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VOTING, OR A PRICE SYSTEM IN A COMPETITIVE MARKET STRUCTURE

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In this brief note it is demonstrated that if all the conditions for the existence of a competitive equilibrium are satisfied, then simple majority voting to determine the distribution of goods may be less efficient than a price system.

The argument here may be somewhat cryptic for those not familiar with the work of Anthony Downs.[1] A considerably more discursive presentation of the background material is given in "A Two Party System, General Equilibrium and the Voters' Paradox."[2] The tax and public goods aspect of the voting problem have been discussed elsewhere in "Notes on the Taxonomy of Problems Concerning Public Goods."[3] The result presented here, nevertheless, stands by itself, hence is presented in this brief form.

[1] Anthony Downs, *An Economic Theory of Democracy* (New York: Harper & Row, 1957).

[2] Martin Shubik, "A Two Party System, General Equilibrium and the Voters' Paradox," *Zeitschrift für Nationalökonomie*, 28 (1968), 341–354.

[3] Martin Shubik, "Notes on the Taxonomy of Problems Concerning Public Goods," (Cowles Foundation Discussion Paper 208).

The political system is modeled at its simplest. We assume the existence of two players called "political parties." The goal of each player is to win an election by as large a vote as possible. A strategy for each player is to name a policy that it will carry out if it is elected. A policy is any point in the set of feasible distributions of final product. It follows immediately that, although any policy may be considered as a strategy, any non-Pareto optimal policies are dominated by some policy that is optimal. A discussion of the reasons for modeling a party in this simple manner is given in detail elsewhere. It is assumed that all voters are passive or "mechanistic," i.e., they do not form groups but merely vote individually, selecting optimally between the two policies offered.

Consider a three-person symmetric market. The three traders have initial endowments of $(a,0,0)$, $(0,a,0)$ and $(0,0,a)$. Each has the same differentiable utility function $\phi_1=\phi_2=\phi_3=\phi(x,y,z)$. It is easy to check that in this case the unique price system will be $(1,1,1)$ and the final imputation to each will be $(a/3,a/3,a/3)$. The worth of this imputation is shown as the point E in Figure 1.

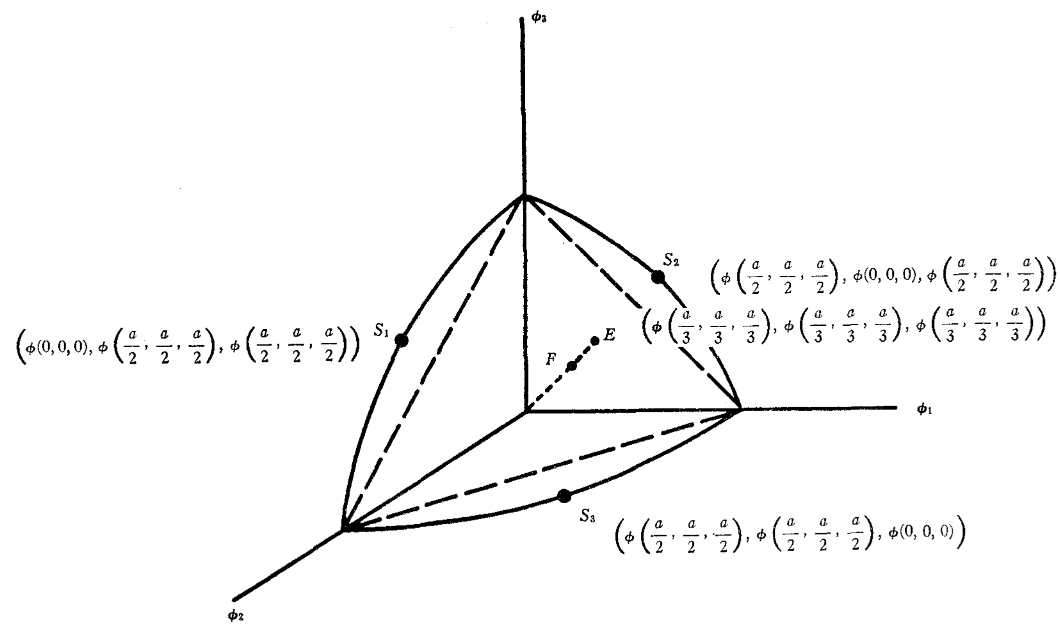

Figure 1

Suppose that instead of using a price system the society decided to adopt a simple majority voting procedure for the distribution of all goods. We must specify a condition concerning the protection of property rights of the minority. For simplicity we can either assume that any taxation up to confiscation of all possessions is sanctioned, or that any redistribution must satisfy conditions of individual rationality as defined by preferences and the distribution at the *status quo ante* the vote. For the purpose of the example we assume that any taxation is possible, hence an individual can end up with nothing. As is pointed out later, the result does not depend on this.

In Figure I, s_1, s_2, or s_3 could be obtained by a majority. They are not preferred to each other, yet one of them is preferred to any other policy that can be named. For example, suppose that the parties limited themselves to seven programs consisting of the three policies noted and the following four others E and $((e,f,g), (g,e,f), (f,g,e))$, $((f,g,e)$ $(e,f,g), (g,e,f))$, $((g,e,f), (f,g,e), (e,f,g))$ where $e+f+g=a$. Suppose that $e>f>g>0$ and $f>a/3$. The resulting 7×7 matrix game is shown below (where 1 indicates a win by one vote, 0 a tie, and −1 a loss by one vote).[4]

	1	2	3	4	5	6	7
1	0	0	0	−1	−1	1	1
2	0	0	0	−1	1	−1	1
3	0	0	0	1	−1	−1	1
4	1	1	−1	0	1	−1	1
5	1	−1	1	−1	0	1	1
6	−1	1	1	1	−1	0	1
7	−1	−1	−1	−1	−1	−1	0

This game has a mixed strategy solution of (0,0,0,1/3,1/3,1/3,0) for each party. As is to be expected, each names the same (mixed) strategy and each stands the same chance of winning the election.

A possible interpretation of the mixed strategy is that the majority rule emphasizes the diversity of interests—any majority wants to benefit by taxing the minority. As this situation is symmetric, the parties wish to appear to be all things to all men at the same time; hence, they mix their strategies over different nonsymmetric outcomes favoring different interest groups.[5]

The individual consumer obtains $1/3$ $(\phi(e,f,g)+\phi(g,e,f)+\phi(f,g,e))$ from this policy. However, if he is risk-neutral or risk-averse, then from the assumption that he has a convex set of preferences $\phi(a/3,a/3,a/3)\geq 1/3(\phi(e,f,g)$ $+\phi(g,e,f)+\phi(f,g,e))$. This is shown as the point F in Fig. 1. If his preference set is strictly convex, then the midpoint solution that is the outcome of the competitive market is strictly preferred to the outcome obtained through the voting procedure.

Had there been a limitation on taxation, the same result would hold with the modification that some of the more extreme policies which discriminate against minorities would be limited.

The result did not depend upon selecting a finite number of policies for the two parties to offer. If all points on the optimal surface are available, then the strategic problem of the parties becomes equivalent to that of a continuous Blotto game, which has been solved by Gross and Wagner.[6] All of the solutions place a zero probability density on the center.

It might be thought that the result depends strictly upon the symmetry of the preferences of the voters. This is not the case. If they are risk-neutral or risk-averse, then whenever the "political noncooperative game" has no pure strategy solution, the result will not be Pareto optimal. Depending on the degree of symmetry and the level of risk aversion, the competitive

[4] Two models can be considered, the first in which a party's goal is to win by as large a vote as possible, the second where its goal is just to win. In this example, the latter assumption appears to be the more reasonable.

[5] Kenneth Arrow has pointed out that one must distinguish between the two cases where (1) the parties announce their (mixed) strategies to the public, or (2) randomize first and announce the the results of the randomization as their policies to the public. If we adopt the first interpretation, we may view the mixed strategy as being a "degree of belief" in the mind of the voter. He perceives some of the contradictions in the statements of the parties; hence he has only a tentative view of what their actions will be. In the second instance we assume that the parties polarize the issues for the voters. In the first case we must assume a cardinal utility scale for the voters who evaluate uncertain outcomes.

[6] O. Gross and R. Wagner, *A Continuous Blotto Game* (The RAND Corporation, RM-408. June 17, 1950).

equilibrium point will lie above the expected value of the vote in a zone of the Pareto optimal surface such that everyone is better off at the competitive equilibrium. This is shown in Figure 2. V is the expected value of the outcome obtained by voting and E the competitive equilibrium. As long as E lies between V^{1} and V^{11} the result is true. This diagram was drawn for ease of exposition in two dimensions where P_1 and P_2 are the evaluations of Voters 1 and 2. The picture is basically the same for higher dimensions.

A possible interpretation of the general result is that if in some sense the original distribution of resources is regarded by the voters as more or less equitable or symmetric, the economic process keeps this property; whereas the political process introduces the possibility of considerable asymmetry as the majority takes from the minority. Without special laws to protect the minority and since anyone could be in the minority, all participants evaluate the expected worth of the mix of outcomes offered by the voting process as worse than the economic outcome.

In the continuous case the policies which are not used are those which give a single individual more than two-thirds of all of the gain. A coalition of the remaining two can always be effective against such an extreme division. The midpoint or even split is also ruled out because any two players can improve their lot by taking from the third.

This model is obviously highly unrealistic and restrictive. Nevertheless, the result is possibly instructive and has a useful interpretation. The outcome is Pareto optimal, but the expected outcome is not. The loss of efficiency appears to be related to the power struggle and the inherent favoring of a nonsymmetric solution by the majority in a political situation. The economic mechanism, *given* the initial distribution of resources is anonymous and egalitarian.

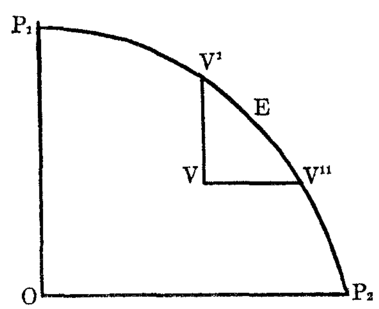

FIGURE 2

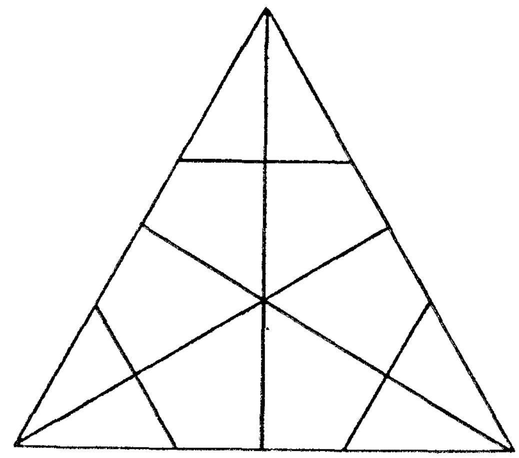

FIGURE 3

It must be stressed that usually a reason for using a voting system is that no price system exists. This is almost always the case when the production of public goods is called for.

APPENDIX

One of the solutions given by Gross and Wagner is:

Colonel Blotto might do the following: He plays a continuous density function p over the regular hexagon (of Fig. 3), i.e. if $d\sigma$ denotes an element of area at point x within this hexagon, the probability that he chooses a point within $d\sigma$ is given by $p(x)d\sigma$. The density function p is determined as follows: p is constant* on the perimeter of the hexagon, zero at the center, and linear along any straight line segment joining the center with an arbitrary point on the perimeter.

* Normalization reveals the value of this constant to be $9/4\sqrt{3}$.