FOREIGN TRADE AND ECONOMIC DEVELOPMENT *

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I

INTRODUCTION.

The literature on dynamic models of international trade and growth is somewhat scanty as compared to the vast literature on the static and comparative static aspects of trade theory (See Bhagwati [1] for a survey of the existing literature). Among the few discussions of the effects of accumulation and/or population growth on international trade are Bensusan-Butt [2], Brems [3], Johnson [4] and Verdoorn [5].

Bensusan-Butt studies the effect of accumulation in a highly suggestive two-country multi-commodity model. There are two processes for producing a commodity in each country, a mechanized and a non-mechanized process. Each country starts with zero past

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accumulation using only the non-mechanized process for production of each commodity. An accident starts off the accumulation process in one country. Bensusan-Butt then traces the course of progressive mechanization, emergence of comparative advantage and export industries in the accumulating country. Capital movement from the accumulating country to the other comes at a certain phase in the growth sequence. The impact of size difference between the two countries on this sequence is also discussed. While his discussion is rich in its insights regarding the possible evolution over time of his international economy, the failure of Bensusan-Butt to state and analyze his model in explicit mathematical terms leaves a student of theory wondering as to which elements of his growth sequence will stand in a rigorous analysis.

Brems [3] and Jonson [4] apply Harrod-Domar type growth theory to the international economy. Brems proposes an elaborate two sector (firms and households) model involving 52 equations. In its technological aspects, Brems' model is similar to a dynamic Leontief model and for that reason can result in negative values for capital stock and gross outputs. The mechanism by which current trade deficits or surplus get adjusted is not explicitly mentioned. It is assumed implicitly that relative prices within a country and terms of trade of traded items remain constant over time. Once again the mechanism by which this is brought about is not mentioned.

Johnson's model is essentially a single sector, single factor one. Constant savings and import propensities are assumed. In Part I of the article Johnson postulates fixed exchange rates with current deficits or surpluses erased by capital movements. After analyzing growth in a single country with exponentially growing exports, Johnson introduces a two-country international economy with each country having its own constant propensities to save and import. Time paths of equilibrium output in the two countries and their rates of growth are analyzed. In Part II of the article exchange rate movements keep trade in balance with no capital movements. Johnson's model also yields time paths involving negative outputs in certain situations. Johnson, however, confines his attention to those intervals of time for which output is non-negative (t).

Verdoorn [5] considers an open economy producing a single commodity using 3 factors of production viz. capital, labor and imports. The factors are assumed to be complementary in production with total output determining uniquely the demand for each. On the supply side, a Keynesian savings function, exponential growth of labor force and an export demand function are postulated. Export prices as a ratio of import prices are assumed to grow exponentially.

(t) This procedure is somewhat unsatisfactory. An alternative and perhaps better procedure will be to modify some of the behavior equations in such a situation.
with the rate of growth being a policy parameter. Part of exports, having an autonomous character, grow exponentially. Besides the rate of growth of export prices, the propensity to save and rate of growth of labor force (net of emigration) are treated as instrument variables. International trade balance is postulated. The problem of deriving an optimum constellation of instruments for achieving (say) maximum national income in a particular year is solved. Thus Verdoorn studies the case of growth of a single country in an international economy.

In Section 2 we present a two-factor, three commodity, dynamic general equilibrium model of international trade. We shall identify on of the countries with an already developed country exporting capital goods and the second 'country with a developing country which imports capital goods and exports consumer goods. In this paper, we shall consider the case where the developing country relies exclusively on international trade and foreign aid for its capital requirement. In a subsequent paper, we hope to consider the case where the developing country established a domestic capital goods sector. Foreign aid in our model will take the form of an agreement by the developed country to meet a fixed proportion of the developing country's import bill indefinitely. In considering two primary factors of production (i.e., labor and capital, and neutral technical change in production our model is more general than those of Johnson and Brems while it differs from Verdoorn's in the sense that unitary elasticity of substitution between factors is assumed. Our model is similar to Johnson's and Brems' in that it takes into account the interaction on each other of the growth of the two trading countries. However, our model is less general in that it leaves out monetary problems altogether. It is concerned with tracing out the time paths of such variables as income (defined in terms of a numeraire), capital, barter terms of tradem, etc.

II

THE MODEL WITH FOREIGN AID.

There are two countries in our international economy. Country 1 produces a consumer good (type 1) and a capital good. Country 2 produces only a consumer good (type 2). Country 2 exports part of its output of its consumer good and imports capital goods produced in country 1. Apart from these two goods traded internationally, no other goods or factors of production move between the two countries. Production of each good takes place under constant returns to scale and neutral technical change in both countries. There are two factors

(1) An important consequence of this is that given the production functions of our model, savings ratio has no influence on the asymptotic growth rates of income.
of production, labor and capital, which is identical with the capital
good produced in country \( i \). Capital once created lasts forever.
Labor force is assumed to grow exponentially in both countries.
Production functions in the two countries are of the Cobb-Douglas
type. A unit of consumer good produced by country \( i \) is taken as the
numéraire.

Country \( i \) devotes a constant proportion \( (1 - s_i) \) of its income for
consumption expenditure. Out of this consumption expenditure a
constant proportion \( \varepsilon \) is spent on domestically produced consumer
goods and the remaining proportion \( (1 - \varepsilon) \) is spent on consumer
goods imported from country \( 2 \). Country 2 exports a constant pro-
portion \( s_2 \) of its output of consumer goods, and imports capital goods
from country \( i \). The relative price of a unit of capital goods exported
by country \( i \) in terms of a unit of consumer goods imported by it
is determined by a balance of payments constraint which says that
apportion \( \lambda \) of the import bill of country \( 2 \) is financed through its
exports, \( \lambda < 1 \) being a case of capital transfer from country \( i \) to
country \( 2 \), \( \lambda = 1 \) one of zero capital flow and \( \lambda > 1 \) one of capital
transfer from country \( 2 \) to country \( i \). This flow is assumed to be
an outright grant of capital by one country to the other with no necessity
of repayment in any form. Transportation costs will be assumed to
be zero. Perfect competition within each country and free trade
between countries will be assumed. Time will be treated as continuous.
The problem is to trace the time paths of income, outputs of various
commodities, factor prices, terms of trade, etc. The effect of changing
some of the parameters of the model on the time paths will also be
evaluated. The following notation will be used:

\[
\begin{align*}
Y_i(t) & \equiv \text{Income at } t \text{ of country } i \text{ in terms of the numéraire} \\
C_i(t) & \equiv \text{Consumption expenditure of country } i \text{ at time } t \\
p_i(t) & \equiv \text{Price per unit of consumer goods produced in country } \\
& \quad i \text{ at time } t \ (p_1(t) \equiv 1 \text{ by choice of numéraire}) \\
q_i(t) & \equiv \text{Price per unit of capital goods at } t \\
r_i(t) & \equiv \text{Rental per unit of time per unit of capital stock in} \\
& \quad \text{country } i \text{ at time } t \\
w_i(t) & \equiv \text{Wage rate per unit of labor at time } t \text{ in country } i \\
Q_{it}(t) & \equiv \text{Output of consumer goods in country } i \text{ at time } t \\
Q_{lt}(t) & \equiv \text{Output of capital goods in country } l \text{ at time } t \\
K_i(t) & \equiv \text{Stock of capital in country } i \text{ at time } t \\
K_i(t) & \equiv \text{Rate of change of } K_i(t) \text{ with respect to } t \\
L_i(t) & \equiv \text{Labor force in country } i \text{ at time } t \\
X_i(t) & \equiv \text{Exports in physical units of country } i \text{ at time } t \\
K_{ij}(t) & \equiv \text{Stock of capital devoted to the production of goods } j \\
& \quad (j = c, k) \text{ in country } i \\
L_{ij}(t) & \equiv \text{Labor services devoted to the production of goods } j \\
& \quad (j = c, k) \text{ in country } i \\
\theta_i & \equiv \text{Proportionate rate of growth of labor force in} \\
& \quad \text{country } i
\end{align*}
\]
\( \sigma_{it} \) = Proportionate rate of growth of neutral technical change in country \( i \) \( (i = 1, 2) \) in the production of goods \( j \) \( (j = c, k) \). \( \sigma_{kt} \) is not defined as country 2 does not produce any capital goods.

\( \alpha_{i} \) = Elasticity of the output of capital goods with respect to capital input in country \( i \).

\( \beta_{i} \) = Elasticity of the output of consumer goods in country \( i \) with respect to capital input.

The following equations summarize the model in algebraic terms:

**Country 1.**

1. \( Y_{1}(t) \equiv C_{1}(t) + q_{1}(t) K_{1}(t) + q_{1}(t) X_{1}(t) - p_{2}(t) X_{2}(t) \)
2. \( C_{1}(t) = (1 - s_{1}) Y_{1}(t) \), \( 0 < s_{1} < 1 \)
3. \( C_{1}(t) \equiv Q_{1e}(t) + p_{2}(t) X_{2}(t) \)
4. \( Q_{1e}(t) = e^{\alpha_{1} t} C_{1}(t) \), \( 0 < \alpha_{1} < 1 \)
5. \( \dot{K}_{1}(t) + X_{1}(t) = Q_{1e}(t) \)
6. \( Q_{1e}(t) = e^{\alpha_{1} t} K_{1e}(t) L_{se}^{1 - \beta_{2}}(t) \), \( 0 < \beta_{2} < 1 \)
7. \( Q_{1e}(t) = e^{\alpha_{1} t} K_{1e}(t) L_{se}^{1 - \beta_{1}}(t) \), \( 0 < \alpha_{1} < 1 \)

8a. \( K_{1e}(t) + K_{1e}(t) = K_{1}(t) \)
8b. \( \dot{K}_{1}(t) = \frac{dK_{1}(t)}{dt} \)
8c. \( L_{1e}(t) + L_{se}(t) = L_{1}(t) \)
8d. \( L_{1e}(t) = e^{\beta_{1} t} \)
8e. \( \beta_{1} e^{\alpha_{1} t} \left[ \frac{K_{1e}(t)}{L_{se}(t)} \right]^{-1 - \beta_{2}} = q_{1}(t) \alpha_{1} e^{\alpha_{1} t} \left[ \frac{K_{1e}(t)}{L_{se}(t)} \right]^{-1 - \beta_{1}} = r_{1}(t) \)
8f. \( (1 - \beta_{2}) e^{\alpha_{1} t} \left[ \frac{K_{1e}(t)}{L_{se}(t)} \right]^{-1 - \beta_{1}} = q_{1}(t) (1 - \alpha_{1}) e^{\alpha_{1} t} \left[ \frac{K_{1e}(t)}{L_{se}(t)} \right]^{-1 - \beta_{1}} = w_{1}(t) \)

**Country 2**

13. \( Y_{2}(t) \equiv C_{2}(t) + q_{2}(t) K_{2}(t) + p_{2}(t) X_{2}(t) - q_{1}(t) X_{1}(t) \)
14. \( C_{2}(t) = (1 - s_{2}) Y_{2}(t) \), \( 0 < s_{2} < 1 \)
15. \( p_{2}(t) X_{2}(t) + C_{2}(t) = p_{2}(t) Q_{2e}(t) \)
16. \( \dot{K}_{2}(t) = \frac{dK_{2}(t)}{dt} = X_{1}(t) \)
17. \( Q_{2e}(t) = e^{\alpha_{2} t} K_{2e}(t) L_{se}^{1 - \beta_{1}}(t) \), \( 0 < \beta_{2} < 1 \)
18. \( L_{2e}(t) = e^{\beta_{1} t} \)
19. \( r_{2}(t) = p_{2}(t) \beta_{2} K_{2e}^{\beta_{1}}(t) L_{se}^{1 - \beta_{2}}(t) \)
20. \( w_{2}(t) = p_{2}(t) (1 - \beta_{2}) K_{2e}^{\beta_{1}}(t) L_{se}^{1 - \beta_{2}}(t) \)

**Balance of Payments Constraint**

21. \( \lambda q_{1}(t) X_{1}(t) = p_{2}(t) X_{2}(t) \), \( \lambda > 0 \)
The above equations are explained as follows:
Equation (1) and (13) define national income of a country as the sum of expenditures on consumption, investment and exports less imports (the sign ≡ is used to indicate equality by definition). Equations (2) and (19) express the relation that consumption expenditures form a constant proportion of income. Equation (3) defines total consumption expenditure in Country 1 as the sum of expenditures on the two consumer goods, viz., domestically produced and imported ones. Equation (4) expresses the relation that the expenditure on domestically produced consumer goods forms a constant proportion of total consumption expenditure. Equation (5) states that the output of capital goods equals the sum of the addition to capital stock at home and exports to country 2. Equations (6), (7) and (17) are production function relations. Equations (8), (9), (10) and (18) are self explanatory. Equations (11) and (12) are the factor allocation equations which express the requirement that marginal value products of capital and labor in the consumer goods industry equal their prices and their corresponding values in the capital goods industry. Equation (15) states that the value of output in country 2 is the sum of sales to domestic and foreign consumers. Equation (16) states that the addition to stock of capital in country 2 equals its import of capital goods. Equations (19) and (20) state that the marginal value products of capital and labor in country 2 equal their prices. Equation (21) states that the value of exports of country 2 forms a proportion λ of the value of its imports.

In solving the model it is convenient to introduce the following factor ratio variables:

\[ \delta_1(t) = \frac{K_1(t)}{L(t)}, \quad \delta_{1\epsilon}(t) = \frac{K_{1\epsilon}(t)}{L_{1\epsilon}(t)}, \]

\[ \delta_{1\kappa}(t) = \frac{K_{1\kappa}(t)}{L_{1\kappa}(t)}, \quad \delta_\eta(t) = \frac{K_\eta(t)}{L_\eta(t)} \]

It is shown in Appendix A that the solution to this model is given by the following:

Let

\[ \eta = \frac{\beta_1 (I - \alpha)}{\alpha_1 (I - \beta_1)}, \quad \mu = \frac{\varepsilon (I - \beta_3) (I - s)}{(I - \alpha_4) [s_1 + (I - \varepsilon) (I - s_3)]}, \]

\[ \Phi(\lambda) = \left( I - \frac{(I - \varepsilon) (I - \alpha_4) \mu}{\lambda \varepsilon (I - \beta_3)} \right), \quad g = \frac{\sigma_{1\kappa}}{I - \alpha_4} + \Theta_1 \]

\[ \bar{\delta}_1(0) = \left[ \frac{\Phi(\lambda)}{g(\lambda + \mu)^{1-\alpha} (I + \eta \mu)^{\alpha_1}} \right]^{1-\alpha_1}. \]

Let \( \delta_1(0) \) be the initial ratio of total capital stock to labor force in country 1. Then
Country 1

(22) \( \delta_1 (t) = \left[ \delta_1 (0)^{1-a_1} - \delta_1 (0)^{1-a_1} + \delta_1 (0)^{1-a_1} e^{\lambda (1-a_1)} \right]^{1-a_1} e^{\theta_1} \)

(23) \( K_1 (t) = e^{\theta_1} \delta_1 (t) \)

(24) \( \delta_{1e} (t) = \frac{\lambda}{\lambda + \eta} \delta_1 (t), \quad \delta_{1s} (t) = \frac{\eta}{\lambda} \delta_1 (t) \)

(25) \( Q_{1e} (t) = \frac{i}{(\lambda + \mu)^{1-a_1} (\lambda + \eta)^{\alpha_1}} e^{\theta_1} \delta_1 (t) \)

(26) \( Q_{1s} (t) = \frac{i}{(\lambda + \mu)^{1-a_1} (\lambda + \eta)^{\alpha_1}} e^{\theta_1} \delta_1 (t) \)

(27) \( q_1 (t) = \left( \frac{1}{\lambda + \mu} \right)^{1-a_1} \left( \frac{1}{\lambda + \eta} \right)^{\alpha_1} e^{\theta_1} \delta_1 (t) \)

(28) \( r_1 (t) = \frac{\lambda}{\lambda + \eta} \left( \frac{1}{\lambda + \mu} \right)^{1-a_1} \left( \frac{1}{\lambda + \eta} \right)^{\alpha_1} e^{\theta_1} \delta_1 (t) \)

(29) \( w_1 (t) = (1-a_1) \left( \frac{1}{\lambda + \eta} \right)^{\alpha_1} e^{\theta_1} \delta_1 (t) \)

(30) \( Y_1 (t) = \frac{i}{\lambda} Q_{1e} (t) \)

(31) \( X_1 (t) = \frac{i}{\lambda} \left[ \frac{(1-e) (1-s)}{s + (1-e) (1-s)} \right] Q_{1e} (t) \)

(32) \( C_1 (t) = -\frac{Q_{1e} (t)}{\alpha_1} \)

(33) \( K_1 (t) = \left[ \frac{1}{\lambda} \right] \left[ \frac{(1-e) (1-s)}{s + (1-e) (1-s)} \right] Q_{1e} (t) \)

Country 2

(34) \( K_2 (t) = K_2 (0) + \left( \frac{(1-e) (1-s)}{\lambda s + (1-e) (1-s)} \right) \{ K_1 (t) - K_1 (0) \} \)

(35) \( Q_{2e} (t) = e^{(\alpha_1 + \gamma_1)} K_2 (t) \)

(36) \( Y_2 (t) = \frac{(1-e) (1-s)}{s} Y_1 (t) \)
\[ X_2(t) = s_2 \, Q_{2e}(t) \]
\[ p_2(t) = \frac{r - \varepsilon}{\varepsilon} \, Q_{1e}(t) \]
\[ r_2(t) = \beta_2 \frac{r - \varepsilon}{\varepsilon} \, Q_{1e}(t) \]
\[ w_2(t) = (1 - \beta_2) \left( \frac{r - \varepsilon}{\varepsilon} \right) e^{-\theta_e t} Q_{1e}(t) \]

**Intercountry Comparisons**

\[ \frac{Y_2(t)}{Y_1(t)} = \frac{(1 - \varepsilon) \, (1 - s_1)}{s_2} \]
\[ \frac{C_2(t)}{C_1(t)} = \frac{(1 - \varepsilon) \, (1 - s_2)}{s_2} \]
\[ \frac{p_2(t)}{q_1(t)} = \frac{(1 - \varepsilon) \, (1 - s_1)}{s_1 + (1 - \varepsilon) \, (1 - s_1)} \times \frac{Q_{1e}(t)}{s_q Q_{2e}(t)} \]
\[ \frac{r_2(t)}{r_1(t)} = \frac{\beta_2 (1 - \varepsilon) \, \eta u}{\beta_1 s \varepsilon (1 + \eta u)} \times \frac{K_1(t)}{K_2(t)} \]
\[ \frac{w_2(t)}{w_1(t)} = \left( \frac{(1 - \beta_2) \, (1 - \varepsilon) \, \mu}{(1 - \beta_1) \, s \varepsilon (1 + \mu)} \right) e^{(\theta_e - \theta_u)t} \]

**III**

**Some Results**

1) Let us first examine the possible range of values for the parameter \( \lambda \). It will be recalled that \( \lambda \) is the proportion of country 2's import bill paid by its own exports. In other words, country 2 receives foreign aid equivalent to \( (1 - \lambda) \) times the value of its imports. Thus a value of \( \lambda > 1 \) will mean, as we noted earlier, that country 2 aids country 1. There is no logical reason to preclude this possibility and we shall not place any upper bound on \( \lambda \). We see from equation (33) that if \( \lambda \) falls short of \( \lambda \) given by

\[ \lambda = \frac{(1 - \varepsilon) \, (1 - s_1)}{s_1 + (1 - \varepsilon) \, (1 - s_1)} \]
\( \dot{K}_1(t) \) becomes negative. This means, in our model, that country 1, in aiding country 2 exports not only its entire output of newly produced capital but also part of its existing stock of old capital. It is natural to require that this possibility is excluded and we shall therefore require that \( \lambda \geq \lambda \). It is clear that the lower the value of \( \lambda \), the easier it is for country 1 to let country 2 to pay a smaller proportion of its import bill through its exports.

2) It is easily seen from (46) that \( \lambda \) decreases as either \( s \) or \( \varepsilon \) increases. The first of these two results can be explained as follows: From (23) and (26) one can show that as \( s \) increases, ceteris paribus, the existing stock of capital in country 1 at any point of time as well as the flow of newly produced capital goods increase. Naturally the effect is to make the problem of subsidizing the import bill of country 2 through export of capital goods from country 1 easier. It is not easy to provide a simple enough explanation to the second result. An increase in \( \varepsilon \) means ceteris paribus, an increase in the demand for domestically produced consumer goods in country 1, and a corresponding decrease in the demand for consumer goods imported from country 2. However, the effect of an increase in \( \varepsilon \) on the capital stock of country 1 at any point in time cannot be unambiguously determined.

3) From the definition \( \Phi(\lambda) \) it is clear that it is an increasing function of \( \lambda \). From (23) we note that \( K_1(t) \), the stock of capital in country 1 at any time \( t \), is larger for a larger value of \( \Phi(\lambda) \). Putting these two together we can state that as \( \lambda \) (the proportion of the import bill of the capital importing country 2 met by its export earnings) increases the stock of capital in the capital exporting country 1 at each point of time \( t \) increases. The increase in \( \lambda \), through its effect on total capital stock, increases also the flow of output of both consumer and capital goods in country 1. However, the effect of an increase in \( \lambda \) on the stock of capital in country 2 is not unambiguous. This effect, as can be seen from (34), consists of the sum of two terms: (a) a positive effect due to the positive effect of a larger \( \lambda \) on \( K_1(t) \) and (b) a negative effect arising from the fact that a larger \( \lambda \) means that ceteris paribus, only a smaller addition to the stock of capital \( K_1(t) \) country 2 at any \( t \), can be financed out of given export bill. The net effect may be positive or negative.

4) From (41) and (42) we note that the incomes and consumption expenditures in one country bear a constant proportion to their corresponding values in the other country. This means that these expenditures in two countries have the same rate of growth over time. Since labor force in each country is assumed to grow exponentially over time, income and consumption expenditures per worker in the country with the faster growing labor force will decline steadily over time as a proportion of their corresponding values for the other
country. It may be worth pointing out that these proportions do not depend upon the factor endowments or rates of technical change. In other words difference in these factors between countries are compensated by appropriate price adjustments. The parameters that do influence the proportions are (a) $s_1$ which affects the volume of the stock and flow of capital goods in country 1 and (b) $s_2$ which affects the demand (supply) in country 1 for consumer goods exported by country 2. Thus an increase in $s_1$ or $s_2$ or $s_4$ decreases the income (or consumption expenditure) of country 2 as a proportion of that for country 1.

5) From (31) it is clear that country 1 exports a constant proportion of its current output of capital goods to country 2.

6) From (23) it is clear that asymptotic rate of growth of capital stock in country 1 is $g$. This does not depend either on the saving ratio $s_1$ or the parameters of the production function of the consumer goods industry. This result is well known in the literature on two-sector models of economic growth. However, as we noted in result (1) above the level of the stock of capital depends upon the saving ratio $s_1$. Another implication of this result on growth rate of capital stock is that in the absence of technical progress in the capital goods industry, capital labor ratios (aggregate and industry wise) approach constant values as time goes to infinity. This can be seen from (22) and (24).

7) Using (22), (25) and (26) it is easily seen that the asymptotic rate of growth of the output of consumer and capital goods in country 1 are respectively $\frac{\beta \sigma_{1\infty}}{1 - \sigma_{1}} + \sigma_{1k} + \theta_1$ and $g$. The first of these two rates is also the asymptotic rate of growth of income in both countries. It is interesting to note that the common asymptotic rate of growth of income in both countries depends only on the parameters relating to the capital exporting country.

8) We note from (27) that the asymptotic rate of growth of $q_1(t)$ the price of a unit of capital good in country 1 in terms of a unit of consumer good produced in that country is $\sigma_{1k} = \left(\frac{1 - \beta_1}{1 - \beta_1}\right) \sigma_{1\infty}$. Thus $q_1(t) \to \infty$, a positive constant or zero according as $(1 - \sigma_1) \sigma_{1k}$ is $>$, $= \text{or} < (1 - \beta_1) \sigma_{1\infty}$. As one would expect the asymptotic rate of growth of $q_1(t)$ is an increasing function of $\sigma_{1k}$ and a decreasing function of $\sigma_{1\infty}$.

9) We can deduce from (34) that the stock of capital in country 2 will approach asymptotically, a constant proportion of the stock of capital in country 1, the common asymptotic rates of growth of
the two stocks being \( \frac{\sigma_{12}}{I - \alpha_1} + \theta_1 \). The asymptotic rate of growth of \( Q_{2e}(t) \), the output of consumer goods produced by country 2 is 
\[ \sigma_{2e} + \theta_2 (I - \beta_2) \left( \frac{\sigma_{12}}{I - \alpha_2} + \theta_1 \right) \].

The relative price of a unit of output of country 2 in terms of a unit of consumer goods produced by country 1 has an asymptotic growth rate of
\[
\left[ \frac{\beta_1 \sigma_{12}}{I - \alpha_1} + \sigma_{2e} + \theta_1 \right] \left[ \sigma_{2e} + \theta_2 (I - \beta_2) + \beta_2 \left( \frac{\sigma_{12}}{I - \alpha_2} + \theta_1 \right) \right]
\]
or
\[
\frac{\sigma_{12}}{I - \alpha_2} (\beta_1 - \beta_2) + \sigma_{2e} - \sigma_{2e} + (I - \beta_2) (\theta_1 - \theta_2)
\]
This is seen from (38).

10) The terms of trade of country 2 i.e., \( \left\{ \frac{p_2(t)}{q_1(t)} \right\} \) (the relative price of a unit of its exports in terms of a unit of its imports) has an asymptotic rate of growth of \( (I - \beta_2) \theta_1 - \theta_2 + \frac{\sigma_{12}}{I - \alpha_2} - \sigma_{2e} \).

Thus \( \frac{p_2(t)}{q_1(t)} \) will approach \( \infty \), a positive constant or zero as time goes to infinity according as \( (I - \beta_2) g \) is \( > \) or \( = \) or \( < \) \( (I - \beta_2) \theta_1 + \sigma_{2e} \).

Dividing through by \( (I - \beta_2) \) we can say that
\[
\lim_{t \to \infty} \frac{p_2(t)}{q_1(t)} = \text{constant} \quad \Rightarrow \quad g \geq g'
\]
where \( g' = \frac{\sigma_{2e}}{I - \beta_2} + \theta_2 \). Now \( g' \) would have been the asymptotic rate of growth of output \( (Q_{2e}) \) of country 2 if the terms of trade were fixed. With variable terms of trade, this rate becomes \( g'' = (I - \beta_2) g' + \beta_2 g \). Now \( g'' - g' \) has the same sign as \( (g - g') \).

Hence the actual asymptotically rate of growth of \( Q_{2e} \), namely \( g'' \), exceeds the potential rate of growth \( g' \) with fixed terms of trade if and only if the actual terms of trade improve over time. In the special case of no technical change in either country, the above result implies that the terms of trade of the country with the faster rate of growth of labor force will deteriorate over time.

11) It can be seen from (23), (34) and (44) that the competitive rate of rental on capital stock in country 2 relative to that in country 1 approaches a constant as time goes to infinity. It can be shown that this asymptotic ratio of rentals per unit of capital stock, increases, ceteris paribus, as either \( (a) \lambda \) increases or \( (b) \) \( s_1 \) increases or \( (c) \) \( s_2 \).
decreases. These results can be explained as follows: A larger \( \lambda \) implies a smaller import of capital goods by country 2 from a given quantity of its exports. This in turn implies smaller rate of investment and total capital stock in country 2, thus raising the rental rate in country 2 asymptotically in relation to the rental rate in country 1. The increase in \( s \) increases the capital stock in country 1 relative to that in country 2. An increase in \( s \) has the opposite effect.

12) It can be seen from (45) that the competitive wage rate in the country with faster rate of growth of labor force falls over time relative to the wage rate in the other country.

**Appendix A**

Let us solve the system of equations representing the model of Section 2. From equations (11) and (12) and using the definitions of the factor ratio variables we get

\[
\frac{1 - \beta_1}{\beta_1} \delta_{1t} (t) = \frac{1 - \alpha_1}{\alpha_1} \delta_{1t} (t)
\]

or

(A 1)

\[
\delta_{1t} (t) = \eta \delta_{1t} (t)
\]

where

\[
\eta = \frac{\beta_1 (1 - \alpha_1)}{\alpha_1 (1 - \beta_1)}
\]

Using (A 1) in (12) we get

(A 2) \( q_t (t) = \left( \frac{1 - \beta_1}{1 - \alpha_1} \right) \eta \delta_{1t} (t) \delta_{1t} (t) \delta_{1t} (t) e^{(\alpha_{1t} - \alpha_{1t})^t} \)

Using (2), (4) and (21) in (1) we obtain

(A 3) \( q_t (t) [\dot{X}_t (t) + (1 - \lambda) X_t (t)] = \frac{\delta_1}{1 - \delta_1} C_t (t) = \frac{\delta_1}{(1 - \delta_1)} \frac{1}{\varepsilon} Q_t (t) \)

Now from (5) and (7) we get

\[
\dot{X}_t (t) = e^{\alpha_{1t}} L_{1t} (t) \delta_{1t} (t) \delta_{1t} (t) \delta_{1t} (t)
\]

or

\[
q_t (t) [\dot{X}_t (t) + (1 - \lambda) X_t (t)] = q_t (t) e^{\alpha_{1t}} L_{1t} (t) \delta_{1t} (t) \delta_{1t} (t) \delta_{1t} (t) - \lambda q_t (t) X_t (t)
\]

Using (21)

\[
q_t (t) e^{\alpha_{1t}} L_{1t} (t) \delta_{1t} (t) \delta_{1t} (t) \delta_{1t} (t) - \lambda q_t (t) X_t (t)
\]

using (3) and (4)
Using (A 3) this can be simplified to yield

\[
(A 4) \quad \left[ s_1 + \frac{(1 - \varepsilon)(1 - s_1)}{\varepsilon(1 - s_1)} \right] Q_{1e}(t) = q_1(t) e^{\sigma_{1e} t} L_{1e}(t) \delta_{1e} a_1
\]

Substituting for \( Q_{1e}(t) \) and \( q_1(t) \) from (6) and (A 2) respectively and using (A 1) we get

\[
\left[ s_1 + \frac{(1 - \varepsilon)(1 - s_1)}{\varepsilon(1 - s_1)} \right] e^{\sigma_{1e} t} L_{1e}(t) \eta_1 \delta_{1e}(t) \beta_1 =
\]

\[
\left[ \frac{1 - \beta_1}{1 - \alpha_1} \right] \eta_1 \delta_{1e}(t) \beta_1^{-a_1} e^{\sigma_{1e} t} L_{1e}(t) \delta_{1e}(t) a_1
\]

Canceling out terms common to both sides we have

\[
\left[ s_1 + \frac{(1 - \varepsilon)(1 - s_1)}{\varepsilon(1 - s_1)} \right] L_{1e}(t) = \left[ \frac{1 - \beta_1}{1 - \alpha_1} \right] L_{1e}(t) \quad \text{or}
\]

\[ (A 5) \quad L_{1e}(t) = \mu L_{1e}(t) , \]

where

\[
\mu = \frac{\varepsilon(1 - \beta_1)(1 - s_1)}{(1 - \alpha_1)[s_1 + (1 - \varepsilon)(1 - s_1)]}
\]

Using (A 5) in (9) and (10) we get

\[ (A 6) \quad L_{1e}(t) = \left( \frac{\mu}{1 + \mu} \right) e^{\Theta_{1e} t} L_{1e}(t) = \left( \frac{1}{1 + \mu} \right) e^{\Theta_{1e} t} \]

We can rewrite (8) as follows

\[ (A 7) \quad \delta_{1e}(t) L_{1e}(t) + \delta_{se}(t) L_{1e}(t) = \delta_1(t) L_1(t) \]

Using (10), (A 1) and (A 6) in (A 7) we obtain

\[ (A 8) \quad \left( \frac{1}{1 + \mu} \right) \delta_{1e}(t) = \delta_1(t) = e^{-\Theta_{1e} t} K_1(t) \]

From (5) and (7) we get

\[ K_1(t) = Q_{1e}(t) - X_1(t) \]

\[ = Q_{1e}(t) - \frac{\beta_1(t) X_1(t)}{\lambda_{1e}(t)} \quad \text{[because of (21)]} \]

\[ (A 9) \]

\[ = Q_{1e}(t) - \left( 1 - \frac{\varepsilon}{\varepsilon} \right) \frac{Q_{1e}(t)}{\lambda_{1e}(t)} \quad \text{[using (3) and (4)]} \]

Using (6) and (7) we can rewrite (12) as

\[ (12)' \quad (1 - \beta_1) \frac{Q_{1e}(t)}{L_{1e}(t)} = q_1(t) (1 - \alpha_1) \frac{Q_{1e}(t)}{L_{1e}(t)} \]
Hence
\[ \frac{Q_{12}}{q_1(t)} = \frac{(1 - a_1)}{(1 - b_1)} \left( \frac{L_{12}(t)}{L_{11}(t)} \right) Q_{13}(t) \]
(A 10)

\[ = \frac{(1 - a_1)}{(1 - b_1)} \cdot Q_{13}(t) \] [using (A 6)]

Using (A 10) in (A 9) we get
\[ \dot{K}_1 = \left[ \frac{(1 - e) \cdot (1 - a_1) \cdot \mu}{\varepsilon \cdot (1 - b_1) \cdot \lambda} \right] Q_{13}(t) \]
\[ = \left[ \frac{(1 - e) \cdot (1 - a_1) \cdot \mu}{\varepsilon \cdot (1 - b_1) \cdot \lambda} \right] e^{\alpha_1 t} L_{12}(t) \delta_{12}(t) \eta^2 \]
\[ = \Phi(\lambda) e^{\alpha_1 t} \frac{1}{1 + \mu} \cdot e^{\Theta t} \left[ e^{-\Theta t} K_1(t) (1 + \eta) \right] a_1 \] [using (A 6) and (A 8)]

where
\[ \Phi(\lambda) = \left[ 1 - \frac{(1 - e) \cdot (1 - a_1) \cdot \mu}{\lambda \cdot (1 - b_1) \cdot \lambda} \right] = \left[ 1 - \frac{(1 - e) \cdot (1 - a_1) \cdot \mu}{\lambda \cdot (1 - e) \cdot (1 - a_1) \cdot \lambda} \right] \]

Hence
\[ (A 11) \quad K_1(t) = K_1(t-1) = \frac{\Phi(\lambda)}{(1 + \mu)^{1-a_1} (1 + \eta\mu)^{a_1}} \]

Equation (A 11) is the fundamental differential equation of our international economy. Solving (A 11) we get
\[ (A 12) \quad K_1(t) = K_1(t-1) + \frac{(1 - a_1) \cdot \Phi(\lambda)}{\sigma_{12} + \Theta (1 - a_1) (1 + \mu)^{1-a_1} (1 + \eta\mu)^{a_1}} \]

Let us define
\[ (A 13) \quad \overline{K}_1(t-1) = \frac{(1 - a_1) \cdot \Phi(\lambda)}{\sigma_{12} + \Theta (1 - a_1) (1 + \mu)^{1-a_1} (1 + \eta\mu)^{a_1}} \]

Then we can rewrite (A 12) as follows:
\[ (A 14) \quad K_1(t) = \left\{ K_1(t-1) + \overline{K}_1(t-1) \right\} + \frac{\sigma_{12} + \Theta (1 - a_1) (1 + \mu)^{1-a_1} (1 + \eta\mu)^{a_1}}{\sigma_{12} + \Theta (1 - a_1) (1 + \mu)^{1-a_1} (1 + \eta\mu)^{a_1}} \]

Using (A 14) we can solve for the rest of the variables of our model:
\[ (A 15) \quad \delta_1(t) = \left\{ \delta_{12}(t) - \delta_1(t-1) \right\} e^{-\Theta (1 - a_1) t} + \frac{\delta_{12}(t) - \delta_1(t-1)}{\sigma_{12} + \Theta (1 - a_1) (1 + \mu)^{1-a_1} (1 + \eta\mu)^{a_1}} \]
Country $i$

(A.16) \[ \delta_{1i}(t) = \left( \frac{1 + \mu}{\bar{\gamma} + \eta \mu} \right) \Delta_{1i}(t) \]

(A.17) \[ \Omega_{1i}(t) = \left[ \frac{\mu \eta \bar{\gamma}}{(\bar{\gamma} + \mu) \bar{\gamma}} \left( \frac{1 + \mu}{\bar{\gamma} + \eta \mu} \right) \right] e^{(\alpha_i + \Theta) \bar{\gamma} \delta_1(t)} \delta_{1i}(t) \]

(A.18) \[ \Omega_{1i}(t) = \left[ \frac{\mu \eta \bar{\gamma}}{(\bar{\gamma} + \mu) \bar{\gamma}} \left( \frac{1 + \mu}{\bar{\gamma} + \eta \mu} \right) \right] e^{(\alpha_i + \Theta) \bar{\gamma} \delta_1(t)} \delta_{1i}(t) \]

(A.19) \[ g_1(t) = \left( \frac{1 - \beta_1}{1 - \alpha_1} \right) \left( \frac{1 + \mu}{\bar{\gamma} + \eta \mu} \right) \delta_1(t) \delta_1(t) \]

(A.20) \[ w_1(t) = (1 - \beta_1) e^{\alpha_1 \delta_1(t)} \delta_1(t) \delta_1(t) \]

(A.21) \[ r_1(t) = \beta_1 e^{\alpha_1 \delta_1(t)} \delta_1(t) \delta_1(t) \]

(A.22) \[ C_1(t) = \frac{I}{\lambda} \Omega_{1i}(t) \]

(A.23) \[ X_1(t) = \frac{I}{\lambda} \rho_1(t) X_1(t) = \frac{I}{\lambda} \frac{1 - \varepsilon}{\varepsilon} \Omega_{1i}(t) \]

\[ X_1(t) = \frac{1}{\lambda} \left( \frac{I - \varepsilon}{\varepsilon} \right) \left( \frac{I - \alpha_1}{I - \beta_1} \right) \left[ \frac{I}{(\bar{\gamma} + \mu) \bar{\gamma}} \right] e^{(\alpha_i + \Theta) \bar{\gamma} \delta_1(t)} \delta_1(t) \]

(A.24) \[ = \frac{1}{\lambda} \left( \frac{I - \varepsilon}{\varepsilon} \right) \left( \frac{I - \alpha_1}{I - \beta_1} \right) \Omega_{1i}(t) \]

\[ = \frac{I}{\lambda} \left[ \frac{(I - \varepsilon)(I - \alpha_1)}{s_1 + (I - \varepsilon)(I - s_1)} \right] L_{1i}(t) \]

(A.25) \[ Y_1(t) = \left( \frac{I}{I - s_1} \right) C_1(t) \]

\[ = \left[ \frac{I}{\varepsilon (I - s_1)} \right] Q_{1i}(t) \]

(A.26) \[ K_1 = \Omega_{1i} - X_1(t) = \left[ 1 - \frac{I}{\lambda} \left( \frac{(I - \varepsilon)(I - s_1)}{s_1 + (I - \varepsilon)(I - s_1)} \right) \right] Q_{1i}(t) \]
\[ K_2(t) = X_2(t) = \left\{ \frac{(1 - \epsilon)(1 - s_2)}{\lambda s_2 - (1 - \lambda)(1 - \epsilon)(1 - s_2)} \right\} K_1(t) \]

(A 27) \[ K_2(t) = K_2(t) + \left\{ \frac{(1 - \epsilon)(1 - s_2)}{\lambda s_2 - (1 - \lambda)(1 - \epsilon)(1 - s_2)} \right\} (K_1(t) - K_1(0)) \]

\[ Y_2(t) = C_2(t) + p_2(t) X_2(t) = (1 - s_2) Y_2(t) + p_2(t) X_2(t) \]

(28) \[ = \frac{s_2}{\lambda} p_2(t) X_2(t) = \frac{\lambda}{s_2} q_1(t) X_1(t) \]

\[ = \frac{\lambda}{s_2} (1 - \epsilon)(1 - s_1) Y_1(t) = \frac{(1 - \epsilon)(1 - s_1)}{s_2} Y_1(t) \]

(29) \[ Q_{2e}(t) = e^{i\pi_2 + \Theta_2}\beta_2 K_2 \]

(30) \[ X_2(t) = s_2 Q_{2e}(t) \]

\[ p_2(t) = \frac{\beta_2(t) X_2(t)}{X_2(t)} = \frac{\lambda q_1(t) X_1(t)}{K_2(t)} \]

(A 31) \[ = \left( \frac{1 - \epsilon}{\epsilon} \right) Q_{2e}(t) \]

(A 32) \[ w_2(t) = (1 - \beta_2) p_2(t) \frac{Q_{2e}(t)}{L_{2e}(t)} = (1 - \beta_2) e^{-\Theta_2} p_2(t) Q_{2e}(t) \]

(A 33) \[ r_2(t) = \frac{\beta_2 p_2(t) Q_{2e}(t)}{K_{2e}(t)} \]
ERRATA

References


CORRECTIONS

\[ \bar{\delta}_1(t) = \left\{ \begin{array}{c} \delta_1(0) \left( 1 - \alpha_1 \right) - \bar{\delta}_1(0) \left( 1 - \alpha_1 \right) \\ \frac{1}{1 - \alpha_1} g(1 - \alpha_1)t e^{-\alpha_1} \\ \bar{\delta}_1(0) e^{-\alpha_1} g(1 - \alpha_1)t e^{-\alpha_1} \end{array} \right\} \]

All 96

\[ K_1(t)^{-\alpha_1} K_1(t) \]

For

\[ K_1(t)^{-\alpha_1} K_1(t) \]

Read

\[ K_1(t)^{-\alpha_1} K_1(t) \]