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A NOTE ON TECHNICAL CHOICE UNDER FULL EMPLOYMENT IN A SOCIALIST ECONOMY[1]

THE problem of choosing the right kind of machines in the course of the development of a socialist economy is indeed a critical one, and we are indebted to both Professor Robinson [2] and Professor Okishio [3] for stressing its importance. Unfortunately, the solutions which they provide which involve switching from one technique of production to another rely crucially on the assumption that the proportion of the labour devoted to the consumption-goods sector be constant; this assumption cannot in any sense be considered justifiable. When this unrealistic assumption is removed, quite different results are obtained, and in fact, the optimum solution will involve continuing changes in the short run in the proportion of labour used in the consumption-goods sector.

We shall show that no matter what the initial endowments and no matter how many techniques are available to the economy, the optimum requires that only one type of machine ever be constructed: the type which minimises the (correctly calculated) labour costs.

1. *The Model*

The economy has available to it a large but finite number of alternative types of machines. A machine of type α requires k_α labourers to construct it, and with one worker, produces b_α units of consumer goods.[4] The economy whose optimal consumption trajectory is analysed here is essentially a discrete capital version of Solow's 1962 model.[5] We assume that α is the most capital-intensive machine available, *i.e.*,

$$k_\alpha > k_\beta \ldots > 0$$
$$b_\alpha > b_\beta \ldots > 0$$

The supply of labour, N, is fixed. To make our analysis as general as possible within the Okishio–Robinson framework, we shall assume a constant

[1] Extensive discussions with my colleague Karl Shell on the questions discussed in this note were invaluable.

[2] Joan Robinson, *Exercises in Economic Analysis* (1960), pp. 38–56.

[3] Nobuo Okishio, " Technical Choice Under Full Employment in a Socialistic Economy," ECONOMIC JOURNAL, September 1966, pp. 585–92.

[4] M. Bruno, in " Optimal Accumulation in Discrete Capital Models," in K. Shell, ed., *Essays in the Theory of Optimal Economic Growth* (M. I. T. Press, 1967), has investigated a similar model in which the production of capital goods of a given type requires capital goods of the same type but in which there are only two techniques available to the economy. For the case of two techniques, the model presented here may be considered a special case of his model, in which the capital requirement of the investment-goods sector are zero.

[5] R. M. Solow, " Substitution and Fixed Proportions in the Theory of Capital," *Review of Economic Studies*, June, 1962, pp. 207–18.

rate of time preference, δ (*i.e.*, consumption goods to-day are worth more than consumption goods to-morrow), an indefinitely long time horizon of planning, and a constant rate of depreciation of capital, μ. Thus the planner wishes to maximize

$$\int_t^{\infty} C(t)e^{-\delta t}dt$$

where $C(t)$ is consumption at time t.

The value of net national output (using consumption goods as numeraire) at time t is simply

$$V = C + p_\alpha \dot{K}_\alpha + p_\beta \dot{K}_\beta \ldots$$

where p_i is the social price of a machine of type i and $\dot{K}_i$ is the change in the number of machines of type i. Total consumption is equal to consumption produced on machines of type α, plus consumption produced on machines of type β, etc., and the change in the number of machines is equal to the new machines built less depreciation of old machines:

$$(1) \qquad V = \min(K_\alpha b_\alpha, N_{c\alpha} b_\alpha) + \min(K_\beta b_\beta, N_{c\beta} b_\beta) \ldots + p_\alpha(N_{I\alpha}/k_\alpha - \mu K_\alpha) + p_\beta(N_{I\beta}/k_\beta - \mu K_\beta) \ldots$$

where $N_{c\alpha}$ is the number of workers on machines of type α and $N_{I\alpha}$ is the number of workers producing machines of type α.

Let us assume that the planner has correctly chosen the prices of the capital goods to represent their true social value. Then to maximise the value of national output at any point of time, subject to the full-employment condition, $N_{I\alpha} + N_{I\beta} + \ldots N_{c\alpha} + N_{c\beta} + \ldots = N$, the planner observes that adding one worker to producing machines of type j increases V by p_j/k_j,[1] and, until the machines of a given type j are fully used, adding one labourer to the jth consumption-good sector increases V by b_j.[2] Hence, the planner finds the maximum p_j/k_j. Call this type of machine σ. There may, of course, be more than one type of machine with $p_i/k_i = \max_j p_j/k_j$, in which case σ represents the set of optimal machine types. All machines for which $b_i > p_\sigma/k_\sigma$ are fully employed. Any left-over labourer is put to work constructing machines of type σ.[3]

The crucial problem for the socialist planner, however, is the choice of the correct social price of each type of machine. The correct social price measures the (discounted) social return over the life of the economy; in

[1] *I.e.*, the marginal product of labour in the jth investment-goods industry is p_j/k_j, and in the jth consumption-good industry (provided it has available capital) is b_j.

[2] Since no capital is required in the investment-goods industry, labour in the investment-goods industry always has a positive marginal product. Hence, full employment is assured.

[3] If the economy is very capital rich, so that all the labourers can be employed on consumption-goods machines, with $b_i > p_\sigma/k_\sigma$, obviously workers are first put on machines of type α; any left over are put on machines of type β, and so on down the line until all the labour is used up. For convenience, we shall label the poorest type machine used as λ.

discrete time, if r_τ is the social return of a machine at time τ, then the "normalised" shadow price of a machine at time t, as viewed at time zero, is given by [1]

$$p_t = \sum_t^\infty r_\tau(1-\delta)^{\tau-t} = r_t + (1-\delta)\sum_{t+1}^\infty r_\tau(1-\delta)^{\tau-t-1}.$$

Since $\quad p_{t+1} = \sum_{t+1}^\infty r_\tau(1-\delta)^{\tau-t-1}$

we have, $\Delta p_t = \delta p_t - r_t$; or, in continuous time,

$$\dot{p} = \delta p - r.$$

The correctly imputed return to capital is, of course, its marginal contribution to the value of national income, or

$$\dot{p}_i = \delta p_i - (\partial V/\partial K_i).^2 \qquad (2)$$

If a machine of type i is used (or would be used if the economy were given some), then one more machine of this type will increase V by b_i, but to man it we must take one labourer off the activity where it is making the smallest contribution. In the case of the capital-poor economy, we must reduce production of investment good σ, by $1/k_\sigma$. If we include depreciation, $dV/dK_i = b_i - p_\sigma/k_\sigma - \mu p_i$. If a machine of type i is not used, then, of course, its marginal contribution would be simply the extra depreciation.[3]

Thus, we can rewrite equation (2) as

$\dot{p}_\sigma = (\mu + \delta)p_\sigma - (b_\sigma - p_\sigma/k_\sigma)$ for the capital good which is produced and used.

(3) $\dot{p}_j = (\mu + \delta)p_j - (b_j - p_\sigma/k_\sigma)$ for the capital goods which are used but not produced.

$\dot{p}_j = (\mu + \delta)p_j$ for capital goods which are neither used nor produced.

A well-known sufficient condition for optimality is the existence of a price system such that

$$\lim_{t\to\infty} p_j(t)e^{-\delta t} = 0.$$

We shall now find such a price system. We shall show, in addition, that there is only one such price system; in fact, we shall show that every other

[1] The "normalised" shadow price of a machine at time t, as viewed at time zero, is simply its shadow price at time t multiplied by (in continuous time) $e^{\delta t}$. In the remainder of the discussion $p_j(t)$ is the "normalised" shadow price.

[2] This is the well-known result from Pontryagin *et al.*, *The Mathematical Theory of Optimal Processes*. $\partial V/\partial K_i$ is the change in V with respect to an increase in K_i, after labour has been optimally reallocated.

[3] For completeness, if an economy is very capital rich,

$$\partial V/\partial k_i = b_i - b_\lambda - \mu p_i \text{ if } b_i > b_\lambda;\ \partial V/\partial K_i = -\mu p_i,\ b_i \leqslant b_\lambda.$$

price system leads eventually to infinite or negative shadow prices. The "correct" pricing system clearly cannot have negative or unbounded prices, since (in discrete time)

$$0 \leqslant \sum_{\tau=t}^{\infty} r_\tau (1-\delta)^{\tau-t} \leqslant \sum_{\tau=t}^{\infty} b_j (1-\delta)^{\tau-t} < \infty.$$

Let us consider what happens as t becomes very large. Machines which are not used must have a social price of 0. On the other hand σ^*, the machine which is eventually produced and used, has a price

$$\hat{p}_{\sigma^*} = \frac{b_{\sigma^*}}{\mu + \delta + 1/k_{\sigma^*}}. \tag{4}$$

For if it had a price greater than p_{σ^*} its price would be rising at a rate faster than δ, and if its price were less than p_{σ^*} its price would be falling continually, eventually becoming negative. To see this note that we can solve $\dot{p}_\sigma = \left(\mu + \delta + \frac{1}{k_\sigma}\right)p_\sigma - b_\sigma$ for

$$p_\sigma(t) = \left(p_\sigma(0) - \frac{b_\sigma}{\mu + \delta + 1/k_\sigma}\right)e^{(\mu+\delta+1/k_\sigma)t} + \frac{b_\sigma}{\mu + \delta + 1/k_\sigma}$$

If $p(0) \neq \frac{b_\sigma}{\mu + \delta + 1/k_\sigma}$, $p_\sigma(t)e^{-\delta t}$ is unbounded (positive or negative). In either case, the condition that p_{σ^*} be positive and finite and the condition that $p_{\sigma^*}e^{-\delta t} \longrightarrow 0$ are not satisfied.

Similarly, it is clear that the capital goods which are not produced, but would be used if they were available to the economy, must have a price

$$\hat{p}_j = \frac{b_j - \hat{p}_{\sigma^*}/k_{\sigma^*}}{\mu + \delta} \tag{5}$$

or otherwise, they, too, would eventually violate the condition that p_j be positive and finite and the condition that $\lim_{t\to\infty} p_j e^{-\delta t} \longrightarrow 0$.

Two things may be observed about those prices. It should be clear that $\hat{p}_j/k_j < \hat{p}_{\sigma^*}/k_{\sigma^*}$ if, and only if, $b_j/[k_j(\mu + \delta) + 1] < b_{\sigma^*}/[k_{\sigma^*}(\mu + \delta) + 1]$. For $\hat{p}_j/k_j < \hat{p}_{\sigma^*}/k_{\sigma^*}$ if, and only if (substituting (5)), $b_j k_{\sigma^*}/[k_j(\mu + \delta) + 1] < \hat{p}_{\sigma^*}$ if, and only if, $b_j/[k_j(\mu + \delta) + 1] < \hat{p}_{\sigma^*}/k_{\sigma^*} = b_{\sigma^*}[k_{\sigma^*}(\mu + \delta) + 1]$. Secondly, if $\hat{p}_j/k_j > \hat{p}_{\sigma^*}/k_{\sigma^*}$, by the reasoning given above (p. 605), j will be produced and used, contradicting our definition of σ. Hence, the capital good which is eventually used and produced must be the one for which $b_i/[k_i(\mu + \delta) + 1]$ is the largest.

We have thus completely characterised the eventual state of the economy: we have found the technique σ^* which is eventually used and produced; those techniques for which $b_j > \hat{p}_{\sigma^*}/k_{\sigma^*}$ are used if machines are available, but as these machines wear out they are not replaced. The prices of these machines remain at $\hat{p}_j$. Those techniques for which $b_j < \hat{p}_{\sigma^*}/k_{\sigma^*}$ are not used—even if there are available machines. Eventually, of course, the machines of type other than σ^* wear out; so that the only machines actually used are type σ^*. The number of labourers in the investment-goods sector is ultimately just enough to replace the machines which wear out in the consumption-goods sector:

$$\dot{K}_{\sigma^*} = \mu K_{\sigma^*} = \mu N_{c\sigma^*} = N_{I\sigma^*}/k_{\sigma^*}$$

or

$$N_{c\sigma^*} = N/(1 + \mu k_{\sigma^*})$$
$$N_{I\sigma^*} = N/(1 + 1/\mu k_{\sigma^*})$$

Consumption per man is, accordingly, eventually given by

$$\frac{b_{\sigma^*} N_{c\sigma^*}}{N} = b_{\sigma^*}/(1 + \mu k_{\sigma^*}) \tag{6}$$

We have yet to tell the history of the economy: how does it get to the configuration described above? We shall now show that as soon as the planning period begins, prices are set at the values given in equations (4) and (5), and the only type of machine ever produced is that which we have denoted by σ^*, *i.e.*, the machine for which $b_i/[k_i(\mu + \delta) + 1]$ is largest.[1] Assume that some other machine, type ρ, were produced. Then there will exist a moment at which the economy switches from producing ρ to producing σ^*. At that time p_ρ/k_ρ must equal $p_{\sigma^*}/k_{\sigma^*}$. But as we have already proved, as soon as σ^* is produced, all prices must take on the values given in equations (4) and (5), *i.e.*, $p_{\sigma^*} = \hat{p}_{\sigma^*}$ and $p_\rho = \hat{p}_\rho$ or $p_\rho = 0$. But $\hat{p}_{\sigma^*}/k_{\sigma^*} > \hat{p}_\rho/k_\rho > 0$; on the other hand, we know from equation (2) that social prices must be continuous.[2] Hence, the only type of machine which is ever produced is σ^*, and prices are immediately set at the values given in equations (4) and (5), and remain there for ever. Employment in the consumption-goods industry and output of consumption goods increase monotonically (if the economy initially is capital poor) to their steady state values.[3]

We thus see that the switching of techniques observed in the Robinson–Okishio analysis was an outcome of the unjustifiable assumption of a constant proportion of the labour force assigned to each sector. There seems to be no reason why a socialist planner would in fact face such a constraint. For purposes of comparison, and as an illustration, it may be worthwhile to tell the story of the development of the simplified economy discussed by Robinson and Okishio. They limit themselves to a situation where there

[1] If there are two types of machines with $b_j/[k_j(\mu + \delta) + 1] = \max_i \{b_i/[k_i(\mu + \delta) + 1]\}$, then the economy is indifferent between them, and will produce either one or both. The rest of the analysis is unaffected by this possibility.

[2] Indeed, this is one of the central theorems of Pontryagin *et al.*, *op. cit.*

[3] Since $N_e = \mu N_e + (N - N_c)/k_{\sigma^*}$, where $N_c(c)$ is employment in consumption good sector at time t, $N_c(t) = [N_c(0) - N/(1 + \mu k_{\sigma^*})]e^{-(\mu + 1/k_{\sigma^*})t} + N/(1 + \mu k_{\sigma^*})$. Similarly $(t) = [C(0) - A - B]e^{-\mu t} + Ae^{-(\mu+1/k_{\sigma^*})t} + B$, where $A = b_{\sigma^*}[N_c(0) - N/(1 + \mu k_{\sigma^*})]$, and $B = b_{\sigma^*}/(1 + \mu k_{\sigma^*})$.

are three types of machines, α (the most capital intensive), β and γ. Initially, the economy has only γ-type machines, but not enough to employ all the available labour force. First, we consider the case where there is negligible time preference,[1] and depreciation is negligible. Hence, the only type of machine which will ever by built is α, since $b_i/[k_i(\mu + \delta) + 1]$ is approximately b_i in that case, and $b_\alpha > b_\beta > b_\gamma$. Prices are set as follows: $p_\alpha = b_\alpha k_\alpha$, $p_\beta = p_\gamma = 0$. Finally, since $p_\alpha/k_\alpha > b_\beta > b_\gamma$, the existing machines of type γ are never used. As α machines are built, workers are immediately employed in producing consumption goods on them. The number of workers working in the consumption-goods industry and the output of consumption-goods industry increases monotonically.

Note that if μ or δ had not been assumed to be negligible, then γ machine might have been used. All the workers for which there were not machines —of any type—available would work in the investment-goods industry. Observe that if

$$\frac{b_\beta - b_\gamma}{k_\beta b_\gamma - k_\gamma b_\beta} > \mu + \delta > \frac{b_\alpha - b_\beta}{k_\alpha b_\beta - k_\beta b_\alpha}$$

type β machine is the only one produced; while if

$$\mu + \delta > \frac{b_\beta - b_\gamma}{k_\beta b_\gamma - k_\gamma b_\beta}$$

γ-type machines are the only ones produced.[2]

A Labour Theory of Value Interpretation

What is perhaps remarkable about these results is that they should have been obvious to an economist trained in the labour theory of value. For the machine which is produced, *i.e.*, the machine for which $b_i/k_i\mu + 1$ is largest,[3] is the one which minimises the labour cost of a unit of consumption, properly measured. The direct labour cost of a unit of output on machines of type i is simply $1/b_i$. To this, we must add the indirect labour cost to replace the depreciating machines, which, per unit of output, is k_i/b_i, so that total labour costs using machines of type i are $(1 + \mu k_i)/b_i$. Thus, the machine for which labour costs are minimised is the machine for which $b_i/(k_i\mu + 1)$ is largest. Moreover, the price of this machine, in terms of labour units, is simply the price given by equation (4), which

[1] For boundedness of the maximand $\int_t^\infty Ce^{-\delta t}dt$ we require $\delta > 0$; but δ may be arbitrarily close to zero, and the following analysis is consequently unaffected.

[2] The results on non-switching will not hold in general for the case where the planner's objective function is $\int_0^\infty U(C)e^{-\delta t}dt$. Detailed discussion of this case would, however, take me beyond the scope of this comment. (It should be noted that in this case prices are not constant.)

[3] For purposes of this section, we shall assume that the rate of time preference is negligible (see footnote 1, above).

is the price in terms of consumption goods, times the price of consumption goods in labour units or

$$\text{Price in labour units} = \frac{b_i k_i}{k_i \mu + 1} \frac{1 + \mu k_i}{b_i} = k_i$$

which is exactly what one would have expected.

Finally, observe that the machine actually produced is the one for which steady-state consumption per man, given in equation (6), is maximised.[1]

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[1] According to the neo-neo-classical theorem, or the golden rule, consumption *per capita* is maximised by employing the technique where net social marginal product is equal to the rate of growth of population. This can be shown to hold in this model. Here, since population is constant, $\partial V / \partial K_{\sigma^*}$ must be zero, and indeed, $b_{\sigma^*} - p_{\sigma^*}/k_{\sigma^*} - \mu p_{\sigma^*} = 0$.