MONEY AND PERMANENT INCOME: SOME EMPIRICAL TESTS*

By James Tobin and Craig Swan
Yale University

According to an increasingly influential school of thought, centered in this city, variation in the money supply is the principal determinant—indeed virtually the exclusive determinant—of variation in money income. The supporting arguments have been more empirical than theoretical. The empirical evidence has included careful historical narrative [6], systematic investigation of cyclical leads and lags among relevant time series [7], and single-equation regression analyses [4]. Less attention has been given to the task of providing a theoretical rationale of the empirical findings, a monetary theory of income determination to set against the neo-Keynesian models of many macroeconomics textbooks. However, Friedman and Schwartz (FS) have presented an explicit model in [7]. Their “permanent income” theory of money demand has testable implications, and in this paper we test some of them.

The Friedman-Schwartz Model

First, a brief outline of the permanent income theory: FS hypothesize that per capita demand for real money balances is related to permanent real income as follows:

\[ M/P_p = \gamma Y_p \]  

(1)

where

- \( M \) = nominal money stock (Currency + Demand Deposits + Time Deposits), per capita
- \( P_p \) = permanent price index of consumer goods
- \( Y_p \) = permanent real income, per capita

The money stock \( M \) is taken to be exogenous; demand must adapt to the supply. Equation (1) is, as a first approximation, always satisfied; the economy is always on its money demand curve.

The permanent value of a variable—price or income—is a weighted average of its current and past actual values, with account taken of trend. For a variable, \( X(t) \), permanent \( X_p(t) \) is defined as follows:

\[ X_p(t) = (1 + \alpha X)^t \prod_{\ell=0}^{\infty} [X(t-\ell)/(1 + \alpha X)^{t-\ell}]^w_t \]

(2)

where

- \( \alpha X \) is the trend rate of growth of \( X \)
- \( w_t \) is the exponential weight of actual \( X_{t-\ell} \):

\[ \sum_{\ell=0}^{\infty} w_t = 1. \]

With several substitutions and simplifications, (2) can be expressed in a more manageable form. First, taking the logarithms of (2) yields

\[ \log X_p(t) = t \log [1 + \alpha X] \]

\[ + \sum_{\ell=0}^{\infty} w_t \{ \log X(t-\ell) - (t-\ell) \log (1 + \alpha X) \} \]

which can be rewritten as:

\[ \log X_p(t) - t \log (1 + \alpha X) \]

(3)

\[ = \sum_{\ell=0}^{\infty} w_t \{ \log X(t-\ell) - (t-\ell) \log (1 + \alpha X) \} \]

Second, FS assume that the \( w_t \) can be characterized as a simple geometrically
declining series of weights which sum to unity; i.e., \( w_i = w_0(1 - w_0)^i \). It is convenient to note that

\[
\sum_{i=1}^{\infty} w_0(1 - w_0)^i = (1 - w_0) \sum_{i=0}^{\infty} w_0(1 - w_0)^i
\]

(4)

This property of the log structure makes possible the following substitution in (3):

\[
\log X_p(t) = t \log (1 + \alpha X)
\]

\[
= w_0[\log X(t) - t \log (1 + \alpha X)]
\]

\[
+ (1 - w_0)[\log X_p(t - 1) - (t - 1) \log (1 + \alpha X)]
\]

or

\[
\log X_p(t) = w_0 \log X(t) + (1 - w_0) \log X_p(t - 1) + (1 - w_0) \log (1 + \alpha X)
\]

(5)

Taking logarithms of equation (1), the equality of supply and demand for real money balances, gives

\[
\log M(t) = \log \gamma + \delta x \log y_p(t) + \log P_p(t).
\]

(6)

FS assume that the \( w_i \) associated with \( y_p \) and \( P_p \) are identical. Using (5) to calculate \( y_p \) and \( P_p \) gives:

\[
\log M(t) = \log \gamma + \delta w_0 \log y(t)
\]

\[
+ w_0 \log P(t)
\]

\[
+ (1 - w_0)[\log (1 + \alpha P) + \delta \log (1 + \alpha P)]
\]

\[
+ (1 - w_0)[\delta \log y_p(t - 1) + \log P_p(t - 1)]
\]

(7)

Since FS assume that the economy is always on its demand curve for money, they can use (6) for period \( t - 1 \) to eliminate the unobserved permanent variables in (7) and get

\[
\log M(t) = w_0 \log \gamma + (1 - w_0)
\]

\[
[\log (1 + \alpha P) + \delta \log (1 + \alpha P)]
\]

\[
+ \delta w_0 \log y(t) + w_0 \log P(t)
\]

\[
+ (1 - w_0) \log M(t - 1)
\]

(8)

Equation (8) can then be solved for \( \log y(t) \):

\[
\log y(t) = -\frac{1}{\delta} \log \gamma - \frac{(1 - w_0)}{\delta w_0}
\]

\[
[\log (1 + \alpha P) + \delta \log (1 + \alpha P)]
\]

\[
+ \frac{1}{\delta w_0} [\log M(t) - \log P(t)]
\]

\[
- \frac{1 - w_0}{\delta w_0} [\log M(t - 1) - \log P(t)]
\]

(9)

Equation (9) is an expression for real per capita income. It could be converted into an expression for money per capita income by adding \( \log P(t) \) to both sides. This would leave \( \log P(t) \) on the right with a coefficient of \( 1 - 1/\delta \). For purposes of estimation, equation (9) has the advantage of being identified. It can be used to derive estimates of \( \delta \), the elasticity of money demand with respect to permanent income, and \( w_0 \), the weight of current information in estimating permanent values. An equation for money income would be over-

\[1\] However, there are problems in using equation (9). First, as indicated below the price level is not really exogenous; there is another structural relationship concerning the division of increases in aggregate demand between real income and prices. Single equation treatment of equation (9) ignores this other relationship. Second, it is by no means obvious that the stochastic elements in the model produce a well-behaved additive error term in equation (9). Suppose that (1) were \( M/P = \gamma y^\epsilon \exp^\epsilon \) where \( \epsilon \) is a normally distributed error. The error in (9), call it \( \eta(t) \), will then be \( \epsilon(t) - (1 - w_0)\epsilon(t - 1) \). For \( \eta(t) \) to be serially independent, \( \epsilon(t) \) would have to be positively serially correlated in a specific manner. If \( \epsilon(t) \) is serially independent, then \( \eta(t) \) will show negative serial correlation (not the high positive serial correlation shown in the residuals from the level regression).
identified. Although equation (9) concerns real income, there is no implication that the effects of an increase in the money supply will affect real income to the exclusion of prices. Actually FS, like other economists, expect short-run changes in money income to be divided between output and prices.

*Estimates from Annual and Quarterly Regressions 1951–66*

Equation (9) was fitted to annual and quarterly data from 1951 through 1966. Results of level and first difference forms of the regression are reported in Table 1.

On the basis of his study of the consumption function and other work, Friedman estimates the weight of current year income in permanent income at 1/3. (This would imply a value of \( \omega \) of .096 in the quarterly regressions.) Friedman also estimates \( \delta \), the elasticity of money demand with respect to permanent income, to be 1.8 (see [7]).

Our estimates of \( \omega \) are higher than the estimates Friedman has reported and our estimates of \( \delta \) are considerably lower than Friedman’s own in [1]. Friedman’s estimates referred to a longer time period. Virtue does not necessarily lie with long time periods; structural changes have occurred. Commercial banks have in recent decades faced much stronger competition for savings from other financial intermediaries than they did in the late nineteenth and early twentieth century. (See [8, p. 105].) It is not surprising, therefore, that Friedman’s estimates of \( \delta \), the long-run income elasticity of money demand, are higher than ours. Furthermore, the major financial reforms of the 1930’s might well have changed these parameters.

The Durbin-Watson statistics suggest that the level regressions show high positive serial correlation of the residuals. Use of first differences meets this problem, but the explanatory power of the model then becomes very low, and in terms of the theory the estimates of \( \omega \) become absurd. (Indeed, they suggest an opposite model,
in which permanent money holdings are related to current income.) A tempting interpretation is that the correlation exhibited in the level regressions reflects common trends in income and money rather than a causal relationship between the variables, and that there are important, serially persistent nonmonetary determinants of income.

A low value of $w_e$ plays an important part in FS's explanation of the observation that short-run fluctuations in income are larger in amplitude than the monetary fluctuations that cause them. The income velocity of money moves pro-cyclically, and the permanent income model is supposed to explain this fact, among others. When money supply increases exogenously, faster than the permanent price level, permanent income must increase sufficiently to absorb the new money. If $\delta$ is 1.8, as FS estimate, then permanent income must rise .55 percent to create demand for an addition of 1 percent to the real stock of money. But the only component of permanent income that can rise is current income; the past incomes that enter the weighted average are irrevocably fixed. With $w_e = \frac{1}{3}$ for annual incomes, current year's income must rise 3 percent to raise permanent income 1 percent, or 1.65 percent to raise permanent income the necessary .55 percent. The calculation illustrates how the model reconciles FS's finding that velocity declines in the long run ($\delta > 1$) with their finding that short-run changes in money stock cause more than proportionate changes in income.

FS recognize in [7] that the model proves too much if it is applied literally to quarterly data. As already noted, the $\frac{1}{3}$ estimate for $w_e$ for annual data implies a weight of .096 for the income of the current quarter. If the entire income adjustment to a change in money stock must occur within a quarter, then it will be more than three times as large as indicated in the previous paragraph. It will take a 5.7 percent rise in current income for the quarter to raise permanent income .55 percent. As Table 1 indicates, our quarterly regressions indicate much larger values of $w_e$ than the FS model, literally applied, would imply. As FS themselves suggest, perhaps we should relax the assumption that money demand adjusts so rapidly as to keep the community on its demand curve every quarter.

In this spirit we introduce the following modification: Assume people do not adjust their current money balances to income and prices but rather adjust a weighted average of the current and preceding quarters' money balances. Consequently we define $M^*(t)$ as $(1-\beta)M(t) + \beta M(t-1)$ and recompute the quarterly regressions with $M^*(t)$ substituted for $M(t)$. Results for $\beta = .25$, .5, .75 and 1.0 are reported in Table 2. The quarterly results in Table 1 are equivalent to $\beta = 0$. While estimates of $w_e$ decline as $\beta$ rises, they are still large. They imply a much larger response of the demand for money to changes in current income than the FS model.

**Interest Rate Effects?**

An alternative explanation of observed pro-cyclical movements of velocity is sensitivity of money demand to interest rates. Given such sensitivity, short-run fluctuations in income can have nonmonetary as well as monetary causes. If the monetary authorities "lean against the wind," then money supply, interest rates, and velocity will all increase in booms and decline in recessions. This would be a Keynesian interpretation of the same observations that the FS model is designed to explain. It would of course have very different policy implications, leaving room for fiscal policy and exogenous changes in
private spending, as well as monetary events, to affect income.

While Friedman has doubted the empirical significance of interest rates, other than expected changes in the value of money, on the demand for money [1], other researchers have found evidence of such influence. (See, for example, [9] [10].) Certainly there is ample theoretical reason to suppose that the holding of money is influenced by its own real yield—which for the FS definition of money depends both on the rate paid by commercial banks on time deposits and on the expected rate of price change—and by the real yields on substitute assets such as Treasury bills.

These effects can be built into the FS equation (1) as follows:

\[
\frac{M(t)}{P_p(t)} = \gamma P_p(t) \left( \frac{P_p(t)}{P_p(t-1)} \right)^v R_{TD}(t) R_T(t)^* \]

where

\[ R_{TD} = \text{rate paid by commercial banks on time deposits}, \]

\[ R_T = \text{market yield on new issue Treasury bills}. \]

is a measure of the change in the permanent price level; it is also, as can be shown by application of equation (5) letting X
equal \( P(t)/P(t-1) \), the permanent value of price change. Substitutions similar to those previously made in equation (1) yield the following formulation:

\[
\log y(t) = -\frac{1}{\delta} \log \gamma - \frac{1 - \omega_s}{\delta \omega_s} \\
\cdot \left[ \log \alpha_p + \delta \log \alpha_e \right] \\
- \frac{\nu}{\delta} \left[ \log P(t) - \log P(t-1) \right] \\
+ \frac{1}{\delta \omega_s} \left\{ \log M(t) - \log P(t) \right\} \\
- (1 - \omega_s) \left[ \log M(t-1) \\
- \log P(t) \right] \\
- \frac{\eta}{\delta \omega_s} \left[ \log R_{TD}(t) \\
- (1 - \omega_s) \log R_{TD}(t-1) \right] \\
- \frac{\epsilon}{\delta \omega_s} \left[ \log R_t(t) \\
- (1 - \omega_s) \log R_t(t-1) \right]
\]

(11)

It is easily seen that equation (11) is over-identified; aside from constants, there are seven coefficients to determine five parameters. But by fixing \( \omega_s \) it is possible to combine terms for \( M(t) \) and \( M(t-1) \), \( R_{TD}(t) \) and \( R_{TD}(t-1) \), and \( R_t(t) \) and \( R_t(t-1) \) and to regard (11) as an equation involving four coefficients to determine four parameters other than \( \omega_s \). Equation (11) was then estimated by varying \( \omega_s \) in steps between 0 and 1.0 and choosing that value of \( \omega_s \) which maximized the \( R^2 \). Results are presented in Table 3.

The estimates of \( \delta \) in this more general version, equation (10), are larger than before. Perhaps the exclusion of relevant interest rate terms biased downward the earlier estimates of \( \delta \). High income may have served as a proxy for high market interest rates, which have a predominantly negative effect on money holdings. More surprising is the positive coefficient on the \( R_{TD} \) term. This result may be in part explained by the strong correlation between the time deposit rate and the Treasury bill rate and by shifts within \( M \)

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* Elasticity of demand for real money balances with respect to permanent price increase.
* Elasticity of demand for real money balances with respect to the rate paid on time deposits.
* Elasticity of demand for real money balances with respect to the yield on new issue Treasury bills.

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**Table 3**

**Annual and Quarterly Regressions of Income on Money and Rates of Return**

<table>
<thead>
<tr>
<th></th>
<th>Intercept Log</th>
<th>Coefficient of:</th>
<th>Log ( R_t(t) ) Log ( R_{TD}(t-1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{M(t)}{\omega_0} ) Log</td>
<td>( \frac{M(t-1)}{\omega_0} ) Log</td>
<td>( P(t)/P(t-1) ) Log</td>
</tr>
<tr>
<td>Annual</td>
<td>.70</td>
<td>.66 (.050)</td>
<td>-.18 (.239)</td>
</tr>
<tr>
<td>Quarterly</td>
<td>.70</td>
<td>.66 (.025)</td>
<td>-.29 (.339)</td>
</tr>
<tr>
<td></td>
<td>( R^2 )</td>
<td>( DW ) ( R^2 )</td>
<td>( R^2 ) ( R^2 )</td>
</tr>
<tr>
<td>Annual</td>
<td>.98</td>
<td>1.62 .99 1.52</td>
<td>.27 -.08 -.08</td>
</tr>
<tr>
<td>Quarterly</td>
<td>.98</td>
<td>.68 .97 1.52</td>
<td>.44 -.08 -.08</td>
</tr>
</tbody>
</table>

* In fact \( \omega_s \) was first varied from .1 to .9 in increments of .1. \( \omega_s \) was then varied by .01 in the region of the previous maximum.
between demand and time deposits. Most surprising, however, is the apparent positive elasticity of money holdings with respect to price changes. Perhaps people do not in fact extrapolate current and past price trends. Finally, these results do not confirm the hypothesis that behavior will be better explained by relating it to permanent values of income, price, and price change. The estimates of \( w_e \) are so close to one as to eliminate almost all difference of permanent from current values.

**Prediction Tests**

Another test of the basic structure of equation (1) is to see how good a predictor it is. Friedman has expressed the view that “the only relevant test of the validity of a hypothesis is comparison of its predictions with experience” [2, pp. 8–9]. Correspondingly, a variant of (9) was used to predict annual and quarterly changes in money income from 1959 through 1968. Expressing \( \log y(t) \) as \( \log Y(t) - \log P(t) \) and taking first difference of (9) yields:

\[
\Delta \log Y(t) = \frac{1}{\delta w_e} \Delta \log M(t) - \frac{1 - w_e}{\delta w_e} \Delta \log M(t - 1) - \frac{w_e(1 - \delta)}{\delta w_e} \Delta \log P(t)
\]

(12)

Let each first difference be expressed as a deviation from its average value; if \( X \) is subject to a geometric trend \( X(0)(1 + \alpha X)^t \), this contributes a constant, \( \log(1 + \alpha X) \), to growth of \( X \) each period. Thus, let

\[
\bar{Y}(t) = \Delta \log Y(t) - \log (1 + \alpha Y)
\]

\[
\bar{P}(t) = \Delta \log P(t) - \log (1 + \alpha P)
\]

\[
\bar{M}(t) = \Delta \log M(t) - \log [1 + \delta(\alpha Y - \alpha P)]
\]

For the present tests, \( \alpha Y \) and \( \alpha P \) were measured as the actual average compound rates of change of money income and prices from 1950 to 1960. Substituting these definitions into (12) yields

**TABLE 4**

**Predictions of Annual Percentage Changes in Money Income**

1959-67

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>FS13</th>
<th>FS14</th>
<th>Naive(^*)</th>
<th>Trend(^†)</th>
<th>Adjusted Trend(^‡)</th>
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<tr>
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<td>8.3</td>
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<td>3.6</td>
<td>1.1</td>
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<tr>
<td>Average absolute error</td>
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<td>2.7</td>
<td>1.9</td>
<td>1.8</td>
<td></td>
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</tbody>
</table>

\(^*\) Actual change last period taken as forecast.

\(^†\) Average annual rate of change, 1950-60.

\(^‡\) Predicted change for year \( t \) is found by solving for \( Y \)

\[
Y(t - 1) = (1 + \tau)^{t-1950} Y(1950).
\]
### TABLE 5
Predictions of Quarterly Percentage Change in Money Income
1959.1–1968.2

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Actual</th>
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* Actual change last period taken as forecast.
† Average annual rate of change, 1950–60.
‡ Predicted change for year $i$ is found by solving for $r$

$$Y(i-1) = (1 + r)^{(i-1950)}Y(1950).$$

Average absolute change: 1.6
Average absolute error: 3.7
The diagram represents the quarterly percentage changes in money income from 1959.1 to 1968.2. The graph shows the actual change in money income over time, along with two sets of predicted changes based on different models.

The equation for $\bar{Y}(t)$ is given by:

$$\bar{Y}(t) = \frac{1}{\delta w_o} \bar{M}(t) - \frac{1 - w_o}{\delta w_o} \bar{M}(t - 1)$$

Use of (13) yields predictions of deviations of money income per capita from its trend. Predictions of actual percentage changes in aggregate money income can then be obtained by adding population change and trend change in income. FS estimates of $w_o$ and $\delta$, .33 and 1.81, respectively, were used. For quarterly predictions $w_o$ was set equal to .096.

In [7] FS indicate that one might expect prices and money income to move together systematically. They consider the elasticity of the measured price level with respect to measured income and assign it a value of .2. Substituting .2Δ log $Y(t)$ for $\Delta \log P(t)$ in (12) yields another predictor of changes in money income,

$$\bar{Y}(t) = \frac{1}{(.2 + .88)w_o} \bar{M}(t)$$

Results of these predictions are presented in Tables 4 and 5. FS13 indicates predictions based on equation (13) and FS14 indicates predictions based on equation (14). Three simple-minded modes of prediction are presented for comparison, in the last three columns of Tables 4 and 5. Figure 1 illustrates actual quarterly percentage changes in money income and the predictions of those changes based on equation (13). The dashed line, FS13 is a graph of the figures reported in Table 5.
The dotted line FS13' uses estimates of \( \omega \) and \( \delta = .73 \) and 1.04— from our original quarterly regression (see Table 1).

A theory that leads to worse error than the naive hypothesis that last year repeats itself is of questionable reliability. The quarterly predictions are even worse than the annual; see Table 5 and Figure 1. This may be partly due to the problems of lag structure discussed above. As our own experiment indicated, quarterly results can be somewhat improved by relaxing the requirement that current income adjust enough to create demand for all the new money, requiring instead that only a fraction of the new money supply be immediately matched by permanent new demand. This can be done, but only at some expense to the power of the FS hypothesis to produce a large money multiplier and to explain how short-run variations in money creation induce reinforcing changes in velocity.

The moral of the exercises of estimation and prediction that we have presented here is simple. Contrary, perhaps, to much popular belief, the evidence does not support the view that there is a simple, direct relationship of income to money. Policy-makers and forecasters would not have much luck in trying to infer movements of money income from changes in money stock. The permanent income hypothesis is an interesting theoretical rationale for certain qualitative features of observed fluctuation of income and money. But it does not fit postwar data very well, and our results certainly provide no reason to prefer the FS model to a Keynesian interest rate interpretation of short-run fluctuations in the demand for money.

**Data Appendix**

\[ M \]
Demand deposits + time deposits + currency per capita (thousands)

\[ Y \]
Net National Product per capita (thousands of current dollars)
From: *National Income and Product Accounts of the U.S. 1929–1965,* Table 1.9, pp. 12–13

\[ P \]
GNP deflator \((1958 = 100)\)

\[ POP \]
Total Population (millions)
No. 357 Jan. 18, 1967
Population for year defined as population on July 1.
Population for quarter defined as average of population for first day of second and third month of quarter.

\[ y \]
Net National Product per capita (constant 1958 dollars)
\[ Y/P \]

\[ R_{TD} \]
Rate paid by commercial banks on time deposits.
Published figures assumed to represent rate paid on June 30.
Quarterly figures from linear interpolation to mid-quarter.
$R_t$ Market yield on new issue 3-month Treasury bills

From: Business Statistics, 1967 (pp. 90 and 237)
Data for predictions through 1968.2 came from the Survey of Current Business, July, 1968, or from more recent sources as listed above. In order to eliminate the problem of data revisions, values for 66.4 were altered in proportion to the more recent data.

REFERENCES