination and the cost of capital, *Quarterly Journal of Economics* 81, 547–78.
- Diamond, P., 1967, The role of a stock market in a general equilibrium model with technological uncertainty, *American Economic Review* 57, 759–76.
- Domar, E. and R. Musgrave, 1944, Proportional income taxation and risk taking, *Quarterly Journal of Economics* 58, 388–422.
- Farrar, D.E. and L.L. Selwyn, 1967, Taxes, corporate policy and return to investors, *National Tax Journal* 20, 444–51.
- Harberger, A., 1962, The incidence of the corporation income tax, *Journal of Political Economy* 70, 215–240.
- Holzman, R.J., 1965, *Tax basis for managerial decisions* (New York).
- Jorgenson, D.W., 1965, Anticipations and investment behavior. In: Duesenberry et al. (ed.), *Brookings quarterly econometric model of the U.S.*
- Markowitz, H., 1959, *Portfolio selection* (New York).
- Modigliani, F. and M.H. Miller, 1958, The cost of capital, corporation finance, and the theory of investment, *American Economic Review* 48, 261–97.
- Mossin, J., 1968, Taxation and risk-taking: an expected utility approach, *Economica N.S.* 35, 74–82.
- Samuelson, P., 1964, Tax deductibility of economic depreciation to insure invariant valuations, *Journal of Political Economy* 72, 604–606.
- Stiglitz, J.E., 1969a, Effects of wealth, income, and capital gains taxation on risk taking, *Quarterly Journal of Economics* 83, 263–283.
- Stiglitz, J.E., 1969b, A re-examination of the Modigliani–Miller theorem, *American Economic Review* 59, 784–93.
- Stiglitz, J.E., 1971, Perfect and imperfect capital markets, *Meetings of the Econometric Society*, New Orleans.
- Stiglitz, J.E., 1972a, On the optimality of the stock market allocation of investment, *Quarterly Journal of Economics* 86, 25–60.
- Stiglitz, J.E., 1972b, On the irrelevance of corporate financial policy, *Cowles Foundation Discussion Paper 339 and American Economic Review* (forthcoming)
- Stiglitz, J.E., 1972c, Some aspects of the pure theory of corporation finance bankruptcy and takeover, *Bell Journal of Economics and Management Science* 3, 458–483.
- Stiglitz, J.E., 1972d, Value maximization and alternative objectives of the firm (mimeo).
- Stiglitz, J.E., 1972e, Taxation, risk, and the allocation of investment in an economy with a competitive stock market. In: Jensen, M. (ed.), *Studies in the theory of capital markets* (New York).

³⁹ These issues are discussed in greater length in Stiglitz (1972b).