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Pecuniary Externalities: A Game Theoretic Analysis

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When one firm or an industry expands, its actions may influence the cost of factors to another firm. The change in the cost of factors represents an externality to the firm which finds its input prices changed. The firm is not the master of its own destiny. An action by an outside agent has influenced its economic well being.

I. Pecuniary Externalities and Markets

Why should a pecuniary externality, i.e., an externality that is manifested in a change in prices, be different from a physical externality such as the production of smoke or other pollution? Modern economic analysis provides the answer that the change in prices is merely a reflection of the maximization process in a competitive market embodied in a general equilibrium system. The apparent externality shows up as a feedback from one market to another in a comparative statics explanation of adjustment to change.

The paradox of the pecuniary externality is caused by the nature of the partial equilibrium analysis of the firm. A real or a pecuniary externality appear to be the same when viewed from the viewpoint of the individual firm. When embedded in a general equilibrium context the differences emerge.

In our attempt to set up the appropriate economic model we find that the paradox of the pecuniary externality is closely related to the implicit assumption of the existence of markets rather than the assumption of the emergence of markets as part of the economic process.

The paradox does not even appear if, instead of viewing the behavior of the individual firm or customer as a mere mechanistic adjustment to a price system, it is viewed as the acts of a player in an economic game.

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Instead of assuming the existence of a price system, we deduce its existence as the limit of the core of an economy with many players (see Shubik). Although the concept of the *core* has gained wide recognition in the past few years, it may still be unfamiliar to some readers. For this reason a definition is supplied in the footnotes.[1]

F. Y. Edgeworth first suggested approaching the formation of the price system via the contracting and recontracting of groups in a market. He indicated the limits of a bargain between traders by means of a contract curve and indicated that as the number of traders is increased the contract curve shrinks and in the limit (with many traders) the price system emerges. In my 1959 article, I noted that the Edgeworth contract curve was the same as the game theoretic concept of the core of a game. I treated a special case; Herbert Scarf treated the general case and proved the convergence of the core to the competitive equilibrium price in a general context. Gerard Debreu worked with Scarf in shortening and strengthening the proof.

The reason for the paradox of the pecuniary externality and why it does not appear when we approach the price system via the core can be explained by means of a simple

[1] The core is composed of that set of imputations of wealth on the Pareto optimal surface which cannot be blocked by any coalition An imputation can be blocked if there exists a coalition that could obtain more for all of its members without cooperating with the others The following simple game illustrates the core Suppose players 1, 2, 3 can together obtain \$30 The Pareto optimal surface consists of a set of imputations of the form (a_1, a_2, a_3) where $\sum_{i=1}^{3} a_i = 30$ Suppose that any individual alone can obtain nothing and that any pair can obtain \$15 The core consists of the imputations satisfying the conditions $a_1 \geq 0$, $a_2 \geq 0$, $a_3 \geq 0$, $a_1 + a_2 \geq 15$, $a_1 + a_3 \geq 15$, $a_2 + a_3 \geq 15$, $a_1 + a_2 + a_3 \geq 30$, and $\sum_{i=1}^{3} a_i = 30$ It is easy to observe, for example, that the imputations (8, 9, 13) or (10, 10, 10) are in the core

Not all games have cores. In the example above, suppose that any pair could obtain \$25 then three of the inequalities would become $a_1 + a_2 \geq 25$, $a_1 + a_3 \geq 25$, $a_2 + a_3 \geq 25$ and no set of (a_1, a_2, a_3) adding to 30 can exist

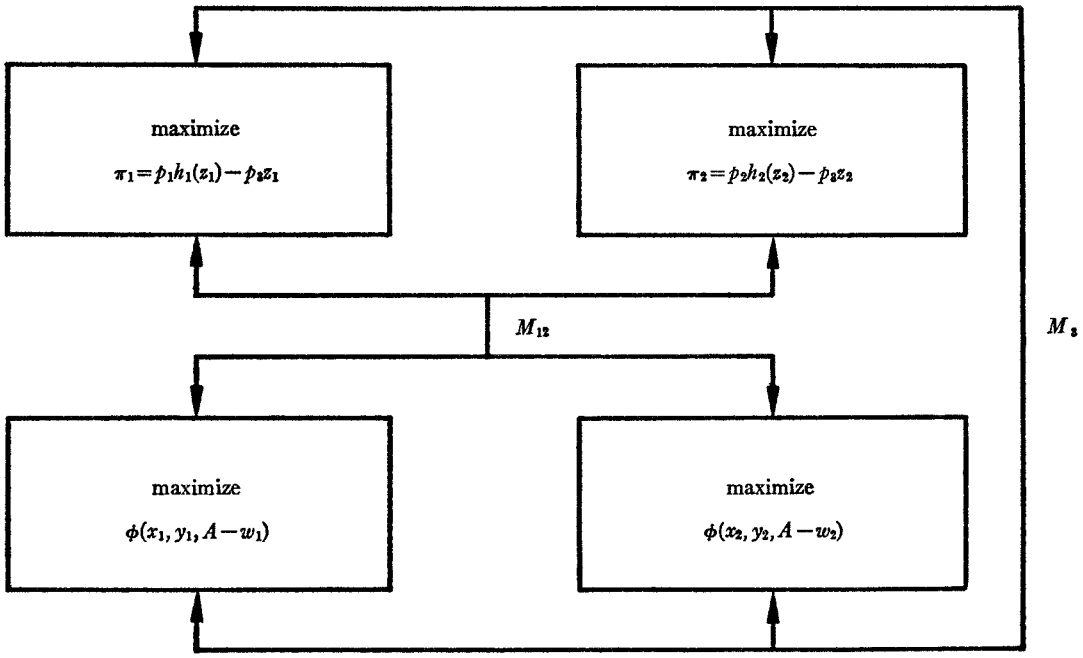

FIGURE 1

example of a closed economy with two industries, customers, two final products, and labor.

Figure 1 shows the general equilibrium market model and Figure 2 shows a closed economy modeled as a game. They are modeled for two firms and two customers. For simplicity, we assume that raw materials are free inputs, hence the profits of firms 1 and 2 can be expressed as:

$$\Pi_1 = p_1 h_1(z_1) - p_3 z_1$$

and

$$\Pi_2 = p_2 h_2(z_2) - p_3 z_2$$

where $x = h_1(z_1)$ and $y = h_2(z_2)$ are the production functions for the two consumer goods. The distributions of the consumer goods to the customers are $x_1 + x_2 = x$ and $y_1 + y_2 = y$, where the subscripts to x and y

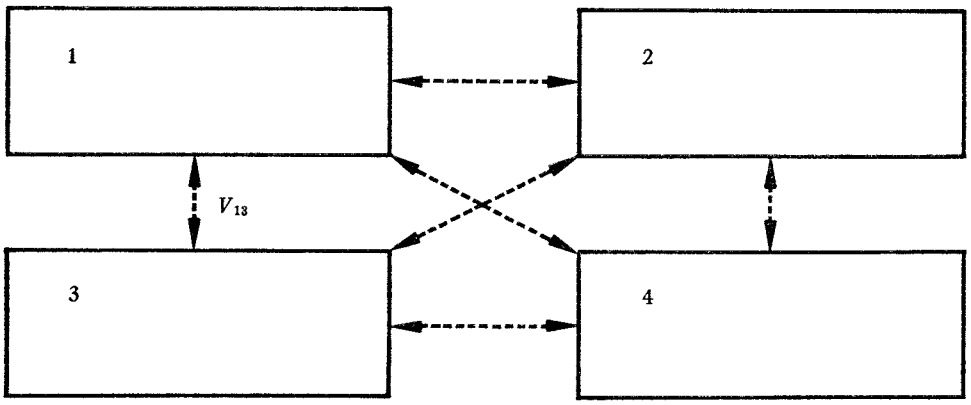

FIGURE 2

denote customers. The amount of labor bought equals the amount sold, i.e., $z_1+z_2=w_1+w_2$. The prices of the first and second consumer goods are p_1 and p_2; the price of labor is p_3. For simplicity we assume that the utility functions of the consumers are the same, ϕ, and their initial endowments are also the same $(0, 0, A)$.

The lines and arrows marked by M_{12} show the presence of a market for the consumer goods (or two interlinked markets). The market structure aggregates and interlinks firms and customers so that any individual trades with an impersonal market rather than indulges in personal face-to-face bargaining and trading. Both firms and customers are assumed to face prices and they maximize accordingly. The lines marked by M_3 denote the labor market.

If we wished we could introduce one more set of lines representing the payments of dividends $D_1+D_2=\Pi_1+\Pi_2$ back to the consumers. For the purpose at hand this is an unnecessary complication. Furthermore if the production functions are homogeneous of order 1 (i.e., constant returns) then profits will be zero and no dividends will be paid.[2]

Figure 2 shows the two firms (as players 1 and 2) and the two customers (as players 3 and 4). The dotted lines indicate that anyone can deal with anyone directly. There is no imposed market structure or price system.

In particular the two firms and customers can form 2^4-1 or fifteen different trading groups. The trades that these different groups can achieve can be denoted by a characterizing function, (see my forthcoming book with Lloyd Shapley).

$V(\{i\})=0$ is the set of outcomes that a coalition of one individual i can obtain if he deals with nobody else. This amount can be assigned the value 0. It is the status quo payoff. There are four one-person coalitions.

$V(\{i, j\})$ stands for the set of outcomes that a coalition of two individuals can obtain.

$V(\{1, 2\})=V(\{3, 4\})=0$. If the firms or the customers try to trade only among themselves they cannot improve over the status quo.

$V(\{1, 3\})=V(\{1, 4\})=$ the Edgeworth contract curve between the first firm and a single customer (where the customers have the same tastes and endowments).

$V(\{2, 3\})=V(\{2, 4\})=$ the Edgeworth contract curve between the second firm and a single customer.

There are six two-person coalitions possible. Similarly there are four three-person coalitions: $V(\{1, 2, 3\})$, $V(\{1, 2, 4\})$, $V(\{1, 3, 4\})$, and $V(\{2, 3, 4\})$; and there is one four-person coalition whose trading possibilities are characterized by $V(\{1, 2, 3, 4\})$.

II. Orthogonal Coalition Games and Decentralization

Despite its frightening name, the concept of an orthogonal coalition game is relatively straightforward and is the key to the analysis of externalities. The basic idea is that in such a game, once the participants have chosen sides, those excluded from a specific coalition have no influence on the fate of the coalition. This is highly related to the view of open trading in a market. When two traders agree upon a bargain, for all intents and purposes they do not care what the rest of the market is doing. This does *not* mean that the availability of other traders has no influence on the market; however, no one is forced to deal with anyone else. The device of recontracting may be regarded as a comparative statics way of checking trading possibilities; however (as is the case in most actual market places), a contract will be honored by both parties and does not depend upon third parties.

When we view the market with two firms and customers, as described in Section I, as one in which any coalition is feasible then as soon as you have signed your contract with some set of participants the remainder of the market is irrelevant.[3] The essence of an

[2] There is a further complication relating to production. Are the production functions individually owned or can any factory produce anything if it has the inputs? Here we assume the former. This might happen if each owner had a nonmarketable talent or capital good used in the process. A fully satisfactory answer to this question is not attempted here as it is not central to discussion of monetary externalities although it calls for care in treating capacity constraints, nonmarket goods and dividends.

[3] It is possible that, for unspecified reasons, the outsiders might wish to give away extra goods to you,

orthogonal coalition game in general and of a market game in particular is "doing your own thing."

Consider the coalition $\{1, 3\}$ which can obtain $V(\{1, 3\})$. This is indicated in Figure 2 by V_{13}, and leaves the first firm and customer utterly independent of the second firm and customer. This is not possible in the setup described in Figure 1. The other firm and customer are always interlinked to everyone else through the impersonal market where a price to one is a price to all simultaneously. The market in Figure 1 forces a feedback upon all participants whether they like it or not. They cannot cut out and form their own community. The market in Figure 2 allows for any grouping anyone wants. Once the groups have been decided upon they are strategically isolated. They have no further interaction with each other. In other words they are completely decentralized.

If an organization can be described as an orthogonal coalition game, this means that it is possible to decentralize it in such a complete manner that the performance of any subdivision will depend only upon itself.[4]

Suppose that we were to increase the number of customers and the number of firms of type 1 and type 2, the market we have described is merely a special instance of the market treated by Debreu and Scarf. As the number of firms and customers increases the core shrinks and a price system emerges.

III. Partial Equilibrium Analysis, Comparative Statics and Externalities

The full force of the paradox of pecuniary externalities comes in when we discuss changes in supply costs to an industry considered in isolation; where the changes have been brought about by a shift elsewhere in the economy. For example, suppose that an industry in the same district has had a change in technology which calls for a different type of labor input. The effect of this change may feed back on the labor costs of the first industry.

Following the description of the two models in Section I we can see immediately the effect of the different treatments. Suppose that there was a shift in technology so that the production function for firm 2 becomes $h_2^*(z_2)$ instead of $h_2(z_2)$. Solving the general equilibrium system indicated in Figure 1 a new set of prices will emerge. In particular the cost of the labor supply to Figure 1 will have changed. Viewed from the position of the head of the first firm looking only at his own market, an external force over which he has no control has affected his costs. The reason for this is because we have implicitly assumed that he has to deal via the overall impersonal price system of the mass market whether he likes it or not.

Let us now consider this change in terms of the model in Figure 2. The only values of the characterizing function that have changed are those involving firm 2. If your group was not dealing with firm 2 before its technological breakthrough, it does not have to deal with it now. If it chooses not to, then it will earn the same amount as before.

The effect of the technological change in firm 2's production has not gone unnoticed. It has increased the value of eight out of the fifteen coalitions. By doing so it has made certain coalitions more attractive to some players than they were before and has changed the pattern of worthwhile recontracts. If we were to take the new characterizing function and add more players, the core will now converge to a new limit which is the new price system that takes into account the change in technology.

IV. Markets, Economic Reality, Pecuniary and Physical Externalities

Suppose that exogenous change in the production process of firm 2 were such that it involves a physical externality. For example it switches to a new process which may be labor saving to itself but which dumps smog on the consumers and on the other firm. Returning to Figure 2 and to the characterizing function; we would have to modify the figure by introducing solid interlinkages

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whether you like it or not. However, if strangers bearing gifts turn up you do not have to take them.

[4] This does not solve the problem of what is the optimal decentralization for the organization as a whole. The answer for a market appears to be: "give up the doing-your-own-thing level of decentralization. Put in a price system in the central office and announce the prices to the otherwise decentralized divisions."

between firm 2 and the three others. Regardless of the market organization they cannot avoid the connection to firm 2. The game is no longer an orthogonal coalition game; we cannot even easily state what a coalition can obtain without first finding out what firm 2 intends to do. It is no longer possible to do your own thing. The characterizing function is not particularly helpful in describing the structure of the threats and interference that the firm with the physical externality can export to all of the other players.

When we try to patch up the characterizing function to take into account the actions of firm 2 we find that the core of the new game may not exist (see Shapley and Shubik 1969). Furthermore if the core were to exist there is no guarantee that it converges as more firms and customers are added. No price system emerges.

In terms of conventional analysis where we *assume* the existence of prices, if we try to solve the general equilibrium system with physical externalities present, although it may be possible to determine prices which clear the markets, they no longer call forth a Pareto optimal outcome.

Returning to Figure 1 we would have to patch up all production functions and the consumer preferences to reflect the role of the smog as an extra commodity. The smog is not an economic good and will not appear with a price, nor can it be traded or avoided. Independent maximizing behavior based upon a price system which does not include a method for pricing and controlling the distribution of the extra commodity will be optimal only by pure coincidence.

In an economy such as ours, markets exist and, at least in some areas, the price system is prevalent. Although the argument in this article has attempted to show that by adopting a more basic approach to trade, based upon the core of an economy and all coalition structures, the phenomenon of pecuniary externalities poses no problem; yet we know that our institutions have markets and we do not deduce their existence *de novo* by calculating the effect of billions of coalitions.

If we assume that firms *must* trade through organized markets, then an analysis of partial equilibrium competition shows that a pecuniary externality may be just as real to a competitor as is smog being dumped on his factory.

If the individual large firm has strategic control over at least part of the environment, its management will not care (unless they are socially motivated) whether costs are going up because a new industry in the area is bidding up the price of engineers or causing costs to rise because it is now more expensive to clean polluted water for the industrial use that the firm has for it. In either case the strategy of interest to the oligopolistic firm is to block the entry of the new industry or to extract a tribute from it to pay for the additional costs it will incur. The firm per se does not care whether (as in the first case) the extra costs are caused by the functioning of a technically efficient price system correctly (from a social point of view) increasing the cost of skilled labor to it; or whether costs are increasing due to a socially undesirable externality which has not been accounted for.

In summary there are three phenomena which must be separated when comparing pecuniary and physical externalities. We may view the whole economy as a mechanistic functioning of the price system. The single price-taking producer will regard a change in costs to him via the price system as bad as a change via a physical externality. However, he is constrained (by being a price taker) to do the right thing when the cost change merely reflects a feedback in the general equilibrium process. He will almost always do the wrong thing when the cost change is caused by a physical externality.

When we view the whole economy as a cooperative game with all coalitions possible, the paradox of the pecuniary externality disappears and is shown to vanish into the overall maximization problem where exogenous change is correctly reflected in the change in the values of some of the outcomes shown in the characterizing function.

When we consider the possibility that an individual firm may be an oligopolist with market power, it is not constrained "to do the right thing" when its costs go up, no matter how they go up. Hence in the sense of an oligopolistic general equilibrium, an indi-

vidual firm may be expected to use its power to noncooperatively fight any type of perceived externality.

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