HYPOTHESIS TESTING AND THE DEMAND FOR CAPITAL GOODS

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I Introduction and Summary

ONE of the basic facts of life confronting econometric researchers is that in order to test any hypothesis it is necessary to assume the validity of other assumptions which cannot be tested. An important part of the art of practical econometrics is knowing how much to include in the maintained hypothesis; if too much is assumed there may be little or nothing left to test, while if too little is assumed it may be impossible to reach any conclusions, or else the analysis may become hopelessly complex.

In a recent article in this Review ¹ Robert Eisner and M. I. Nadiri have examined critically one of the essential maintained hypotheses used by Dale W. Jorgenson, James A. Stephenson, Robert E. Hall, and Calvin D. Siebert in a substantial body of empirical research on the demand for capital goods.² This assumption maintains that the long-run partial elasticity of the flow of capital services, the stock of capital, the flow of gross investment demand, or the flow of net investment, with respect to the price of output (p) divided by the price of capital services (c) should be unity. By re-specifying Jorgenson's model in a logarithmic form, Eisner and Nadiri have produced tests of the hypothesis that the long-run price elasticity of demand for capital stock is unity. Not only do they find that the estimated elasticity with respect to (p/c) is significantly less than one, but all of their preferred point estimates of this parameter are less than 0.16 and in some cases do not differ significantly from zero. The first of the seven conclusions summarized by Eisner and Nadiri is that "the role of relative prices, the critical element in the neoclassical approach, is not confirmed."³

In principle, the Eisner-Nadiri goal of relaxing and testing crucial maintained hypotheses is a laudable one. Their conclusions, if they can be sustained, have far-reaching implications. If their estimated elasticities are correct, then fiscal and monetary policy-makers have little, if any, direct influence on investment expenditures. A cautious approach to the importance of the Eisner-Nadiri conclusions would seem justified, however, in view of the fact that others have also undertaken the task of critically examining the maintained hypotheses in the Jorgenson model. While none of the other critics of Jorgenson has defended the precise manner in which he has specified his model, without exception the results have been favorable to the essence of the "neoclassical approach" to investment functions — the assumption that relative prices do matter.⁴

The next section of this paper is essentially an exercise in detective work aimed at finding out why Eisner and Nadiri obtained results contrary to the body of other research. The analytical method used is to carry the goal of Eisner and Nadiri — relaxing and testing maintained hypothesis — one step further. The maintained hypothesis I relax and test involves the assumption of serially independent errors.⁵

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³ [7]. Eisner's criticisms have been amplified in [5] and [6].

⁴ This body of research includes [12] [13] [16] [17] [19] [20] [21] [22].

⁵ [7], p. 380.

⁶ See [2] [3] [4] [9]. Some of this evidence is discussed briefly in section III below. The evidence on demand for factors other than capital, and on direct estimation of CES production functions, is also relevant, at least indirectly. See [13] for discussion of this evidence.

⁷ As I note below, the stochastic assumption I make — that the errors are a first order autoregressive process — is only one step more general than that used by Eisner and Nadiri. I do not wish to imply that this stochastic assumption is anything more than a minimal improvement; the only reasons for not using other types of assumption was my desire to minimize computational problems.

[354]
In a sense, the exercise succeeds only partially; some of the empirical results are even more peculiar than those reported by Eisner and Nadiri.

In the context of a maintained hypothesis about the errors which is only slightly more general than the one used by either Eisner and Nadiri or Jorgenson and Stephenson, but otherwise using the Eisner-Nadiri model, the empirical results are somewhat startling. The summary of conclusions drawn by Eisner and Nadiri includes seven separate paragraphs. For one of two similar sets of data, four of these conclusions simply no longer follow, if the more general stochastic specification is applied. For the second set of data at least two of the conclusions are reversed. Unfortunately, however, the noise level for the data and model analyzed becomes so high that the final conclusion of the second section must be completely agnostic with respect to what really influences the demand for capital.

In an attempt to draw conclusions less ambiguous than those derived from the data Eisner and Nadiri have analyzed, I have applied the same model to a different set of data in the third section. For reasons which may have to do with the construction or quality of the time series, the sample period, or both, the noise level is much lower when the model is applied to the economy-wide demand for equipment for the period 1949–1966. Both output and relative prices appear to have statistically significant effects on equipment demand and the point estimates of parameters are almost precisely what one would expect if the data had been generated by a model including a Cobb-Douglas production function with constant returns to scale.

None of this, of course, should be interpreted as positive support for the policy conclusions emphasized by Jorgenson and Stephenson on the basis of the lag patterns reported in their first article. What is being defended is the “neoclassical approach” to the study of investment behavior.

II A Re-examination of the Eisner-Nadiri
Conclusions about the Demand for Capital in Manufacturing

The model of the demand for capital goods which Eisner and Nadiri employ involves the assumption that, in the long run, capital stock, $K$, will be related to output, $Q$, the price of output, $p$, and the rental price of capital services, $c$, according to

$$ K = A(p/c)^{E_p}Q^{E_q}, $$

(1)

where $A$ is a constant and $E_p$ and $E_q$ are the elasticities of capital stock with respect to the price ratio $p/c$ and output, respectively.

If firms are constrained by a CES production function, and if the flow of capital services is proportional to the stock of capital, then $E_p$ may be interpreted as an estimate of the elasticity of substitution, $\sigma$, and $E_q = \sigma + (1 - \sigma)/\nu$, where $\nu$ is the degree of homogeneity of the function. Because Jorgenson develops his model from an assumption that the production function is Cobb-Douglas, both $E_p$ and $E_q$ in his model are specified to be equal to 1, and this holds regardless of whether or not the production function is homogeneous of degree one.

In the short run, it is not assumed that a relationship such as (1) will always hold; instead a distributed lag relationship is postulated by Eisner and Nadiri in which

$$ \Delta \ln K_t = \sum_{i=m}^{\infty} [\mu_{pi}\Delta \ln (p/c)_{t-i} + \mu_{qi}\Delta \ln Q_{t-i}], $$

(2)

where $K_t$ represents capital stock at the end of period $t$,

$$ \sum_{i=m}^{\infty} \mu_{pi} = \bar{E}_p, $$

and

$$ \sum_{i=m}^{\infty} \mu_{qi} = \bar{E}_q. $$

Equation (2) also should include some sort of disturbance term. Note that after taking logarithms, all of the variables have been first-differenced, so that the variable on the left-hand side is the natural logarithm of one plus the ratio of net investment in period $t$ to capital stock at the end of period $t - 1$.

As it stands, equation (2) is too general to permit parameter estimation; it has an infinite
number of parameters. Following Jorgenson, Eisner and Nadiri assume the $\mu_{pi}$ and $\mu_{qi}$ weights may be approximated by a general rational lag distribution. The equation which emerges is one of two which they employ for their empirical tests, but the other equation is a special case of this one:

$$\Delta \ln K_t = \sum_{i=m}^{n} \left[ \gamma_{pi} \Delta \ln (p/c)_{t-i} + \gamma_{qi} \Delta \ln Q_{t-i} \right] + \sum_{j=1}^{s} \omega_j \Delta \ln K_{t-j} + \eta_t. \quad (3)$$

The lag parameters $\mu_{pi}$ and $\mu_{qi}$ in (3), as well as the elasticities $E_p$ and $E_q$ may be derived on the basis of estimates of the parameters $\gamma_{pi}$, $\gamma_{qi}$, and $\omega_j$.

Eisner and Nadiri estimate the parameters of (3) by ordinary least squares. There is no discussion of assumptions about the distribution of the disturbance term $u$. However, as is well-known, ordinary least squares estimates of equations with lagged endogenous variables on the right-hand side will not even be consistent unless the errors are serially independent. It should be noted that in this equation, in addition to the lagged values of $\Delta \ln K$, the lagged values of $\Delta \ln (p/c)$ and $\Delta \ln Q$ must also be considered as endogenous variables in the system of equations of which (3) is a part.

In this context, then, the assumption of independent disturbances is a crucial one if meaningful results are to be obtained. Ideally, in order to justify application of standard test statistics, one would hope that the disturbance vector would have a multivariate normal distribution, with all disturbances having mean zero and the same variance, and each disturbance being independent of all the others. Implicitly, these properties have no doubt been assumed. In doing this, Eisner and Nadiri are following standard procedure; in the published research on distributed lag investment functions with lagged dependent variables on the right-hand side, only Koyck has given explicit consideration to the possibility of nonindependent disturbances.

It is not difficult, of course, to develop a rationale for the assumption that the disturbances in equation (3) should be independent. One might hypothesize, for example, that the rate of investment (relative to capital stock) is partially adjusted each quarter from the rate which prevailed in the previous quarter towards the rate of net investment necessary to match increments in $p/c$ and $Q$. The errors in such an adjustment process might well be thought to be independent from quarter to quarter. At the same time, an equally convincing (or unconvincing) rationale could be developed for the assumption that the disturbances $w$ in

$$\ln K_t = a + \sum_{i=m}^{n} \left[ \gamma_{pi} \ln (p/c)_{t-i} + \gamma_{qi} \ln Q_{t-i} \right] + \sum_{j=1}^{s} \omega_j \ln K_{t-j} + w_t. \quad (4)$$

should be serially independent. This equation is identical to (3) except for the fact that the first-differencing operation has not been performed. Equation (4) may be interpreted as a capital stock demand equation, or a “capital stock adjustment” model.

Unfortunately, it is not possible for both (3) and (4) to have independent errors, at least for the same values of the indices $m$, $n$, and $s$. For given values of these indices

$$u_t = w_t - w_{t-1}. \quad (5)$$

If the disturbances $w$ are independent, then the disturbances $u$ will be a moving average of independent errors, leading to biased and inconsistent parameter estimates. On the other hand, if the disturbances $u$ are independent and of constant variance, then the disturbances $w$ will be generated by an unstable autoregressive process, and will be very badly autocorrelated.

In general, ordinary least squares parameter estimation will be inappropriate for at least one of the two specifications (3) or (4). It is quite possible that neither will have independent errors.

Since each specification is theoretically plausible, the choice between them must be made

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8 See [18] for a discussion of properties and estimation techniques for lag distributions of this class.  
9 My equation (5) corresponds to equation (8) in [7]. The other specification used by Eisner and Nadiri is given in their equation (7).  
10 See for example, Griliches [10].  
on empirical grounds. Fortunately, it is easy to formulate a general model which includes both (3) and (4) as special cases. Using this slightly more general model provides an appropriate test to discriminate between (3) and (4) and perhaps to reject them both.

The model is a standard one. If the disturbance \( w \) is defined by the equation

\[
\omega_t = \rho \omega_{t-1} + \epsilon_t
\]

in which \( |\rho| \leq 1 \) and \( \epsilon \) is normally and independently distributed with mean zero and variance \( \sigma^2 \), ordinary least squares estimation of the parameters of (3) will be optimal only if \( \rho = 1 \); ordinary least squares estimates using (4) will be optimal only if \( \rho = 0 \).

Hildreth and Lu [15] have shown that in an equation of the form

\[
y_t = X \beta + w_t
\]

where \( X \) represents a vector of independent variables and \( w \) has the distribution indicated above, then maximum likelihood estimates of \( \rho \) and the coefficient vector \( \beta \), conditional on a fixed value of \( w_t \), may be obtained by minimizing \( \epsilon \), the sum of squared residuals in

\[
y_t - \rho y_{t-1} = (X_t - \rho X_{t-1}) \beta + \epsilon_t
\]

Minimizing the sum of squared residuals in such an equation is a nonlinear problem, and solution of it by standard nonlinear estimation techniques involves the risk that there may be more than one local minimum of the function. This difficulty may be circumvented by simply specifying a number of trial values of \( \rho \) in the admissible range \(-1 \leq \rho \leq 1\), and transforming each of the variables as in equation (8). This method will provide at least approximate maximum likelihood estimates, and the approximation may be improved by successive searches with smaller and smaller increments in \( \rho \).

The precise specification of the equation whose parameters I estimate is completed by rewriting equation (4) in the form of equation (8), and choosing values of \( m, n, \) and \( s \), the parameters which relate to the degree of the numerator and denominator polynomials in the rational lag distribution. I have chosen \( m = 1, n = 8, s = 2 \). Except for the stochastic specification (and the consequent estimation of \( \rho \) the equation differs from the one on which Eisner and Nadiri base most of their conclusions only in being a bit more general; I use eight lagged values of \( p/c \) and \( Q_t \), instead of seven. The equation is thus

\[
\ln K_t - \rho \ln K_{t-1} = \frac{1}{\gamma} \ln (p/c)_{t-i} - \rho \ln (p/c)_{t-i-1} + \epsilon_t
\]

As noted above, with \( \rho = 1 \), (9) corresponds to (3); with \( \rho = 0 \), (9) corresponds to (4).

The sample includes quarterly data from the second quarter of 1949 through the end of 1962. The dependent variable, \( \ln K_t \), is the natural logarithm of a constant dollar measure (in 1954 dollars) of the net stock of manufacturing capital at the end of period \( t \), calculated by Jorgenson. The output measure \( Q_t \) is derived by using the wholesale price index to deflate gross value added by manufacturing firms, also calculated by Jorgenson. The price index \( p_t \) is the wholesale price index, rebased to 1954 = 100 by Eisner and Nadiri. Two different measures of the rental price of capital services are used — \( c_t \), in which the discount rate for investment decisions is defined as corporate profits plus net monetary interest divided by the value of outstanding securities, and \( c_t \), in which the yield on long-term government bonds is used to approximate this discount rate. These data, with the exception of the wholesale price index, not required for their model, had been used by Jorgenson and Stephenson, although only \( c_t \) was used as a measure of the price of capital services.

The sources of the data are described in the Appendix to [7].

See [20] [21]. Jorgenson, however, did use \( c_t \) in [16] and [17].
I have estimated the autoregressive parameter, $\rho$, using a grid of twenty-one trial values, spaced at intervals of length 0.1 in the domain $-1 \leq \rho \leq 1$. The results for a selection of trial values, including those closest to the maximum of the likelihood function, are given in Table 1. For the data set using $c_1$, the first set of trial values showed two local maxima of the likelihood function, at $\rho = .9$ and at $\rho = .2$. The maximum at $\rho = .2$ appeared to be the better one, but to verify this the intervals $.8 \leq \rho \leq 1.0$ and $.1 \leq \rho \leq .3$ were explored using grids with intervals of length .01. This second stage established that the global maximum, to the nearest .01, was at 0.20, while the maximum at 0.92 was an inferior one.

Only a cursory glance at Table 1 is required to see that some of the point estimates look very peculiar. The data set using $c_1$ provides the best explanation of the dependent variable, by a margin of about ten percent in terms of the sum of squared residuals. The point estimates of the parameters, however, appear quite nonsensical. The maximum likelihood estimate of $E_s$, 0.028, if correct would indicate that in the long run the level of capital stock is virtually independent of output. Using the underlying marginal productivity condition and solving back for the implied parameters of the supposed underlying CES production function, the implication is that the elasticity of substitution is 0.358 and that the function is homogenous of degree-1.95. This is not a sensible production function!

Before jumping to any conclusions on the basis of the point estimates, the question of sampling variability must be considered. The burden of the next few paragraphs is that neither of the nonsensical point estimates $E_s = .028$ and $E_p = .358$ is, using conventional significance levels, significantly different from unity; if one started with the Cobb-Douglas model as a null hypothesis it could not be rejected. Before establishing this, a brief excursion is required into the realm of hypothesis testing and confidence interval derivation for nonlinear models.

First, consider the question of statistical inference about the autoregressive parameter $\rho$. In the context of the error specification adopted, estimates conditional on values of $\rho$ very close to 1 (and also between $-.2$ and $-1.0$) may be rejected because they result in significant increases in the sum of squared residuals.

An exact test of the null hypothesis that $\rho$ takes a particular value is not possible without knowledge of the exact distribution of the likelihood ratio between the value of the likelihood function under the null hypothesis and the maximum value of the function under the alternative hypothesis that $-1 \leq \rho \leq 1$. Asymptotically, if the null hypothesis is true the statistic

$$-2(L_0 - L_1),$$

where $L_0$ is the log of the likelihood function under the null hypothesis and $L_1$ is the log of the likelihood function using the maximum likelihood estimate of $\rho$, will have a $\chi^2$ distribution with 1 degree of freedom. For finite samples this test is not exact, and it may be very badly biased. An alternative approach is suggested by Beale, based on the fact that for linear models the exact test involving the likelihood ratio uses the $F$ distribution. Following Beale's suggestions, it appears that for this particular model his recommended procedure for small sample asymptotic tests involves using exactly 1, 15.
the same \( F \) ratio which would be appropriate if the model were completely linear.\(^{16}\)

Proceeding, then, as if the model were linear, for the first data set the hypothesis \( \rho = 1 \) may be rejected at the 0.05 significance level; the test statistic \( F \) is 4.45, or slightly larger than 4.12, the critical value of \( F(1, 35) \). An approximate 95 per cent confidence interval for \( \rho \) would include \( .97 \approx \rho \approx -2 \), for this data set. For the data set using \( c_2 \) a similar confidence interval would include \( 1.0 \approx \rho \approx -2 \).

Thus, my conclusion is that a wide range of values for \( \rho \) are consistent with the data, although the "level" specifications give somewhat better explanations of the data than do the "first-difference" specifications. The evidence, especially for the second data set, would not be strong enough to reject either the hypothesis \( \rho = 1 \), or the hypothesis \( \rho = 0 \), if there was any good reason for giving either one the strong presumption that is usually accorded a null hypothesis. My point, however, is that no such reason exists, except for computational convenience. If the true value of \( \rho \) is somewhere between 0 and 1, then the errors entering the level specification will be autoregressive and the errors entering the first difference specification will be a mixed autoregressive moving average process. In neither case will ordinary least squares be optimal for providing point estimates of parameters or providing the raw material for hypothesis testing. Instead, it seems desirable to me to proceed only on the maintained hypothesis that the errors are a first order autoregressive process. Nevertheless, if only in order to be able to compare my results to those of Eisner and Nadiri, I will also test various models within the contexts of two other maintained hypotheses: (1) \( \rho = 1 \), and (2) \( \rho = 0 \).

Proceeding conditionally on each of these three stochastic specifications there are a number of interesting hypotheses about the other parameters which may be tested. The hypotheses which Eisner and Nadiri attempt to test formally are:

(a) \( E_p = 1 \) (by seeing whether \( 1 - \Sigma (\hat{\gamma} + \hat{a}) \) differs significantly from 1);

(b) \( E_q = 1 \) (by seeing whether \( 1 - \Sigma (\hat{\gamma} + \hat{a}) \) differs significantly from 1);

(c) \( E_p = E_q = 1 \), conditional on the hypothesis that \( \gamma_{pi} = \gamma_{qi} \) for all \( i \) (by seeing whether \( 1 - \Sigma (\hat{\gamma} + \hat{a}) \) is significantly different from 1 in regressions where all of the \( \gamma_{pi} \) are constrained to be equal to the \( \gamma_{qi} \).

The test statistics which Eisner and Nadiri use are all \( t \) statistics which are appropriate for testing a single linear restriction on the coefficients. All of the above hypotheses, and a number of others which require more than one linear restriction may be tested using statistics which will have \( F \) distributions in the event the hypothesis is true.

For the purpose of testing a wide range of hypotheses which various investigators, including Eisner or Jorgenson, might want to put forth as null hypotheses, I have computed sums of squared residuals for twelve alternative models in addition to the basic one specified in equation (9) above. Several of the models are special cases of other ones, and any particular investigator probably would not want to use more than one of them as a null hypothesis.\(^{17}\)

Eisner and Nadiri, who are worried about (a) the possibility that relative prices may have no effect; (b) the possibility that even if there is an effect, it may only be on the timing of investment expenditures;\(^{18}\) or (c) the possibility that the underlying function is CES with constant returns to scale but \( \sigma < 1 \), would presumably wish to use either (a) Model 3, in which no relative price terms appear, or (b) Model 5, in which \( E_p = 0 \), or Model 11 in which \( E_p = 0 \) and \( E_q = 1 \), or (c) Model 8 in which \( E_p = 1 \) and \( E_q \) is not restricted.

None of the models corresponds exactly to the Jorgenson-Stephenson model, since their

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\(^{16}\) A fuller discussion of why this is the case is contained in a more extensive multilingual version of this article, available from the author.

\(^{17}\) To conserve space, only eight of the alternative models are discussed in this paper, and only three of them are considered in this section. Readers interested in the other models are referred to the more extensive version of the paper which may be obtained from the author.

\(^{18}\) The notion, that relative prices may affect capital stock in the short run but not in the long run, was formally developed by Eisner and Strotz in their study for the Commission on Money and Credit (81), pp. 63-87. The theoretical model is one in which the speed of adjustment of actual to "desired" capital stock is a function of the interest rate.
specification is linear in the variables rather than linear in the logarithms of the variables. However, Model 10, in which \( E_p = E_q = 1 \) and the lag distribution for the relative price terms is the same as the lag distribution for output, comes closest to being the embodiment of all of their maintained hypotheses. Model 9, with \( E_p = E_q = 1 \), includes the Cobb-Douglas part of their hypothesis but relaxes the assumption of identical lag distributions. Model 4 maintains the assumption of identical lag distributions, but relaxes the assumption that the elasticity of \( K \) with respect to \( pQ/c \) is unity. Model 6 has \( E_p = 1 \) with \( E_q \) unrestricted; the interpretation of this model is unclear for \( E_q = 1 \) the underlying model implies that \( E_q \) should also be 1.

Only models 1, 6, 8, and 9 are applied to the data for manufacturing capital stock. Because the results are so inconclusive, extensive exploration of all the other models does not seem justified with these data. In section III the other models are applied to a set of data which permits sharper discrimination between them.

Applying these models to the question of statistical inference about the elasticities \( E_p \) and \( E_q \), table 2 gives information about: (a) whether or not a one-dimensional 95 per cent confidence interval about the maximum likelihood estimate of \( E_p \) would include the point \( E_p = 1 \); (b) whether or not a one-dimensional confidence interval drawn about the maximum likelihood estimate of \( E_q \) would include the point \( E_q = 1 \); and (c) whether or not a two-dimensional confidence interval would include the point \( E_p = 1, E_q = 1 \). Because the Cobb-Douglas specification fixes both elasticities, I feel that (c) is the most meaningful question to ask with respect to Jorgenson’s maintained hypothesis. This question was never examined by Eisner and Nadiri, but examination of part (1) of table 2 indicates that if it had been asked the answer would have been no, within the context of their maintained hypothesis about the errors. Thus, a researcher who only examined the first differenced data would feel quite secure in concluding, as they did, that “the role of relative prices is not confirmed,” “the elasticity of capital stock with respect to output is reasonably high . . . .”, and “results contradict Jorgenson’s assumption of a Cobb-Douglas production function . . . .”

By way of contrast, a researcher examining only the regressions using levels of all the variables (see part (2) of table 2) would conclude, for the first set of data, that the estimated elasticity of capital stock with respect to relative prices was substantial (.629), apparently confirming their role in the investment process. Furthermore, this point estimate is not even close to being significantly different from one. Finally, he would be able to conclude that the Cobb-Douglas assumption was not contradicted, for the test statistic for Model 9 is well below the critical level for rejection of the hypothesis at conventional levels. Of course, the point estimate of output elasticity (-.434) might cause some concern; it is even more nonsensical than the maximum likelihood estimate. Certainly one might suspect something peculiar about the model or the data. On the other hand, one could blame the sample, in view of the large confidence intervals. In this situation, not much empirical discrimination about the long-run elasticities is possible.

Using the second set of data, the point estimate of \( E_q \) is much more respectable (.773), but although the point estimate of \( E_p \) is much smaller (.117) the rest of the story would be the same; the confidence region for the long-run elasticities is very large. In particular, with either \( \rho = 0 \) or the first order autoregressive assumption, Model 9 is not rejected.

Again, my point is that neither of these procedures is defensible. In order to avoid mistaken conclusions a stochastic specification at least as general as the autoregressive one used here would seem to be the minimum required. Ideally, one should specify a variety of behavioral models for generating the underlying lag distributions, and test a variety of stochastic

\[\text{References:}\]

1. [7], pp. 380–381. Table 2 covers only the data set using \( e_\tau \), but the same conclusion holds for the other data set. The test statistic for Model 9 is 4.05, which exceeds the critical value of \( F(2, 36) \) at the 0.05 level. Model 6 is also rejected, but Model 8 is not.

2. Although the results are not shown in table 2, Model 13, in which both elasticities are zero, is not rejected either. If one started out believing in such a model, analysis of these data would not contradict that belief.

3. With \( \rho = 0 \), the test statistic is 1.45, must less than 3.26, the critical value of \( F(2, 36) \) at the 0.05 level. With the assumption of autoregressive errors, the test statistic for Model 9 is 2.15.
### Table 2.—Test Statistics for Alternative Models of Demand for Manufacturing Capital, Given Alternative Maintained Hypotheses about Stochastic Specification

<table>
<thead>
<tr>
<th>Model No.</th>
<th>$\Delta^2 \times 10^4$</th>
<th>$F_{statistic}$ for $H_0$: Model $i$; $H_1$: Model $j$</th>
<th>Critical Value of $F$ (0.05 level)</th>
<th>Result of Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Maintaining the hypothesis $\rho = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (no restrictions)</td>
<td>0.56848</td>
<td>6.02</td>
<td>4.11$^b$</td>
<td>Reject Model 6</td>
</tr>
<tr>
<td>6 ($E_0 = 1$)</td>
<td>0.5635</td>
<td>.07</td>
<td>4.11$^b$</td>
<td>Do not reject</td>
</tr>
<tr>
<td>8 ($E_0 = 1$)</td>
<td>0.58386</td>
<td>3.87</td>
<td>3.26$^a$</td>
<td>Reject Model 9</td>
</tr>
<tr>
<td>9 ($E_0 = E_0 = 1$)</td>
<td>0.62903</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Maintaining the hypothesis $\rho = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (no restrictions)</td>
<td>0.51389</td>
<td>.04</td>
<td>4.11$^b$</td>
<td>Do not reject</td>
</tr>
<tr>
<td>6 ($E_0 = 1$)</td>
<td>0.51447</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 ($E_0 = 1$)</td>
<td>0.57286</td>
<td>4.13</td>
<td>4.11$^b$</td>
<td>Reject Model 8</td>
</tr>
<tr>
<td>9 ($E_0 = E_0 = 1$)</td>
<td>0.58162</td>
<td>2.37</td>
<td>3.26$^a$</td>
<td>Do not reject</td>
</tr>
<tr>
<td>(3) Maintaining the hypothesis that errors are a first order auto-regressive process</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (no restrictions)</td>
<td>0.50426 ($\beta = 2$)</td>
<td>.27</td>
<td>4.12$^a$</td>
<td>Do not reject</td>
</tr>
<tr>
<td>6 ($E_0 = 1$)</td>
<td>0.50822 ($\beta = 3$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 ($E_0 = 1$)</td>
<td>0.56220 ($\beta = 2$)</td>
<td>4.62</td>
<td>4.12$^a$</td>
<td>Do not reject</td>
</tr>
<tr>
<td>9 ($E_0 = E_0 = 1$)</td>
<td>0.56416 ($\beta = 3$)</td>
<td>2.08</td>
<td>3.27$^a$</td>
<td>Do not reject</td>
</tr>
</tbody>
</table>

* Data set using $c_1$ as price of capital goods.

$^a$ Significance level is only approximate for this test.

$^b$ $F(1,15)$, $F(2,30)$, $F(1,15)$, $F(1,15)$.

Specifications which make sense in view of the range of plausible hypotheses. It is wellknown that partial adjustment or cost of adjustment distributed lag models suggest much different stochastic specifications than do expectational or "permanent something" models. In addition, distributed lag models for investment demand could arise simply because a single decision (a capital budget made up once a year; an order to begin construction leading to expenditures spread over several quarters) affects the dependent variable in several periods. This third type of distributed lag provides a third type of plausible stochastic specification, and perhaps the most difficult type to deal with because the errors would seem to be inherently a moving average process of perhaps the fourth order.

With due recognition, then, for the fact that the assumption of errors generated by a first order autoregressive process may still be too special for use as a maintained hypothesis in this case, the results summarized in part (3) of table 2 indicate that the conclusion a researcher would have drawn by focussing on the special case $\rho = 0$ are essentially the same as those which emerge from the more general specification. The conclusion—that in conventional statistical terms these data are not inconsistent with the assumption of a Cobb-Douglas production function—is diametrically opposed to the one which seems to be correct if only the special case $\rho = 1$ is considered.

All of this, of course, is at best backhanded support for the maintained hypothesis of a Cobb-Douglas production function employed in the Jorgenson model. Large confidence intervals or not, the point estimates of elasticities are nothing to write home about. The explanation may involve peculiarities of the sample period, or it may involve measurement errors in the data, but it cannot be said that the results reported in this section provide evidence for the model, or for the role of relative prices in investment demand. All that I have attempted to do here is refute the statement that they provide convincing evidence against this model or this role.

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$^{22}$ Zellner and Geisel [25] have recently provided a very interesting example of this sort of exploration using consumption functions.

$^{23}$ Without fiddling with either the specification or the sample period, Model 3, the model in which no relative price terms at all appear, cannot be rejected at any significance level smaller than 0.25. If, however, the number of values of $(\rho/c)$ is reduced to four, or if the sample is reduced to exclude the period before the Treasury-Federal Reserve accord, the hypothesis of no relative price effects may be rejected at the 0.05 level.
To make clear exactly what this re-examination of the work of Eisner and Nadiri does show, it is useful to quote from several of their conclusions, and contrast them to what I have found. Eisner and Nadiri summarize their conclusions in seven paragraphs. Four of these conclusions are at issue. They state:

1. The role of relative prices . . . is not confirmed. The elasticity of capital stock with respect to the ratios of the wholesale price index to Jorgenson's measures of the rental price of capital is far below the value of unity which he has assumed in his papers . . . .

My finding is that, for one of the two sets of data, this is incorrect if \( \rho \neq 1 \). Using \( c_1 \) as the cost of capital, if \( \rho = 0, E_r \) is estimated to be equal to 0.629. The hypothesis \( \rho = 0 \) cannot be rejected, while the hypothesis \( \rho = 1 \) can. The maximum likelihood estimate of \( E_r \), 0.358, is far below unity, but not significantly so.

Eisner and Nadiri also conclude:

3. The elasticity of capital stock with respect to output is reasonably high, in a number of instances approaching unity. This is consistent with flexible accelerator models.

Using the first-order autoregressive specification, with \( c_1 \) as the cost of capital, \( E_q \) is estimated to be 0.028, far below unity. No model I know of would predict this.

With regard to the specific issue of hypothesis testing, the Eisner-Nadiri conclusion is:

4. Results contradict Jorgenson's assumption of a Cobb-Douglas production function . . . .

The basis for this conclusion is the fact that Model 9 can be rejected given the stochastic specification \( \rho = 1 \). With an improved stochastic specification, the results do not contradict the Cobb-Douglas assumption using either measure of the cost of capital.

Finally, Eisner and Nadiri attempt to cast doubt on a particular hypothesis about the lag distributions for the \( p/c \) and \( Q \) terms, which I have advanced elsewhere. This hypothesis is dubbed the "putty-clay hypothesis." If it can be validated, the hypothesis would provide a serious criticism of some of the policy conclusions Jorgenson and others have drawn from their studies. It has not yet been fully tested, but the essential prediction is that relative prices should affect investment expenditures in a much different way, and with a generally much longer lag than output. The idea behind this hypothesis is that factor proportions are not variable after fixed capital has been put into place.

On the basis of lag distributions reported in their tables 5 and 6, Eisner and Nadiri concluded:

6. The role of relative prices suggested by the "putty-clay hypothesis" is not confirmed by estimates using the general Pascal lag distribution.

Again to conserve space, I am not including tables to conform to the ones Eisner and Nadiri included; one such table is included in the next section. Such tables, however, have been constructed, using the maximum likelihood estimates of the parameters of Model 9, for both sets of Jorgenson's data. They conform almost exactly to the qualitative predictions on the "putty-clay model"—the adjustment to changes in relative price is more gradual than the adjustment to changes in output.

Eisner and Nadiri have started toward the very laudable goal of testing Jorgenson's maintained hypothesis, and also peripherally, to explore the "putty-clay hypothesis." Their attempt has floundered because of an inadequate maintained hypothesis of their own about the errors entering their equation. If they had started with the hypothesis \( \rho = 0 \), or with the first order autoregressive hypothesis, it seems doubtful that their paper ever would have been written.

III Application of the Eisner-Nadiri Model to the Demand for Equipment

With respect to the value of the "neoclassical approach," the results of section II are inconclusive. The case for or against the Jorgenson model must rest on other evidence, including the predictive power of the model, with as many built-in assumptions as desired, relative to the predictive power of other models. The case for relative price effects—the effects of

See [2], pp. 4–24, for an extensive discussion developing the basis for expecting such a pattern of lagged effects.

[7], pp. 378–379.
capital goods prices, wages, output prices, the cost of capital to firms, depreciation rules, profits tax rates, tax credits, etc. — must rest on other evidence than analysis of the single aggregate time series which Eisner and Nadiri re-examined and which I have re-examined. Other studies have been made, as mentioned above. These studies are not inconsistent with the view that the price elasticity of capital demand might be unity, although several of them suggest values slightly below unity.28

In a study of equipment demand for the period 1951/3–1965/4, I assumed a CES production function with constant returns to scale, and obtained an estimate of 1.022 for $E_p$.

I also concluded that the lag distributions conformed to the qualitative suggestions of the putty-clay hypothesis. Eisner has argued that this conclusion about the lag distributions is a statistical artifact, produced by “the particular constraints introduced into the Almon lag estimator.” 80

Because the issues raised by Eisner and Nadiri are much broader than the narrow question of whether or not various maintained hypotheses of Jorgenson are correct, and because additional data may help to clarify what implications, if any, their model has for the question of the actual role of relative prices in the investment function, I have used data derived from my earlier study to estimate an economy-wide equipment demand function in the form of equation (9) above.

The data to be analyzed in this section are as follows. The dependent variable, $\ln K_t$, is the natural logarithm of a constant dollar measure of the net stock of producers’ durable equipment for the private sector of the United States economy. The output measure, $Q_t$, is constant dollar gross product of the business sector. The relative price term, $(p/c)_t$, includes several coefficients estimated in my previous study (including a trend term which is somewhat difficult to interpret, but which is relatively unimportant). The variable labelled $p/c$ in this section is defined as:

$$ (p/c)_t = e^{\delta t} (p'_t/c'_t), $$

where $p'_t$ is the implicit price deflator for business gross product and $c'_t$ is the rental price of capital services, defined according to

$$ c'_t = \frac{q_t (r_t + \delta) (1 - k_t - u_t z_t)}{1 - u_t}. $$

In (11) $q_t$ is the implicit price deflator for producers’ durable equipment, $r_t$ is an approximation to the cost of capital of firms, based on the industrial bond yield and the dividend-price ratio.81 $\delta$ is the rate of decline of the value of services provided by a unit of equipment, $k_t$ is the effective rate of tax credit against equipment purchases, $u_t$ is the general rate of income taxation for corporations, and $z_t$ is the present value per dollar of new equipment of the income tax deduction which can be taken for depreciation over the lifetime of the equipment.

As before, trial values of $\rho$ spaced at intervals of length 0.1 in the domain $-1 \leq \rho \leq 1$ are used. The results are presented in table 3. By fitting a quadratic to the three smallest values of $e'e$, it can be estimated that $e'e$ reaches a minimum at about $\rho = -0.068$, and that the minimum of $e'e$ is about $0.9956 \times 10^{-4}$.

As table 3 shows, estimates of the price elasticity are extremely sensitive to the value of $\rho$. For values of $\rho$ close to 1, the estimated price elasticity is very low. In the context of the error specification adopted, however, estimates conditional on large values of $\rho$ may be rejected

The recent work of Michael Evans [9] covering quarterly time series for 13 two-digit industries and the manufacturing aggregate, for the period 1952/1–1967/4, includes a term reflecting the inverse of $p/c$. The point estimate of elasticity of equilibrium capital stock with respect to $p/c$ appears to be in the range between 0.5 and 0.8 (the model Evans uses is not a constant elasticity model, and the elasticity varies over the sample period). In Robert Coen's work [3] [4], two models (for aggregate manufacturing, 1950/1–1966/3) are presented. The relative price term is the average wage divided by $c$. For Coen's preferred (and best-fitting) model the elasticity (measured at the point of means of all variables) of equilibrium capital with respect to $(w/c)$ is only 0.29. In his less-preferred model, which corresponds most closely to the Jorgenson model, this elasticity is 0.58. In comparing these elasticities to those for terms involving $p/c$, it should be noted that in a Cobb-Douglas model the expected elasticity of $K$ with respect to $w/c$ would be only $1/(1 + \alpha/\beta)$ where $\alpha$ and $\beta$ are the elasticities of output with respect to capital and labor, respectively.

28 [2], p. 66.

29 [7], p. 379.

80 The form of this approximation is $r_t = (\alpha + \alpha \delta R_t + \alpha R_t + \alpha \delta) (1 - \alpha \mu_t)$, where $R_t$ is the bond yield and $R_t$ is the dividend-price ratio. In [1], the coefficients $\alpha$, $\alpha$, and $\alpha$, and $\alpha$ were estimated by nonlinear methods, $\alpha$ was set equal to zero, and $\alpha$ was specified on the basis of casual empirical arguments. In this study all of these coefficients are taken as a priori information.
Table 3.—Estimation of Autoregressive Parameter, Demand Equation for Equipment, Total Private Economy *

<table>
<thead>
<tr>
<th>ρ</th>
<th>Σρ^2</th>
<th>E_p</th>
<th>E_q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.7384 × 10^{-4}</td>
<td>.1355</td>
<td>1.0344</td>
</tr>
<tr>
<td>1.0</td>
<td>1.7353</td>
<td>.3159</td>
<td>.8945</td>
</tr>
<tr>
<td>0.0</td>
<td>1.0002</td>
<td>1.0125</td>
<td>1.0277</td>
</tr>
<tr>
<td>-0.1</td>
<td>.9984</td>
<td>1.0199</td>
<td>1.0294</td>
</tr>
<tr>
<td>-0.2</td>
<td>1.0067</td>
<td>1.0203</td>
<td>1.0320</td>
</tr>
</tbody>
</table>

* The specification used is given in equation (9). The sample period extends from 1949/2 through 1966/4.

because they result in significant increases in the sum of squared residuals. The hypothesis ρ = 1 may be rejected at the 0.001 significance level. Using the regression with the constant suppressed, the test statistic for the hypothesis ρ = 1 is 19.0, much greater than the critical value of F(2, 51) for the 0.001 significance level (8.0). An asymptotic 95 per cent confidence interval for ρ would extend from about +0.35 to about -0.42.

Since estimating ρ in every case is a very time consuming process, and since the estimate seems to be not at all significantly different from zero, all further results are based on the assumption that ρ = 0, and that the “level” form of the equation is the most appropriate one. This is not the best statistical procedure, for I am shifting the hypothesis ρ = 0 into the maintained hypothesis, and I have argued above that this is not legitimate. The results indicate, however, that almost all of the models rejected here would almost certainly still be rejected under the more general hypothesis; in view of the large saving on computation, I feel that this procedure may be defensible.

Proceeding conditionally on the stochastic specification ρ = 0, I have estimated the parameters of all of the alternative models mentioned above. Table 4 lists the sums of squared residuals derived by fitting the various restricted models to quarterly data on the stock of equipment, for the sample period 1949–1966. Estimates of E_p and E_q are also included; * a indicates that the value represents an assumption built into the model in question. Every one of the alternative models is a special case of Model 1. Thus, each of the restricted models may be used as a null hypothesis, with the alternative hypothesis being Model 1. Values of the F statistic for each of these comparisons are given in column 5; critical values for rejecting the null hypothesis at the 0.05 significance level are given in column 6.

Adding the first quarter of 1949 to the sample results in a somewhat lower estimate of E_p (0.8276 compared to 1.0002) and a slightly higher estimate of E_q (1.0339 compared to 1.0125), but these estimated elasticities are not in any case significantly different from one (the test statistics are far below the rejection level for Models 6, 8, and 9). All of the other alternative models may be rejected at the 0.01 significance level. The conclusion is that this body of data may well be consistent with a variety of hypotheses about E_p (it seems likely that hypothesized values of E_p at least over the range from 0.5 to 1.2 could not be rejected), but it is not consistent with E_q = 0. Furthermore, the hypothesis of no relative price effects in the short or long runs (Model 3) is rejected, although the test statistic shown in table 4 should be adjusted in some way to allow for preprocessing of the data; as mentioned above, the relative price term p/c is not really known a priori but includes four coefficients estimated in a closely related regression (for the time period 1951/3–1965/4). Making a crude allowance for this, the hypotheses of no relative price effects is still decisively rejected.

The portion of the Jorgenson-Stephenson hypothesis which is rejected is the hypothesis of identical lag distributions. Model 10 may be rejected in favor of Model 1. If, instead, Model

1951/2–1966 sample. None of these estimated elasticities differed significantly from 1.

* The four coefficients in question are the time trend and the three coefficients which feed into the definition of r_t. If I had re-estimated them for each model, the results could only have been to make all models except Model 3 fit better. This suggests that in comparing Model 1 and Model 3, an asymptotic test which would, if anything, be biased in favor of Model 3, would be to proceed as if all of the nonlinear coefficients had been re-estimated in this study. The F ratio for the comparison would then be (0.8743/12)/(1.0557/49) = 3.35, which still exceeds the critical value of F(12, 49) at the 0.01 significance level (2.57).
<table>
<thead>
<tr>
<th>Model Number</th>
<th>$\Sigma d \times 10^4$</th>
<th>$\xi_p$</th>
<th>$\xi_q$</th>
<th>$F$ Statistic for $H_0$: Model $j$</th>
<th>Critical Value of $F$ (0.05 level)</th>
<th>Result of Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0657</td>
<td>.828</td>
<td>1.034</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.9400</td>
<td>0.0 *</td>
<td>1.106</td>
<td>5.44</td>
<td>2.12 *</td>
<td>Reject Model 3</td>
</tr>
<tr>
<td>4</td>
<td>2.4034</td>
<td>1.071</td>
<td>1.071</td>
<td>8.32</td>
<td>2.12 *</td>
<td>Reject Model 4</td>
</tr>
<tr>
<td>5</td>
<td>1.6068</td>
<td>0.0 *</td>
<td>1.099</td>
<td>26.01</td>
<td>4.02 *</td>
<td>Reject Model 5</td>
</tr>
<tr>
<td>6</td>
<td>1.0680</td>
<td>1.0 *</td>
<td>1.026</td>
<td>.114</td>
<td>4.02 *</td>
<td>Do not Reject Model 6</td>
</tr>
<tr>
<td>8</td>
<td>1.0780</td>
<td>1.199</td>
<td>1.0 *</td>
<td>.509</td>
<td>4.02 *</td>
<td>Do not Reject Model 8</td>
</tr>
<tr>
<td>9</td>
<td>1.0825</td>
<td>1.0 *</td>
<td>1.0 *</td>
<td>.419</td>
<td>3.17 *</td>
<td>Do not Reject Model 9</td>
</tr>
<tr>
<td>10</td>
<td>2.4592</td>
<td>1.0 *</td>
<td>1.0 *</td>
<td>7.70</td>
<td>2.06 *</td>
<td>Reject Model 10</td>
</tr>
<tr>
<td>11</td>
<td>1.8250</td>
<td>0.0 *</td>
<td>1.0 *</td>
<td>18.88</td>
<td>3.17 *</td>
<td>Reject Model 11</td>
</tr>
</tbody>
</table>

* The maintained hypothesis for all of these models includes $p = 0$. The sample extends from 1949/1 through 1966/4.

* Indicates parameter value is assumed as part of Model $j$.

$F(8.53)$, *$F(1.55)$, *$F(2.55)$, *$F(3.55)$.

9 is used as the alternative hypothesis, Model 10 is rejected in favor of Model 9, a Cobb-Douglas model with separate lags for output and relative prices. The $F$ statistic for this last comparison is 9.99, substantially exceeding $F_{0.01}(7, 55) = 2.98$. The difference between the two lag distributions may be examined in Table 5, which displays the distributions derived from Model 1 in a format comparable to tables 5 and 6 of the Eissner-Nadiri article. The qualitative features of the distributions are in accord with the suggestion of the "putty-clay" hypothesis—that the relative price effect on investment should be more gradual than the effect of output. Eissner's suggestion, that in earlier work I obtained this result on the basis of constraints built into the distributed lag estimation technique I used, is incorrect, for the result shows up in the present estimation when a general rational distributed lag specification is substituted for the modified Almon distributed lag specification used earlier.

The results of analyzing these data are extremely favorable to the role of relative prices in the investment function. The point estimates of elasticities conform almost exactly to the predictions of the Cobb-Douglas model. The noise level in the data is much lower than the noise level in the Jorgenson-Stephenson data. I can only speculate about why this should be the case. One possibility is that there may be significant differences between models which cover all fixed capital and models, like this one, which include only equipment. The measure of output may be better (deflating gross value added by the wholesale price index does not seem to me the best method to get at real output). The measure of rental price of capital incorporates improved specifications of the way taxes enter into the model, developed independently by Hall and Jorgenson and Coen.
Finally, the longer sample period, extending through 1966, includes a period in which major tax experiments substantially increased the variation in the $p/c$ measure. For all of these reasons, the elasticities derived from these data are likely to be more reliable than those reported in section II, but further research is needed to pin down the influence of the various factors.

My conclusions, from the analysis in this section may be summarized as follows:

1) The stochastic specification is again crucial to the conclusions, but the specification which leads to a first-difference transformation of all variables is decisively rejected.

2) Point estimates of long-run elasticities of the stock of equipment with respect to either relative prices or output are very close to unity.

3) The hypothesis that either or both of these elasticities is zero may be rejected.

4) The hypothesis of no relative price effects may be rejected.

5) The lagged effect of relative prices on the stock of equipment operates more slowly than the lagged effect of changes in output, and the hypothesis that both act with the same lag distribution may be rejected.

IV Concluding Remarks

The implications of this paper go far beyond the question of the validity of the Jorgenson model, narrowly defined. The more basic

$^a$ See [23] for discussion of some of the questions relating specifically to this model.
questions involve whether or not relative prices play a role in factor demand, and how the dynamic mechanism works. There are also questions of how empirical research in this area ought to be carried out, and I have tried to illustrate some of the possible pitfalls of aggregative time series analyses of the type pursued by some researchers.

Two trite lessons are illustrated by the results. The first is the simple point, emphasized by Zvi Griliches in his survey article on distributed lags,† that when using distributed lags, even more than in other time series work, the stochastic assumptions are crucial. The second point is a homely note of caution. As we all know but perhaps too often forget, all statements about confidence intervals are only as good as the underlying assumptions—correctness of the functional form, no errors-in-variables, normality, independence and the rest. Statements about accepting and rejecting hypotheses may be useful ways to communicate, in a somewhat conventional way, certain information about the results of a regression analysis. We must not forget, however, the necessary pound of salt.

REFERENCES


