Some aspects of the pure theory of corporate finance: bankruptcies and take-overs

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This paper considers the implications of bankruptcies, take-overs, and divergent expectations for the financial policy of the firm; we argue that, under reasonable assumptions, there is an optimal debt-equity ratio. Previous studies have shown that under very general conditions, if there is no chance of bankruptcy, then financial policy has no effect on the value of the firm; there is no optimal debt-equity ratio. Under certain very restrictive conditions, the no-bankruptcy condition may be removed. We show that when these restrictive conditions are not satisfied, and when there is a real possibility of bankruptcy if the firm issues too much debt, the firm's valuation will depend on its debt-equity ratio; the real decisions of the firm (e.g., its investment and choice of technique) cannot be separated from its financial decisions; and the real decisions of the firm may not be productively efficient. Finally, the implications of the possibility of a take-over for the financial policy of the firm are considered.

1. Introduction

This paper argues that, under reasonable assumptions, there is an optimal debt-equity ratio. Previous studies have shown that, under very general conditions, if there is no chance of bankruptcy, then financial policy has no effect on the value of the firm; there is no optimal debt-equity ratio. Under certain very restrictive conditions, the no-bankruptcy condition may be removed. But what happens when these restrictive conditions are not satisfied, when there is a real possibility of bankruptcy, if the firm issues too much debt? We shall show that the firm's valuation will depend on its debt-equity ratio (Section 3); that there will be, as a consequence, an optimal debt-equity ratio (Section 4); that the real decisions of the firm (e.g., its investment and choice of technique) cannot be separated from its financial decisions (the two must be made simultaneously); and that the real decisions of the firm may not be productively efficient.

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1 See Modigliani and Miller [14] and Stiglitz [21, 24].
In addition to its treatment of bankruptcy, the detailed example considered in this paper differs in one other respect from most other recent models of capital markets: it explicitly takes into account differences in expectations. In our view, the assumption of identical expectations is unrealistic, some of its implications are untenable, and it leaves unexplained some important phenomena of the capital market. One of these, take-overs, will be discussed in some detail in Section 5.

The reason that the assumption of identical expectations has been so extensively employed is partly that it is apparently difficult to introduce heterogeneous expectations into such models and still obtain simple results, partly a feeling that one can explain too much too easily in terms of differing expectations, and partly that many of the phenomena which differing expectations might be used to explain can be explained in other terms. For instance, differences in portfolios held by different individuals can be explained by differences both in attitudes towards risk and in expectations. Both elements are difficult to observe directly; thus it is difficult to ascertain the relative importance of each.

Earlier literature tended to focus on differences in expectations, more recent literature on differences in attitudes towards risk. The reason that different models focus on one or the other is not that either is the “true explanation”; rather, to understand how each of them works they are best studied in isolation—for example, to understand the effects of differences in attitudes towards risk on portfolio allocation we assume that individuals have the same expectations, and conversely. In the example which we shall present later, we assume that all individuals have the same risk attitude, and for simplicity we assume risk neutrality.

In this section, we attempt to define what we mean by bankruptcy and to suggest why it plays an important role in the analysis of firm behavior.

Definition of bankruptcy. In a two-period model in which the firm invests in the first period and makes its return in the second period (and thereupon dissolves), bankruptcy is easy to define: the income of the firm is less than the fixed obligations to bondholders.

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2 In particular, those growing out of the Sharpe-Lintner model. See [13], [15], and [18].

3 Not only do the mean-variance models with identical expectations imply the irrelevance of financial policy, but they imply that the reason that financial policy is irrelevant is that there is a single mutual fund containing all corporate bonds and stocks. As in the Arrow-Debreu model (see [2] and [4]), the only risks that would not be diversified out would be risks such as those associated with the business cycle. The value of a firm formed by the merger of two other firms would be just the sum of the values of the constituent firms. For a more extended treatment of these points, see [23].

4 Cf. Lintner [12].

5 It is interesting to note that both Keynes, in explaining the liquidity preference schedule, and Knight [11], in explaining the continuation of entrepreneurship in spite of its apparent zero return, placed heavy emphasis on the role of differing expectations. See also Tobin [28].
In an ongoing firm, however, income can be less than the obligations of the firm and yet the firm is not bankrupt; it can simply borrow more. Clearly, a firm is bankrupt if the value of its equity is zero (it cannot be negative under conditions of limited liability), or equivalently, if the value of its future income streams, assuming it does not go bankrupt, is less than the value of its outstanding debt. It is this definition which I used in my proof of the irrelevance of financial policy in a multi-period model. (The argument required that no individual believed that in any state of nature at any date the value of equity would be zero.) Two questions are raised by this definition. First, should the debt be evaluated at market prices or at maturity values? One can imagine a situation where market prices exceeded maturity values and the firm was able to meet its debt obligations in the next period when they became due. Is it possible that the value of debt at market prices equals or exceeds the value of the firm in this period, and hence the value of the equity of the firm is zero? The answer is no; since in the next period, in all states of nature, there will be a residual left-over for the shareholders, the value of the equity in this period must be positive, and hence the value of the firm in this period must exceed the value of bonds (at market prices) in this period. Second, although this is a sufficient condition for bankruptcy, is this a necessary condition? An investment plan in which there is a possibility of bankruptcy necessitates some individual believing that at some date in the future and in some state of nature the value of the firm would be less than the value of its bonds (valued as if they were “safe bonds”) were the firm not to go bankrupt today, but at the actual date of formal bankruptcy the value of the equity of the firm may be positive. The reason for this may be demonstrated by example.

Let the firm have a single asset, cash, say $975,000. The firm has outstanding $1,050,000 in bonds, each promising to pay one dollar a period from now. The interest rate is 10 percent, so the present value of the bonds is $953,637, which is less than the value of cash. The firm has a single project; it pays $1,150,000 with probability 0.25 and $960,000 with probability 0.75. There is thus a large probability that the firm will not be able to meet its debt obligations. Yet the value of the equity in this period is positive; if the market were risk neutral, the value of the equity would be $(100,000 × 0.25)/1.1 = $22,727. The expected return to the bondholders, however, is not $1,050,000 but only $1,050,000 × 0.25 + $960,000 × 0.75 = $982,500 with a present value of $(982,500)/1.1 = $893,181, which is much less than the value of the bonds today if they were redeemed in this period (i.e., $953,637). One might imagine a debt contract in which, if the bondholders could show that the policies of the firm would with significant probability

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6 If there are Arrow-Debreu securities, we can easily calculate the value of future income streams; however, if there are not this may be hard to ascertain.

7 See Stiglitz [21].

8 In [21] we show that if the value of equity is not zero at any date in any state of nature then the value of equity plus the market value of bonds of the firm is independent of the financial policy of the firm.

9 I am indebted to Robert Merton, both for originally raising this question and for providing this example.
result in a default of their bonds in the near future, they could declare the firm “bankrupt” in this period. In that case, the above firm could be declared bankrupt in this period even though after bankruptcy the value of the equity is positive ($975,000 - $953,637 = $21,363).

Bondholders may not be able to make the firms declare bankruptcy unless the firm has defaulted on their obligations to them; that is, even though there is a significant prospect of the firm’s not being able to meet its debt obligations in the future and indeed, even though the value of the assets of the firm today are less than the present value of its outstanding debt, the bondholders cannot force bankruptcy today. For instance, as a slight modification of the above example, assume that the firm had only $800,000 cash but investment opportunities still yielded $1,050,000 with probability 0.25 and $960,000 with probability 0.75. The present value of debt obligation ($953,637) is greater than the cash (the capital of the firm), so one might say that the firm has negative net worth, i.e., is “bankrupt”. Indeed, one might even observe that if the market were risk neutral, the present value of the entire income of the firm

\[
\frac{960,000 \times 0.75 + 1,050,000 \times 0.25}{1.1} = 915,908
\]

is less than the present value of the outstanding debt obligations. Yet the value of the equity is the same as in the previous example, $22,727, so long as the bondholders cannot force bankruptcy. (If they can, clearly the value of the equity in this example is zero.)

These examples satisfy our condition for bankruptcy: at some date at some state of nature the firm is unable to meet its debt obligations; the value of equity is zero. But they also make clear that our definition does not resolve the timing of bankruptcy, i.e., whether the possibility of default in the future can allow the bondholders to force a bankruptcy now. Presumably, this is a matter of how the debt contract is written. Note that this problem is not so serious with short-term bonds as it is with long-term debt. With short-term bonds the individual can decide in each period whether, in effect, to demand his cash back. With long-term bonds there may be a change in the management of the firm after the individual has purchased the bond: the new management may undertake a policy which in the judgment of the bondholder—and perhaps of the market as a whole—will lead to bankruptcy with a high probability. Yet there may be nothing the bondholder can do.\(^{10}\)

These problems are exacerbated by the clear conflict of interest between the bondholders and the equity owners. Because of the limited liability provided by bankruptcy, the firm is concerned only with the return in those states in which it does not go bankrupt.\(^{11}\)

\(^{10}\) There are, of course, certain legal constraints on what the firm can do.

\(^{11}\) Before the bonds are issued, the firm must clearly worry about returns in all states of nature, since that will affect the terms at which it can borrow; once having obtained the loans, however, the firm is only concerned about returns in those states in which the firm does not go bankrupt. (Of course, a firm which must have continual recourse to the capital market must continue to worry about returns in all states.)
For instance, in both of the examples presented above it was in the interest of the stockholders for the firm to undertake the project. If it does not, they receive in the first example $21,363, in the second nothing; if it does, they receive an amount whose present value (assuming risk neutrality) we have calculated at $22,727. In both cases the bondholders receive more if the firm dissolves in the first period, even though in one of the examples bondholders receive less than the present value of the debt obligations (but this is still larger than the present value of the expected return if the firm undertakes the project). Notice that in the first example, where the firm had $975,000 initial cash position, the expected real return to the investment was negative, (−8 percent) and yet, if the stockholders were making the decision, the project would be undertaken. It is clear that this may result in productive inefficiency.12

- **Implications of bankruptcy for financial decisions of the firm.**

Bankruptcy is important in the financial decisions of the firm in three respects. First, no bankruptcy means that the nominal rate of interest which the firm must pay on its bonds is independent of its debt-equity ratio. If there is a chance of bankruptcy, bonds become risky assets; the nominal rate of interest13 will rise as the firm borrows more. There is no reason to suppose14 that the nominal rate should be the same function of the debt-equity ratio for all firms or individuals; indeed, quite the contrary. A complete theory must provide an explanation of the determination of the nominal rate for any given firm. We shall attempt to do this below.

One feature of the bond which is important in determining the optimal debt-equity ratio should be noted here: the bond is a promise to pay a fixed amount provided the firm does not go bankrupt, and to pay a pro rata share of the assets of the firm if it does. The borrower and the lender may, and in general will, have different attitudes about the mean and variance of the return to a risky bond. Borrowers usually will be more confident of repaying. From their view, the mean amount they are paying is larger than the mean amount the lender believes he is receiving (and the variance is smaller); the divergence between the two increases as the probability of bankruptcy, in the judgment of the lender, increases, (as the debt-equity ratio increases). In other words, the nominal interest rate which the borrower has to pay rises faster as the debt-equity ratio increases than the borrower feels is "justified."

Bankruptcy is also important to the financial decisions of the firm in that, as we have shown,15 without bankruptcy the consumption opportunity sets of the individual are unaffected by the debt-equity ratio; with bankruptcy, except under special circumstances, this is no longer true.

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12 When there is a finite probability of bankruptcy, the rule of firm value maximization is not equivalent to maximizing the value of equity, and it is clearly the latter with which firms are concerned.

13 The nominal rate of interest is that rate which the firm pays in all those states of nature in which it does not go bankrupt.

14 As Modigliani and Miller seemed to have done in [14]. As we shall note below, their result, that the debt-equity ratio does not affect the value of the firm even with bankruptcy, depends crucially on this assumption.

15 In [21, 24].
Consider, for example, the standard two-period model in which the firm invests in one period and makes its returns in the second. Consider two alternative financial policies—in one the firm issues B bonds, in the other no bonds (and hence there is no chance of default). It is easy to see that the individual buying stock on margin in the latter case can exactly replicate (if the value of the firm remains unchanged) the returns in the former case in those states of nature where the firm does not go bankrupt. But if the firm goes bankrupt in some state, \( \theta' \), in the one case his return is zero, while in the other his return (after repaying his debt) is negative. In other words, the firm which issues a risky bond is issuing two securities. When the firm does not go bankrupt it is only issuing one security not already provided on the market; the bonds it issues are just like those of any other firm. Bankruptcy may, by creating a new security, change the consumption opportunity set of society.

The conditions, then, under which the possibility of bankruptcy does not affect the market valuation of a firm are of two kinds:

1. Where bankruptcy does not in fact result in a new security, and
2. Where the creation of the new security by bankruptcy makes no difference.

Condition (1) will occur under the following conditions:^16

1. If there already exist as many securities as states of nature (the Arrow-Debreu model, which has been discussed elsewhere and shown to be “unrealistic”).^17
2. If the particular securities created by bankruptcy could have been created by individuals. This will be the case if individuals can purchase securities on margin^18 and create limited liability arrangements whereby (again in our two-period model) if profits of the firm are less than the nominal payments due on the loan, the shares are defaulted without further consequences to the individual.

It should be clear, however, that the scope for such limited liability arrangements is limited, and thus the presumption that bankruptcy does result in a new security remains.^19

There is only one general situation in which the new securities created by bankruptcy make no difference; that is, when all individuals purchase the risky securities in the same proportion in their portfolio. The conditions under which this can occur are given by the so-called portfolio separation theorem and have been discussed elsewhere.^20 The theorem requires very restrictive conditions on either the utility function and/or the distribution of

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^16 A third, but unlikely case, arises if there are a large number of firms with perfectly correlated returns (i.e., in the terminology of Modigliani and Miller, they belong to the same risk class). Then if it should happen that there is a firm in the same risk class at the debt-equity ratio our given firm is contemplating, our firm's decisions to choose that debt-equity ratio will not change the set of available securities, even if it involves bankruptcy.

^17 See [22] and [23].

^18 That is, borrow funds using the securities as collateral.

^19 Transactions and information cost are clearly important in explaining why conditions (1a) and (1b) are not satisfied.

^20 See Cass and Stiglitz [3].
returns of the set of available securities. In addition, it requires that all individuals have the same expectations about the returns to the firm. It is the latter assumption that we find most objectionable.

Finally, transactions costs and information costs associated with bankruptcy also will make the market valuation of the firm depend on the probability of bankruptcy. In the subsequent discussion we shall ignore these costs—in spite of their importance—because we wish to establish that even within the “idealized” world of Modigliani and Miller in which there are no transactions costs, market valuations will in general depend on the debt-equity ratio if there is a finite probability of bankruptcy.

3. The optimal debt-equity ratio with given investment

In this section we shall show, by means of a simple example, the dependence of the firm valuation on its financial policy. We postpone until the next section the derivation of the optimal financial policy.

Assume that there are two groups of individuals, denoted by superscripts $a$ and $b$. There are two firms in the economy, denoted by subscripts 1 and 2. Group $a$ originally owns all the shares in firm 1, the profits of which both groups agree are risky. Similarly, both groups agree that firm 2 is perfectly safe; it has a constant returns-to-scale technology. This fixes the rate of interest on a perfectly safe asset at $r^*$. We employ the conventional two-period model; investment occurs in the first period, returns occur in the second, and the firm then “dissolves”. Our interest here is on the financial decisions, so we assume that the real decisions, i.e., the levels of investment, are already fixed.

We focus our attention on the first firm. It has a return in state $\theta$ of $X(\theta)$. The two groups of individuals disagree on the probability of any given state's occurring. We define $F(X)$ as the distribution function of $X$, in group $i$'s judgment. It is convenient to define $\pi(X) = 1 - F_i(X)$. Since bankruptcy, in a two-period model, occurs when debt obligations exceed income, i.e., when $X(\theta) \leq (1 + \hat{r}(B))B$ where $\hat{r}(B)$ is the nominal interest rate that firm 1 pays if it issues $B$ bonds, $\pi_i[(1 + \hat{r})B]$ is the probability that the firm will not go bankrupt, in $i$'s judgment.

We define

$$\mu_{X \geq Y}^i \equiv \int_Y X dF_i(X).$$

(1)

$\mu_{X \geq Y}^i/\pi_i(Y)$ is the mean of $X$ conditional on $X$ being greater than $Y$ as viewed by group $i$. In other words, $\mu_{X \geq (1 + \hat{r})B}/\pi_i((1 + \hat{r})B)$ is the mean return of the firm when it does not go bankrupt.

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21 All of the members of each group are assumed to be identical.

22 For some purposes it may be preferable to think of these as two industries, since we shall assume that they act competitively.

23 For the moment, we need not specify how the initial ownership claims are distributed among the members of firm 1.

24 Provided there is any investment in the second industry at all.

25 As set forth, for example, in [24].

26 We accordingly drop the subscript 1 on $X_i(\theta)$, $\hat{r}_i$, etc.

27 See p. 459 above and [24].
For simplicity, we assume that members of both groups are risk neutral, and that individuals are not allowed to sell shares short. We assume that for all \( Y, \mu_{X > Y}^X > \mu_{X > Y}^Y \), i.e., the first group, the original owners of the \( i^{th} \) firm, have unambiguously more optimistic expectations about the returns to the firm than do the other group. The return to the shareholders after paying the bondholders is simply
\[
X^i(\theta) - (1 + \hat{r})B \quad \text{if} \quad X^i(\theta) > (1 + \hat{r})B, \quad \text{and} \quad (2)
\]
where
\[
\hat{r} \geq r^*.
\]
The mean return is then (in \( i^{th} \)'s judgment)
\[
\mu_{X \geq (1 + \hat{r})B}^i = (1 + \hat{r})B\pi^i((1 + \hat{r})B) \quad (3)
\]
and the value of the equity, \( E \), which is entirely owned by \( a \), is simply
\[
E = \frac{\mu_{X \geq (1 + \hat{r})B}^i - (1 + \hat{r})B\pi^i((1 + \hat{r})B)}{(1 + r^*)}. \quad (4)
\]

We must now determine the function \( \hat{r}(B) \); i.e., how does the nominal rate depend on the borrowings of the firm? The second group is risk neutral. Therefore, all that it requires is that (in its judgment) it receives the same average return as on a safe asset \( (r^*) \). The return on the risky bond is
\[
(1 + \hat{r}) = \begin{cases} 
1 + \hat{r} & \text{if} \quad X \geq (1 + r)B \\
X/B & \text{if} \quad X < (1 + r)B.
\end{cases} \quad (5)
\]
Thus, letting \( \delta X^b \) be \( b^{th} \)’s expectation of \( X \),
\[
\delta[(1 + \hat{r})B] = (1 + \hat{r})B\pi^i((1 + \hat{r})B) + \delta[X^b|(X(\theta) < (1 + \hat{r})B)]
\]
\[
= (1 + \hat{r})B\pi^i((1 + \hat{r})B) + \delta X^b - \mu^b_{X \geq (1 + \hat{r})B}
\]
\[
= (1 + r^*)B. \quad (6)
\]

Adding (4) and (6), we obtain
\[
V = E + B = \frac{\mu_{X \geq (1 + \hat{r})B}^i - (1 + \hat{r})B - (\pi^a - \pi^b)(1 + \hat{r})B + \delta X^b}{1 + r^*}. \quad (7)
\]
If the two groups have identical expectations:
\[
\pi^a((1 + \hat{r})B) = \pi^b((1 + \hat{r})B) \quad (8a)
\]
and
\[
\mu_{X \geq (1 + \hat{r})B}^a = \mu_{X \geq (1 + \hat{r})B}^b \quad (8b)
\]
for all \( B \), in which case the value of the firm reduces simply to
\[
V = \frac{\delta X^b}{1 + r^*} = \frac{\delta X^a}{1 + r^*}, \quad (9)
\]
so the value is independent of the debt-equity ratio, as we have already argued it would be in this case (since this is just a special form of the mean-variance model).
But note what happens when the two groups differ in their
expectations. As long as there is no bankruptcy:
\[
\mu^a_{X > (1 + \hat{r})B} = \delta X^a
\]
and
\[
\mu^b_{X > (1 + \hat{r})B} = \delta X^b
\]
so
\[
V = \frac{\delta X^a}{1 + r^*}.
\]
Again, the debt-equity ratio makes no difference. But once
\[
B > X^b/1 + r^* \quad \text{for some } \theta,
\]
the second group will think the firm may (with a positive
probability) go bankrupt. The derivative of the value of the firm with
respect to a further increase in the debt is given by
\[
(1 + r^*) \frac{dV}{dB} = \left[ \mu^a_{X > (1 + \hat{r})B} - \mu^b_{X > (1 + \hat{r})B} \right]
\]
\[- \left( \pi^a - \pi^b \right)(1 + \hat{r})B - \left( \pi^a - \pi^b \right) \frac{d(1 + \hat{r})B}{dB}.
\]
Under our assumptions, the first group is more optimistic than
the second, so that even when the second group thinks there is a
positive probability of bankruptcy, the first group may not. In
particular, defining
\[
X^a_{\min} = \min_{\theta} X^a(\theta),
\]
if
\[
X^a_{\min} > (1 + \hat{r})B > X^b_{\min},
\]
the first group thinks that there is no chance of bankruptcy while
the second group thinks that there is. Then, at the point where
\[
B(1 + r^*) = X^b_{\min},
\]
\[
\frac{dV}{dB} = (X^b_{\min} - (1 + \hat{r})B)f^b(X^b_{\min}) \frac{d(1 + \hat{r})B}{dB} = 0,
\]
where \(f^b(X^b_{\min})\) is the probability of the state where \(X = X^b_{\min}\) (in
the second group’s estimation). But
\[
\frac{d^2V}{dB^2} = - f^b(X^b_{\min}) \left( \frac{d(1 + \hat{r})B}{dB} \right)^2 < 0.
\]
Thus the value of the firm with fixed investment will decrease as
it increases its debt beyond that point where the lenders think
there is any chance of bankruptcy.

The result that the value of the firm will decrease if the firm has
a high debt-equity ratio—sufficiently high that in the view of
lenders there is a positive probability of bankruptcy—was derived
assuming that there were no real costs involved in bankruptcy.
Taking these costs into consideration would reinforce our result.
On the other hand, the creation of a “new security” (the risky
bonds) might increase the value of the firm if individuals were
risk averse (rather than, as we assume here, risk neutral). It is not
because the interest rate is an increasing function of the debt-equity ratio that the value of the firm decreases as it issues more bonds, but because there is a divergence in the estimation of the chances of bankruptcy between the lender and the borrower. What limits the amount the borrower borrows is that the amount which the lender must be compensated, using his probabilities, to leave himself indifferent is more than the borrower gains from borrowing, where the borrower is using his own probabilities.

In the previous section, we assumed that the level of investment (output) of the risky firm was already fixed, and focused on the financial decision alone. Now that we know that the financial decision may affect the valuation of the firm, it becomes clear that the financial decision may affect the level of investment of the firm. We shall show that maximization of earnings per dollar invested yields both an optimal debt-equity ratio and an equilibrium level of investment in the firm, and that the two decisions cannot in general be separated. In addition, it is noted that the economy may not be productively efficient.

Constant returns to scale. We assume that there are stochastic constant returns to scale in our first firm: doubling the inputs doubles the output in every state of nature. We further assume that each of the two groups has a fixed amount to invest, \( I^1 \); we are concerned with the competitive allocation of this investment. The representative firm in the first industry decides on what percentage of its investment to finance by means of debt. From the analysis of the previous subsection, it should be clear that the nominal rate of interest is simply a function of that percentage, since doubling the number of bonds and doubling the level of investment leaves unchanged the return per bond in any state of nature. A competitive firm believes that it can double its value if it doubles its size, financing a fixed proportion of its growth by bonds and the rest by equity. Hence, because of our assumption of constant returns to scale, competitive equilibrium requires the total value of the firm to be equal to its level of investment. Otherwise the firm would expand or contract.

From our present point of view, however, we are primarily concerned with the choice of a debt-equity ratio. The firm wishes to maximize the expected return per dollar invested by its equity owners, \( \mathcal{E}e(\theta) \), where

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28 In the discussion at Hakone, several participants in the seminar suggested that what was crucial in my analysis was not the bankruptcy assumption but the assumption that the nominal rate of return was an increasing function of the debt-equity ratio. I hope that the above argument convinces the reader that the latter by itself is not sufficient. Moreover, I have not been able to find any convincing argument why the nominal rate of interest should be increasing apart from the considerations discussed here.

29 The meaning of competitive equilibrium in the presence of uncertainty is not unambiguous, as discussed at length in [22]. As pointed out there, the particular approach taken is, in general, not completely satisfactory, since it does not seem applicable to cases where there are no constant returns to scale (or at least multiplicative uncertainty) and where there is a choice about projects (alternative distributions are feasible).
\[ c'(\hat{\theta}) = \frac{X'(\theta) - (1 + \hat{\rho})B}{E} = \begin{cases} \frac{x' - (1 + \hat{\rho})z}{1 - \alpha} & \text{if } x' > (1 + \hat{\rho})\alpha \\ 0 & \text{otherwise} \end{cases} \]  

(13)

where

\[ \alpha = B/I, \]  

(14)

the proportion of investment financed by bonds, and

\[ x' = X'/I', \]  

(15)

the return per dollar invested. The probability of not going bankrupt is simply a function of \( \hat{\rho} \) and \( \alpha \). We thus define \( \mu_{x \geq (1 + \hat{\rho})z}^d \) and \( \pi'(1 + \hat{\rho})z \) just as in Section 3 [equation (11)]. If \( G'(x) \) is the distribution of \( x \) in group \( i \)'s estimation, \( \pi'(1 + \hat{\rho})z = 1 - G'(1 + \hat{\rho})z \) is the probability that the firm with debt-equity ratio \( \alpha/1 - \alpha \), paying a nominal interest rate of \( \hat{\rho} \), will not go bankrupt, in group \( i \)'s estimation,

\[ \mu_{x \geq (1 + \hat{\rho})z}^d = \int_{x(1 + \hat{\rho})z}^{\infty} x dG'(x). \]

\[ \mu_{x \geq (1 + \hat{\rho})z}^d/\pi'(1 + \hat{\rho})z \]  

is the mean return to the firm when it does not go bankrupt. The firm wishes to maximize

\[ \mathbb{E}c'(\hat{\theta}) = \frac{\mu_{x \geq (1 + \hat{\rho})z}^d - (1 + \hat{\rho})z}{1 - \alpha}; \]

that is,

\[ \mathbb{E}c'(\hat{\theta}) = (1 + \hat{\rho}) + \frac{d\hat{\rho}}{dz}. \]  

(16)

The value of \( \alpha, \alpha^* \), satisfying (16) is the optimal value of \( \alpha \), and

\[ \frac{\alpha^*}{1 - \alpha^*} \]

is the optimal debt-equity ratio.

Equation (16) has a natural interpretation: the average return to equity investment (equals marginal return if equity and debt are expanded proportionately) must equal the marginal cost of borrowing (i.e., of increasing the proportion financed by debt).

If the opportunity alternative to investing in the risky bonds of the first firm for the \( b \) group is to buy safe bonds (invest in the safe industry), and if \( b \) is risk neutral, then \( \hat{\rho} \) must be such as to make the mean return from holding the first firm’s risky bonds equal to \( (1 + r^*) \). Denoting the mean return on the risky bonds by \( \hat{\rho} \) (in group \( b \)'s estimation):

\[ (1 + \hat{\rho}) = (1 + \hat{\rho})_n^b + \frac{\mu_{x \geq (1 + \hat{\rho})z}^b}{\alpha}. \]  

(17)

Then \( \hat{\rho} \) must be such as to make

\[ (1 + \hat{\rho}) = (1 + r^*). \]  

(18)

Hence

\[ \frac{d\hat{\rho}}{dz} = \frac{\mu_{x \geq (1 + \hat{\rho})}^b}{\pi^b \alpha^2}. \]  

(19)
Substituting (15) and (17)–(19) into (16), and letting \( y = (1 + \hat{r})x \), we obtain
\[
\mathcal{E}x^a = (1 + \hat{r}) + \frac{d\hat{r}}{dx} \alpha = \frac{(1 + r^*)}{\pi^a(y)}.
\]
(20)

Equation (20) together with (17), which may be rewritten as
\[
\alpha(1 + r^*) = y\pi^b + \mathcal{E}x^b - \mu_{x \geq y}^b,
\]
gives us two equations with two unknowns, \((y, \alpha)\). We can thus solve for the equilibrium debt-investment ratio, \( \alpha \) (or debt-equity ratio \( \alpha / (1 - \alpha) \)) and the nominal interest on bonds, \(1 + \hat{r} = y/\alpha\).

A numerical example may be helpful. Assume \( b \) believes that \( x \) is uniformly distributed over the interval \([1, 2]\). Then
\[
\pi^b = 2 - y, \quad y \geq 1,
\]
\[
\mathcal{E}x^b - \mu_{x \geq y}^b = \frac{y^2 - 1}{2}, \quad y \geq 1.
\]
Let \( \mathcal{E}x^a = 1.67, 1 + r^* = 1.5 \). Then
\[
\alpha = 0.73
\]
\[
y = 1.1
\]
\[
(1 + \hat{r}) = 1.51;
\]
that is, equilibrium involves 78 percent of investment financed by debt, a 10-percent probability of bankruptcy, and a nominal interest rate of 51 percent.

Until now, we have assumed that the second group always invests its resources either in the safe asset or in the bonds of the first firm. But they also have the option of buying shares in the first firm. This they will do if the mean return from the shares of the first firm is less than the return on the safe asset or on the bonds of the first firm. The mean return from the shares in the first firm as viewed by the second group \( \mathcal{E}e^b \) is given by
\[
\mathcal{E}e^b = \frac{\mu_{x \geq (1 + \hat{r})x}^b - (1 + \hat{r})x\pi^b}{1 - \alpha}.
\]

Subtracting (17), we observe that (after some simplification)\(^3\)
\[
1 + \hat{r}^b \geq \mathcal{E}e^b \quad \text{as} \quad 1 + \hat{r}^b \geq \mathcal{E}x^a.
\]
Thus, whether \( b \) prefers investing in the risky bonds of the firm to its shares, depends simply upon whether the bonds pay, in \( b \)'s judgment, more or less than the total mean return per dollar invested in the firm.

We now are prepared to describe all possible equilibria for this economy. First, it is clear that all the capital of the first group is invested in the equity in firms of the first industry, provided only

\(^3\) If the firm issues a sufficiently large number of bonds that even its stockholders believe that there is a finite probability of bankruptcy, then (16) becomes
\[
\mathcal{E}e(\theta) = -\left(\mu_{x \geq (1 + \hat{r})x}^a - 1\right) \frac{d(1 + \hat{r})x}{dz} \frac{d\hat{r}}{dx}
\]
with a similar interpretation to that above.
that $\delta x^a > 1 + r^a$, the return on the safe asset; \(E^a\) if $E^a$ is the value of $a$'s equity in firms in the first industry, and $I^a$ is the amount of resources $a$ has to invest, then

$$E^a = I^a.$$  

What the second group does depends on the relationship of $\delta x^b$ and $1 + r^a$.

(1) If $\delta x^b > 1 + r^a$, then all of the resources of the second group are also invested in the first industry. This will take the form of either bonds or shares. There are two cases. Define $z^*$ as the solution to (16), where $r(z)$ is given by (17) with $1 + \delta x^b$ (i.e., investors in the second group must be indifferent between holding bonds and equity):

(a) If $z^*I^b/1 - z^* < I^b$, then $B^5$, holdings of the "b" group of risky bonds, are just $B^5 = z^*(I^a + I^b)$. The amount invested in equity then will be just $(1 - z^*)(I^a + I^b)$.

(b) If $z^*I^b/1 - z^* > I^b$ then at a mean return of $\delta x^b$, there is an excess demand for borrowing; this drives up the mean return on bonds. The equilibrium then is characterized by the second group holding all of its assets in the form of bonds in the first industry, with a mean return $(1 + \delta x^b)$ between the expected return on equity as viewed by the two groups: \(\delta x^b < 1 + \delta x < \delta x^a\).

(2) On the other hand, if $\delta x^b < (1 + r^a)$, then investment in the first industry is given by

$$I^b(1 + z^*),$$

where $z^*$ is again found from (16) where $r(z)$ is given by (17) with $\delta x^b$ is the safe rate of return. Again, this result is conditional on $z^*I^a/1 - z^* < I^b$. The case where $z^*I^a/1 - z^* > I^b$ follows exactly along the lines sketched above.

The important point to observe here is the interdependence between the real allocation of resources and the financial decisions of the firm. We could not solve for the former until we described the latter. This interdependence is even clearer in the case of diminishing returns to scale.

\(\Box\) **Diminishing returns to scale.** We now assume that the returns to the risky firm are described by $X(\theta) = h(I)\phi(\theta)$, where $h' > 0$, $h'' < 0$, and $\delta \phi = 1$; i.e., there is multiplicative uncertainty but diminishing returns to scale. Bankruptcy occurs whenever $X(\theta) \leq (1 + \delta)B$; i.e., whenever

$$\frac{h(I)}{I} \leq (1 + \delta)z.$$  

\(1\) If this assumption is not satisfied, both $a$ and $b$ invest in the safe industry and the problem is uninteresting.

\(2\) The exact value is found as follows. From (17), we can solve $\delta x^b$ as a function of $\delta x^a$ and $z$. Substituting into (16), and letting $z^* = I^a/I^b$, we find that value of $\delta x^b$ which satisfies (16).

\(3\) That is, if $z^*I^a/1 - z^* > I^b$, then $1 + r^a < 1 + \delta x^b < \delta x^a$. 470 / JOSEPH E. STIGLITZ
If \( h(\hat{I})/I \) is a constant (there are stochastic constant returns to scale), the probability of bankruptcy depends simply on \( 1 + \hat{r} \): but in the case of decreasing returns to scale, at any given debt-equity ratio and nominal interest rate the probability of bankruptcy decreases with the scale of the firm. The consequence of this is that the nominal rate of return on bonds will in turn depend both on the debt-equity ratio and the scale of the firm; the firm will take this into account in making its investment decisions. The result is that the economy will not, in general, be productively efficient; that is, if in the first industry there are several different firms with different production functions,

\[
X^j(\theta) = \varphi(\theta)h^j(I),
\]

where the \( j \) denotes the \( j \)th firm in the first industry, efficiency requires that

\[
h^i = h^j
\]

for all \( i, j \) in the industry. The market equilibrium will not in general satisfy (22).

To see this, we observe that in equilibrium the return per dollar invested in the equities of all firms in the industry must be the same:

\[
\frac{\delta(X^j)^\rho - (1 + \hat{r}^j)B^j}{E^j} = \rho;
\]

the same is true for all \( j \). Assume that the original shareholders wish the firm to maximize their net worth the first period,

\[
\max_{(I^1, I^2)} \frac{\delta(X^j)^\rho - (1 + \hat{r}^j)B^j}{\rho} - (I^1 - B^j),
\]

where \( I^j \) is the value of investment, so \( I^1 - B^j \) is the new capital raised through issuing new equity, \( I^1 \) and \( B^j \) are chosen so that\(^\text{34}\)

\[
\frac{\partial(\delta X^j)^\rho}{\partial I^j} = \rho + \frac{B^j}{1 + \hat{r}^j} \frac{\partial \hat{r}^j}{\partial I^j},
\]

\[
\rho = (1 + \hat{r}^j) \left[ 1 + \frac{B^j}{1 + \hat{r}^j} \frac{d(1 + \hat{r}^j)}{dB^j} \right].
\]

Equations (23) and (24) must be solved simultaneously: the real investment decision cannot be separated from the financial decision.

Upon solving explicitly for the interest rate \( \hat{r} \) which firms have to pay as a function of both the level of investment and the level of debt, (as we did in the previous subsection) and substituting the results into (23) and (24) we can show that (22) will not in general be satisfied. The details are left to the reader.

\section*{On managers and shareholders.} In the discussion so far, we have assumed that the only nonfinancial decision is the level of investment. We now consider what happens when there is a choice of alternative projects (alternative techniques of production, alternative commodities to be produced, etc.) and what happens if there are disagreements concerning which projects should be undertaken. Our focus is on the consequent possibility of a take-

\(^{34}\) (23) and (24) together imply that the firm is also maximizing its return per dollar invested by the shareholders.
over and the implications this has for the financial decisions of
the firm. In order for a firm to make a decision, it must ascertain
(1) the probabilities of alternative states; (2) the consequences to
the market value of alternative courses of action; and (3) the avail-
able set of possible courses of actions. 35

These are all matters in which professionals (in particular,
managers) are likely to be more informed than the typical stock-
holder. As a consequence, it is difficult for the shareholder to
judge whether the manager made the "correct" decision. In the
event of an unsuccessful project, did the manager assess the prob-
abilities correctly, but have bad luck? Or was the manager inept
in judging the distribution of outcomes of the project? Or did the
manager choose the best project of the available set, but all the
available projects had low returns? Or was the manager pursuing
some objective other than that of value maximization (e.g., he was
more interested in the size of the firm)?

If we had repeated trials of the same experiment, (the world
next year is just as it was last year), where the same opportunities
were open to the firm, the manager made the same decision as
before, but the outcome, as before, depended on the throw of a
die (on the state of nature), then we could discriminate between
"bad luck" and "incompetence": on the average, the unlucky
manager would yield higher returns to his firm than the inept one.
Unfortunately, we do not have repeated trials of the same exper-
iment. Not only are there changes in the set of opportunities avail-
able to the firm, but, even if there were not, the outcome of the
previous trial will affect our estimate of the distribution of outcomes
from the available set of opportunities. Even if the managers of
the firm have had a good record in the past, the stockholder must
decide, in the instance of a bad outcome, whether this is a matter
of "bad luck," whether investment opportunities in the industry are
"drying up," or whether the manager is no longer "competent" either
due to circumstances that have changed or because the manager
has "changed" (because of old age, for example).

Two important conclusions emerge from this analysis. First,
unlike conventional neoclassical analysis, in which a firm is nothing
more than the factors of capital and land which it owns and the
book of blueprints which it confronts and in which the entre-
preneur's sole job is to find, at the given factor prices, the correct
page of the book of blueprints; here, the entrepreneur plays a
crucial role. For instance, the book of blueprints may only be
known to him; although factor prices today are given, he must
guess what they will be next period. Thus the market value of
the firm is not just the market value of its factors (apart from the
managers); it depends on the market's estimate of what the man-
agers will do with those factors as well.

Second, individuals may differ not only in their judgment of the
probability distribution of the possible outcomes of alternative
projects available to a firm but also on their evaluation of the
judgment of the managers in making the correct (the "desired")

35 If there were a complete set of Arrow-Debreu securities, as soon as one knew
the Arrow-Debreu prices, one could easily determine the consequences to the
market value and would not have to pay attention to the probabilities of alternative
states.
decisions. Indeed, dissatisfaction with the decisions of the firm is not likely to take the form of the shareholders directing the firm to take a particular alternative course of action; rather, it almost invariably consists of replacement of the management. In practice, the instigation for the replacement of a particular management will come not from the shareholders directly, but from an alternative proposed management.\textsuperscript{36}

\textbf{On the determinants of take-overs.} If you own a firm and I observe that you are making the wrong decisions (in my judgment) i.e., you are earning, on the average, a smaller return from your fixed capital than I estimate you could earn (ignoring for the moment any risk aversion), I could offer to buy you out. If this happened everywhere in the economy, all firms would be managed by the individual who (in his judgment) obtained the highest return for that particular firm. However, there are at least three limitations on this process:

(1) Even though I believe I could, on the average, earn a higher return than the present management, my estimates have greater variance than the management's estimates because the management has more "information," and therefore I do not undertake a take-over. This is a formal way of saying that although I think I can do better than the present management on the average, I am not "very sure about it."

(2) Transactions costs. This is a generic name for all the costs associated with a take-over, not the least important of which is the fact that if I fire the manager, he may take with him certain key personnel who have information and whom it would be costly for me to replace.

(3) I may not have the capital to buy a controlling interest in the firm.

Take-overs are of concern to the present shareholders of the firm for at least two reasons:

(1) The possibility of take-overs probably increases the rate of interest which the firm must pay on its bonds. When the original bondholders lend money to a firm, they take cognizance not only of the returns in the different states of nature of the firm under its present management but also of the chance of a take-over bid, a new management which would make an alternative set of decisions. They must take into account all the possible take-over bids, and the resulting dispersion in the possible returns from the firm may be very large indeed. Thus, the return which the original bondholders demand depends not only on the possibility of bankruptcy under the present managements, but also on the possibility of bankruptcy under alternative managements and the probability of these alternative managements' occurring.

\textsuperscript{36}There are good reasons for this: (1) The decision of whether a particular management had bad judgment is a relative one; it may be "bad," in the sense that each of the shareholders believes he could do better, but of the alternative management teams it may be the best; and (2) The organization of a take-over requires management itself.
(2) Although the majority of shareholders in the firm are better off as a result of the take-over—they would not otherwise have sold their shares—the valuation of the minority may be greater than the market price. They have been, as it were, enjoying some consumer surplus which they will lose if the new management changes its policies. Thus they may be worse off as a result of the take-over.

That firms may wish to avoid take-overs seems clear. The question is, what actions can they take which make it less likely that they will be taken over. Returning to our list of the limitations on take-overs, we observe that the chance of take-overs may be reduced:

(1) By taking actions which make it less likely that an alternative management can argue that it will do a better job; actions which are "risky" _ex ante_ always entail the risk of failure, and there is some tendency therefore for firms to take more conservative actions than they might if they were not faced with the possibility of a take-over.\(^{27}\)

(2) By taking actions which increase the amount that an individual or group must borrow in order to take it over, which decrease the attractiveness of the terms at which they can borrow or the return from the firm once it has been taken over. Three considerations seem to be relevant here.

(a) For any given total market valuation, the larger is the debt-equity ratio, the more likely is a take-over. To take over control of the firm, the individual needs only to raise enough capital to buy a majority share in the equity. Thus, for a firm with a given market value, a take-over bid is much easier if there is a large debt-equity ratio than if there is a small debt-equity ratio.

There is one important problem with this argument: if individual borrowing\(^{30}\) is a reasonably good substitute for firm borrowing, the smaller is the debt-equity ratio of the firm, the more the individual can borrow, to exactly offset the (lack of) borrowing of the firm. There are, however, two reasons why this is not so. Most obvious is the fact that regulations limit the amount an individual can borrow using securities as collateral. Second, bondholders are likely to demand a higher rate of interest from the individual than from the firm, and the rate they demand is likely to be an increasing function of the amount borrowed. The individual may very well have other liabilities, so that the individual may go bankrupt in some states of nature when the firm does not;\(^{39}\) offsetting this, of course, is the fact that the individual has other assets, so that he may not

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\(^{27}\)This cannot be taken to an extreme, for then an alternative management can argue that "good risks" are being passed by.

\(^{30}\)Since many take-overs are by other firms, the argument is even stronger: we simply substitute direct borrowing of the firm taking over for the "indirect" borrowing of the firm taken over.

\(^{39}\)Indeed, as a consequence, individuals may be unwilling to borrow extensively, unless the kinds of limited liability arrangements discussed earlier in the previous section exist.
go bankrupt even when the firm does. Moreover, the lender's estimate of the probability of bankruptcy of the firm depends on who is managing the firm, because now there are real consequences to the control of management. Management may very well insist on a higher nominal rate of return from the individual attempting a take-over than from the original management, for they know less about the decisions he is likely to make with respect to this given firm (even if they have high "hopes" of his doing well, they are likely to have a greater dispersion in their estimate of the outcomes, and hence a higher estimate of the probability of bankruptcy). Of course, it is possible that the individual attempting a take-over has a "good" reputation, in which case he may be able to borrow more cheaply than the firm itself for any given level of borrowing; this will facilitate take-over bids. 40

(b) If, in the view of those taking over, the nominal interest rate on outstanding loans is lower than would prevail after the take-over (for the level of debt planned by the take-over group), then the high debt-equity ratio increases the attractiveness of the take-over. If, on the other hand, the nominal interest rate is greater than would prevail after the take-over (in the take-over group's estimation) then a high debt-equity ratio (the high interest payments) discourages a take-over. 41

(c) It is likely that the larger the firm, the more difficult a take-over will be. Consider two firms identical in every respect (including debt-equity ratio) but that one is twice as large as the other. The group attempting the take-over must mobilize more resources in the latter case than in the former. In particular, the group will have to borrow more. Since the given percentage increase in the requisite debt of the take-over group constitutes an increasing percentage of its total assets (provided that the individual or firm planning the take-over originally had some assets, as it usually has), 42 the larger is the total amount it has to borrow the riskier the borrowing will be in the eyes of the lender and the higher rate it will demand. But, just as in the preceding two cases, the normal presumption may be reversed. If in the view of the lender the firm being taken over has a less risky return than the firm taking over (even taking account of the uncertainty associated with the new management) then the larger is the firm being taken over, the lower will be the interest rate at which the firm taking over

40 Indeed, many of the successful take-over bids in the United States by the big conglomerates were based on large borrowings; the market seemed to have a great deal of confidence in these managers. Subsequent events have thrown some doubt on whether this confidence was in fact justified.

41 One has to distinguish between the interest rates the take-over group must pay in order to succeed in the take-over (which involve the obligations of the company taking over) and the interest rates at which the firm which has been taken over can borrow after the take-over.

42 The valuation of the assets that is relevant here is the lender's valuation of the worth of the assets at the time that the loan comes due. The net worth of the firm taking over may (from this point of view) be negative.
can borrow, and the more attractive the investment opportunity. This is typically the case when a small firm attempts a take-over of a larger firm.

What emerges from the analysis is that there is no single policy which avoids the possibility of all take-overs; an action which thwarts one take-over bid, from a group with one set of expectations of what it can borrow at, with one set of resources, and with one set of production plans, encourages a take-over bid from another group. Nonetheless, it would seem that there is some presumption that, in general, actions which increase the value of the equity of the firm will make a take-over bid less easily managed. But even here there is an ambiguity. Actions which, in the view of the present management will result in a high market valuation (with a high probability), in the next period, thus making a take-over in the next period less likely, may decrease the market valuation in this period, thus making a take-over in this period more likely.

An example. The following example, a slight modification of that presented earlier, shows how the threat of take-over affects the actions of firms.

Assume that there are three groups of individuals. All the members of each group are identical and are risk neutral. Group \(a\) initially owns industry 1. Industry 1 has two available projects, both of which exhibit stochastic constant returns to scale but differ in the probability distribution of outcomes. In \(a\)'s estimation, project \(a\) has a much higher return than the return on a safe asset; in \(b\)'s estimation it has a zero expected return; and in \(c\)'s estimation, it has a positive return somewhat higher than \(r^*\). Project \(b\) has a zero return in \(a\)'s and \(c\)'s estimation, and a very high return in \(b\)'s.

First, we consider the market equilibrium which would result if group \(a\) and group \(c\) ignored the possibility of take-over. It would be essentially that described in Section 3. Project \(a\) would be undertaken. Group \(b\) would invest in the safe industry. The debt-equity ratio in firm 1 would be

\[
\frac{z^*}{1 - z^*} = \frac{\gamma I^c}{I^c + (1 - \gamma)I^c}
\]

(25)

(where \(\gamma\) is the proportion of \(c\)'s investment held in the form of bonds), provided

\[
\frac{I^c}{I^a} \geq \frac{z^*}{1 - z^*},
\]

(26)

as we shall assume. But now consider the possibility of take-over. Group \(b\) will take over the firm, provided:

1. His expected return is greater than \(r^*\); since it will be impossible for him to issue bonds, this implies that

\[
\frac{\mu_{X_1 > (1 + r)B_1} B_1}{E_{1/2}} - (1 + \hat{r}_1)B_1 \pi_1^{ob}(1 + \hat{r}_1)B_1 > 1 + r,
\]

(27)

where \(\mu_{X_1 > (1 + r)B_1}^{ob}\) is \(b\)'s conditional expectation of the profits of industry 1 if project \(b\) is undertaken. Note that to obtain
control only half the shares need be purchased, but once control is obtained, the half remaining in the hands of a shareholder become valueless in a's eyes, and hence can be obtained costlessly if b acted collusively.

(2) It is feasible for them to take over the firm, i.e. their resources collectively are greater than half of the value of the firm:

$$\frac{E_i}{2} < I^b.$$  \hfill (28)

The value of equity, from (25) is just

$$I^a + (1 - \gamma)I^c,$$

and this may very well be less than $I^b$, ensuring that a take-over is feasible, and for sufficiently optimistic values of $\mu^{bk}_{-1} > (1 + \delta)B$ it will be desirable.

Although as a result of the take-over b is better off, groups a and c are worse off:

(1) Group c's bonds are now (in its view) essentially worthless.

(2) Although half of the shareholders of group a are indifferent to the new arrangement (since otherwise they would not have sold out their shares) the other half sustain a large capital loss (since in their view the shares are not likely to yield any return and group b may not buy out all the shareholders at the old price of a share). Thus as a group they are worse off.

As a consequence, we have argued that both bondholders and shareholders will take cognizance of the possibility of a take-over. In our example, this will take the following form. Group c will refuse to lend anything if $E_i$ is less than 2$I^b$. This means that there is a sharp discontinuity in the interest rate function. Moreover, unlike the previous case where the nominal interest rate was just a function of the debt-equity ratio, here it is a function of both the debt-equity ratio and the absolute level of equity.\(^{43}\) Thus the equilibrium debt-equity ratio in our example will be given not by (25) but by

$$\frac{I^a + I^c - 2I^b}{2I^b}. \hfill (29)$$

\(\Box\) **Further comments on take-overs.** There are two key question areas which have been deliberately avoided in the discussion thus far. The first class of questions concerns the existence of a competitive market equilibrium in economies where take-overs may be important. Indeed, in view of the fact that "control" is vested in 51 percent of the shareholders, the very meaning of "competitive equilibrium" must be called into question. But although it is easy to formulate extended definitions of market equilibria, taking into account the nonmarket aspects of economies with stock markets

\(^{43}\) Recall that in Section 4 we assumed that the firm was acting competitively, in the sense that it assumed that if it doubled its debt and its investment, its market value would double. We expressed some misgivings about this assumption at the time; now that the level of equity as well as the debt-investment ratio affects the interest rate, we must express still stronger misgivings about this assumption.
(the shareholders meetings or the possibility of take-overs by purchase of a 50-percent share), there is nothing to ensure the existence of such equilibria. Consider the following minimal requirements for a market equilibrium:

(1) Of all possible plans, the given plan must be preferred by a majority of the original shareholders weighted by shareholdings.

(2) The same plan must be preferred by a majority of shareholders after the plan has been announced: now that they have purchased their shares, it must not pay them to hold a new shareholders meeting to adopt an alternative plan.

(3) It must not pay any group of individuals, whether original shareholders, shareholders after the plan is announced, or not shareholders at all, to purchase a majority of the shares and adopt an alternative plan: it does not pay any entrepreneur to organize a “holding company” the purpose of which is to buy shares of a given company to take over control, promising to adopt an alternative plan once it has done so.

The requirements may not be satisfied by any plan for a number of reasons:

(1) Except under very stringent conditions, we encounter in both the first and second requirements the usual voting problems: there may exist no plan which is preferred against all other plans by a majority (cyclical majorities). But even if there were only two groups of individuals in the economy, we would encounter difficulties.

(2) Consider, for instance, a situation where there are two possible production plans for the firm, one of which in one group’s estimation has a very high return, but in the other group’s estimation has a very low return, and conversely for the other plan. Since only a majority of the shareholders need to be “bought off,” it is clear that if the firm is controlled by the first group, the second group can “buy off” a majority of the first group, and if the second group controls the firm, the first group can “buy off” a majority of the second group. Thus the third requirement is not satisfied. Only by acting collusively can such take-overs be stopped.

(3) Indeed, not even a formal take-over is required for these difficulties to be encountered. In the above example, if originally the firm is owned by the first group, as a result of the high expected return of the project (accompanied by a relatively low variance) the individual will have a high level of expected utility. Accordingly, the individual may allocate a larger proportion of his assets to the safe firm than he would have allocated if the return were low (at the same variance). It is clearly possible then that when the first group originally owns a majority of its shares and adopts its preferred project, with a high expected return, it ends up a minority shareholder, and conversely when the second group originally owns a majority of the shares. Thus the first and the second requirements may not be satisfied simultaneously.

(4) Still a fourth difficulty is encountered when there is more
than one factor or commodity. To establish existence of equilibrium, the excess demand curves for the different commodities and factors need to be continuous (or upper-semicontinuous). Production today depends on expected prices in the different states of nature in the next period, which in turn depend on production plans tomorrow. Normally, we would expect slight changes in prices today to have slight effects on production plans and expected prices tomorrow, and hence to have a slight effect on production plans undertaken today—and accordingly only a slight effect on excess demands today. But if as a result of a slight change in the price, the firm goes from having a profit equal to its obligations to bondholders to having less than enough to pay its bondholders (it goes bankrupt), then the decisions taken by the new management group—whose expectations about returns in the future for alternative projects may be quite different from those of the original management—are likely to be discretely different from those taken at the previous price. But this means that there is no basis for arguing that the excess demand curves in this period will be continuous.

We have only been able to hint here at the difficulties involved simply in establishing the existence of equilibrium for a market economy exhibiting a few of the features which we seem to observe in modern capitalist countries. I have explored some nonmarket concepts of equilibrium, in particular, the core, but these seem equally unsatisfactory. Indeed, we can construct examples where the core is empty.

The second class of questions has to do with information, the determination of expectations of returns, and the value of a firm. The return to an individual from holding a share in a firm is not only the dividends but also the capital gains. Thus the price today depends on the expected price tomorrow. The expected price tomorrow depends not so much on what we think the firm’s present discounted value of returns will be tomorrow, but what the market assesses as its value. That this may give rise to speculative booms is clear and has been discussed elsewhere. It has been shown that (in a deterministic model) “every speculative boom must come to an end:” in finite time the price must become infinite, and somebody winds up holding the shares when the boom busts. One might argue that the market would “learn” to avoid such behavior; but without futures markets, there seems to be no mechanism for avoiding these difficulties. What I want to point out here are the further difficulties arising in the presence of uncertainty. For when the share no longer yields the high returns expected, the individual must decide whether (1) he was unlucky, in the sense that he recognized that in some states of nature the price would be low, and ascribed a low probability to the occurrence of those states, but nonetheless they sometimes occur; (2) further information has become available affecting other individuals’ expectations about returns in the future and hence the price of the share; or (3) the

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44 And, perhaps equally important, not making use of the Arrow-Debreu securities which do not in fact exist.
original price was "too high." There are no sure grounds for differentiating among these.

The fact that individuals must rely in part upon other's estimates of the value of the firm has two important implications for take-overs.

First, the fact that an individual or group of individuals estimates that the result from controlling this company is greater than its present price is information to the stockholders; not unlikely, as a consequence, they will revise upwards their estimate of the returns to the firm. Indeed, if the firm or individual attempting the take-over offers only slightly more than the present market price, the stockholders may revise their estimate of the market value as a result of his offer so that it now exceeds his offer. That is at least part of the reason why take-over bids are almost all substantially above the market price of the firm at the time they are made.

Second, in borrowing, the lender may have little information about the probability of bankruptcy. If he observes like-minded people insisting on the more than safe return, he may use this as evidence that others think that there is a large probability of bankruptcy. Thus he may revise his estimate of the probability of bankruptcy, insisting on an even larger nominal return. The effect of all of this is to make the supply curve of loan capital very sharply upward sloping beyond some point, and make it more difficult for individuals and firms to acquire the funds required for a take-over by borrowing.

6. Concluding comments

We have shown in this paper that the possibility of bankruptcy has very strong implications for firm behavior: it may not be possible to separate out financial and real decisions and there may indeed be an optimal debt-equity ratio. Moreover, we have demonstrated the further consequence that the economy may not be productively efficient.

Some economists have argued that because bankruptcies are relatively infrequent, they cannot be important; that a model which assumes they cannot occur is more "realistic" than one, as presented here, in which they play a central role. We would argue that even if bankruptcies are relatively infrequent, the possibility of bankruptcy may have an important effect on firm behavior; for instance, they may choose a debt-equity ratio which avoids having a very high probability of bankruptcy. Moreover, one ought to include some take-overs and mergers in the same category as bankruptcy; rather than go through the high costs associated with actual bankruptcy, a firm which is approaching insolvency is likely to prefer these methods of disappearance. These have been quite frequent both in the United States and the United Kingdom in recent years.

We would argue that in the models we have investigated here, not only are the assumptions more realistic than those of the conventional models but the predictions of the models seem more in accord with how firms behave. In any case, further exploration of models with bankruptcy and divergent expectations seems called for.
References

