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III

The Optimal Rate and Direction of Technical Change

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Economists have long suspected that there is a bias in technical change.[1] Kennedy [4] allowed the entrepreneur to choose the desired bias by introducing the innovation possibility curve (IPC), which shows the trade-off between labor-augmenting and capital-augmenting technical change. The Kennedy model of induced innovation is the combination of the IPC with the neoclassical growth model.

Kennedy and others[2] have described the behavior of the model under the assumption that the economy is competitive and is composed of firms which maximize the instantaneous rate of cost reduction. The equilibrium of the model displays constant relative shares of the two factors, but the bias of technical change varies according to the savings assumption. When the savings rate is a constant and the elasticity of substitution is less than one, the long-run equilibrium has Harrod-neutral technical change.

A natural extension of the Kennedy model is to determine the optimal plan in an economy where the planning authorities choose the savings rate as well as the rate and direction of technical change. Sections 1 through 3 of this essay analyze the planned economy in which only the savings rate and

[1] This work was done during the tenure of a National Science Foundation Cooperative Fellowship. I should like to thank Karl Shell for his many comments.

[2] See the early verbal discussion of Hicks [3]. The IPC was first introduced in the published literature by Kennedy [4], with subsequent analysis by Samuelson [7] and Drandakis and Phelps [2].

the direction of technical change are controlled. Section 4 allows the planning authorities to control the rate of technical change. Finally, in section 5 it is argued that the descriptive model of the competitive economy labors under a theory of the firm that is so dubious as to vitiate the descriptive analysis.

1. The Kennedy Model of Induced Innovation

The economy that we are analyzing is identical to that of Samuelson [7]. There are two productive inputs, labor L and capital K, which are combined to produce a homogeneous output. Production is described by a constant returns to scale, neoclassical production function. Output can either be consumed or invested as capital. Thus, suppressing time subscripts where possible, we have

$$Y = F(K, L; t), \tag{1}$$

$$Y = I + C, \tag{2}$$

where Y, I, and C are instantaneous output, gross investment, and consumption. The function F is homogeneous of degree one in K and L, is twice differentiable, and has diminishing returns. The t parameter in F represents the shifts of the production function over time due to changes in technology. In particular, technical change is disembodied and takes the form of labor augmentation and capital augmentation. We can then rewrite Equation 1 with a time-invariant F as

$$Y = F(\lambda K, \mu L), \tag{3}$$

where λ and μ are the capital- and labor-augmentation factors, respectively.[3]

The augmentation factors are generated according to an innovation possibility curve (IPC) similar to that of Kennedy and Samuelson. Thus the rates of factor augmentation follow the rule:

$$\frac{\dot{\lambda}}{\lambda} = g\left(\frac{\dot{\mu}}{\mu}\right), \tag{4}$$

where $g' < 0$ and $g'' < 0$. A typical IPC is pictured by the solid line in Figure 1.[4]

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[3] This characterization of technical change is general in the sense that in a competitive economy (where only one technique is used at a given point of time) one cannot use observed magnitudes to identify between factor-augmenting technological change and any other form of technical change. For planning purposes, however, we need to know more than just the local properties of technical change.

[4] It is assumed for convenience that the IPC extends into the second and fourth quadrants in Figure 1. This is not strictly necessary for the argument but it is necessary for the generality mentioned in footnote 3. In an early draft Drandakis and Phelps [2] argue that the IPC should remain in the first quadrant, because in any other case isoquants will cross giving the appearance of technical regress. Their argument overlooks

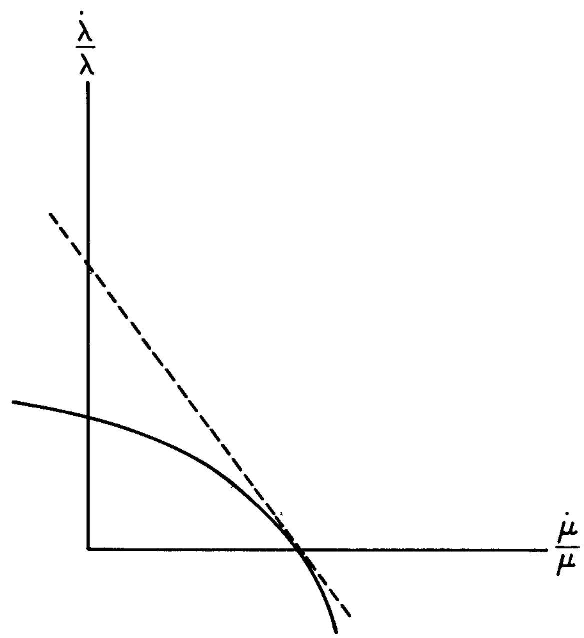

FIGURE 1. The innovation possibility curve.

Labor grows exponentially at rate n. Capital is accumulated according to the savings identity

$$\dot{K} = sY - \delta K, \tag{5}$$

where δ is the rate of depreciation of capital.

It will be helpful to put all variables in efficiency units. Thus, define new variables y, k, and x:

$$y = \frac{Y}{L}, \quad k = \frac{K}{L}, \quad x = \frac{\lambda K}{\mu L}.$$

Then rewrite Equations 3 and 5 as follows:

$$y = \mu f(x), \tag{6}$$

$$\dot{k} = s\mu f(x) - (\delta + n)k. \tag{7}$$

The planning authority of our stylized economy controls the aggregate savings rate s and the direction of technical change $\beta = \dot{\mu}/\mu$. It is assumed to maximize a utility function that is linear with respect to per capita consumption and that discriminates between generations by discounting future

the fact that for a strictly convex technology set the firm is concerned with only local changes in the production function. As long as the time path of λ and μ is efficient, the economy will never be in the region where isoquants are crossing.

consumption at the constant positive rate ρ. Thus, assuming the integral converges, the planning authority chooses the path that maximizes the integral:

$$J = \int_0^\infty \frac{C}{L} e^{-\rho t}\, dt = \int_0^\infty (1 - s)\mu f(x) e^{-\rho t}\, dt. \tag{8}$$

The problem of the planning authority can be solved by the "maximum principle" of Pontryagin and his associates.[5] By applying the maximum principle we can derive the necessary conditions for a maximum of our preference functional, Equation 8; these necessary conditions are described by Equations 9 through 21.

The state variables of the system are k, λ, and μ, whose motion is described by the following differential equations:

$$\dot{k} = s\mu f\left(\frac{\lambda k}{\mu}\right) - (\delta + n)k, \tag{9}$$

$$\dot{\lambda} = g(\beta)\lambda, \tag{10}$$

$$\dot{\mu} = \beta\mu. \tag{11}$$

The control variables are s and β. We can form the Hamiltonian of the system:[6]

$$H = e^{-\rho t}\left\{(1 - s)\mu f\left(\frac{k\lambda}{\mu}\right) + p_1\left[s\mu f\left(\frac{k\lambda}{\mu}\right) - (\delta + n)k\right] + p_2 e^{ht} g(\beta)\lambda + p_3\beta\mu\right\}, \tag{12}$$

where the p_i are the conjugate (or Lagrange) variables of the system, and h is the rate of Harrod neutral technical change; that is, h is the solution to the equation $g(\beta) = 0$.

From Theorem 3 in [6] we know that if a program $[s(t), \beta(t)]$ is optimal, then there exist continuous functions $p_1(t)$, $p_2(t)$, and $p_3(t)$ that satisfy

$$\dot{p}_1 = (\rho + \delta + n)p_1 - f'(x)\gamma\lambda, \tag{13}$$

$$\dot{p}_2 = [\rho - h - g(\beta)]p_2 - f'(x)k\gamma e^{-ht}, \tag{14}$$

$$\dot{p}_3 = (\rho - \beta)p_3 - \gamma[f(x) - xf'(x)], \tag{15}$$

where $\gamma = (1 - s + sp_1)$.

The analogous conditions to the first-order conditions in the ordinary calculus are that $[s(t), \beta(t)]$ maximizes H at every point of time. This implies that

$$s(t) \text{ maximizes } (1 - s + p_1 s)$$

or

$$\gamma = \max(1, p_1). \tag{16}$$

[5] See Pontryagin *et al.* [6] and the discussion of the maximum principle by Shell [8].

[6] Note that the original conjugate variable of λ is now $p_2 e^{ht}$, which explains the appearance of e^{ht} in Equations 12 and 14, as well as the h in the first term of Equation 14.

In addition, $\beta(t)$ satisfies the equation

$$\frac{\partial H}{\partial \beta} = 0 = p_2 g'(\beta)\lambda e^{ht} + p_3\mu. \tag{17}$$

The concavity of g and the requirement that prices be nonnegative imply that $\partial^2 H/\partial\beta^2 \leq 0$, which ensures that the solution to Equations 16 and 17 is maximum.

Finally, it is necessary that the system satisfy the initial conditions:

$$k(0) = k_0, \tag{18}$$

$$\lambda(0) = \lambda_0, \tag{19}$$

$$\mu(0) = \mu_0, \tag{20}$$

and that it meet the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}p_1(t) = \lim_{t\to\infty} e^{(h-\rho)t}p_2(t) = \lim_{t\to\infty} e^{-\rho t}p_3(t) = 0. \tag{21}$$

The only parts of the system that need interpretation are Equations 13 through 15. The p_i represent shadow prices of the state variables. Equation 13 has the market interpretation that there must exist a price for capital such that all assets have a real rate of return equal to the social rate of return, ρ. Equations 14 and 15 have the usual dynamic programming interpretation of shadow prices, but (for reasons that will become apparent in section 5) they do not have market interpretations since technical change is not a marketable commodity.

2. The Optimal Direction of Technical Change

The first step of the analysis is to determine whether there is a long-run equilibrium of the system of necessary conditions of Equations 9 through 21. Assume that the shadow price of capital p_1 is constant. Using the definition that $x = \lambda k/\mu$, constancy of p_1 implies from Equation 13 that

$$\lambda f'(x) = \frac{p_1}{\gamma}(\rho + \delta + n). \tag{22}$$

Differentiating Equation 22 totally with respect to time we have

$$\begin{aligned} 0 &= f''(x)\lambda\dot{x} + f'(x)\dot{\lambda} = \frac{f''(x)\dot{x}}{f'(x)} + \frac{\dot{\lambda}}{\lambda} \\ &= \frac{f''(x)xf(x)}{f'(x)[f(x) - xf'(x)]}\,\frac{[f(x) - xf'(x)]}{f(x)}\,\frac{\dot{x}}{x} + \frac{\dot{\lambda}}{\lambda} \\ &= -\frac{1-\alpha}{\sigma}\,\frac{\dot{x}}{x} + g(\beta) \end{aligned}$$

or

$$\frac{\dot{x}}{x} = \frac{\sigma g(\beta)}{1 - \alpha}, \tag{23}$$

where σ = the elasticity of substitution $= -f'(x)[f(x) - xf'(x)]/[xf(x)f''(x)]$ and α = the share of capital $= f'(x)x/f(x)$. The constancy of the effective capital-labor ratio x implies from Equation 23 that

$$g(\beta^*) = 0, \tag{24}$$

$$\beta^* = h, \tag{25}$$

where asterisks represent long-run equilibrium values of the variables.

Similarly, using Equations 24 and 25, we can determine the stationary values of Equations 14 and 15:

$$p_2{}^* = \frac{\gamma f'(x)x\mu e^{-ht}}{\lambda(\rho - h)},$$

$$p_3{}^* = \frac{\gamma(1 - \alpha)f(x)}{\rho - h}.$$

Putting these into the maximum condition (Equation 17) we have

$$g'(\beta^*) = g'(h) = -\frac{1 - \alpha(x^*)}{\alpha(x^*)}, \tag{26}$$

which defines the equilibrium value of x^*. Thus the only equilibrium of the system is the equilibrium where technical change is purely labor augmenting. If σ is bounded away from unity, we know the equilibrium in Equation 26 exists and is unique, since α ranges from 0 to 1 and $[-(1 - \alpha)/\alpha]$ ranges from 0 to $-\infty$.

In this equilibrium it is easily seen that $p_1 = 1$, since if $p_1 > 1$, then $s = 1$, consumption is 0, and we have a minimum of Equation 8; while if $p_1 < 1$ then $\dot{x} = -(\delta + n + h)x < 0$, violating the stationarity of x.

From Equation 22, since $p_1 = \gamma = 1$, we know:

$$\lambda^* = \frac{\rho + \delta + n}{f'(x^*)}. \tag{27}$$

From Equation 9 we know that $\dot{x} = 0$ implies that

$$s = \frac{\delta + n + h}{\lambda f(x)} x$$

or, using Equation 27 and the definition of α,

$$s^* = \frac{\delta + n + h}{\lambda f'(x)} \frac{f'(x)x}{f(x)} = \frac{\delta + n + h}{\delta + n + \rho} \alpha^*. \tag{28}$$

Since $\rho > h$ is a necessary condition for convergence of the integral in Equation 8, we know that $0 < s^* < 1$. Finally, $\mu(t) = \mu^* e^{ht}$, where μ^* is determined by initial conditions.

Bringing all results together we have

$$\begin{aligned} g'(\beta^*) &= -\frac{1 - \alpha^*}{\alpha^*}, \\ \lambda^* f'(x^*) &= \rho + \delta + n, \\ \beta^* &= h, \\ s^* &= \frac{\delta + n + h}{\delta + n + \rho} \alpha^*. \end{aligned} \tag{29}$$

The equilibrium exists and is unique if $\sigma(x) \neq 1$.

We can call Equations 29 the *Harrod equilibrium*, which is the unique long-run equilibrium with a constant effective capital-labor ratio. Unlike most models of optimal accumulation, the equilibrium effective capital-labor ratio is independent of tastes, depending only on technology. When there is no technical change ($h = 0$), the savings rate is exactly that of the usual model (cf. [8]); and as a limit for $h = \rho = 0$ we get the golden rule savings rate of $s^* = \alpha^*$.

3. Optimality of the Harrod Equilibrium

It was shown in section 2 that for $\sigma \neq 1$ there is a unique stationary equilibrium satisfying the necessary conditions. We called this the Harrod equilibrium. If we stay forever in the Harrod equilibrium, then the necessary conditions of Equations 9 through 21 are satisfied irrespective of the value of σ.

We can prove the rather remarkable result that if $\sigma < 1$, then an economy that remains in the Harrod equilibrium maximizes the preference functional in Equation 8; while, if $\sigma > 1$, the Harrod equilibrium is certainly not the optimal path.[7]

Before proving the proposition, we introduce some additional notation. Let $z = (x, \lambda, \mu, \beta, s)$. When $z = z^* = (x^*, \lambda^*, \mu^*, \beta^*, s^*)$, the system is in

[7] This surprising result is due to the effect of capital deepening on capital's productivity. If $\sigma < 1$ then the elasticity of output with respect to effective capital (α) approaches zero as capital deepens Under this condition it does not pay to continue to deepen capital past x^* Rather, it is optimal to expand effective labor. On the other hand, if $\sigma > 1$, then α approaches unity as effective capital is deepened, and it is possible to have very large growth rates by further capital deepening. Thus in the case where $\sigma > 1$ it is suboptimal to limit the system's growth rate to the pedestrian $n + h$ associated with the Harrod equilibrium

Harrod equilibrium as described by Equations 29. The condition $z_0 = z^*$ implies that the system starts in Harrod equilibrium.

The proof of the proposition requires a slight modification of the original system outlined in section 1. We modify by linearizing $g(\beta)$ at $\beta = h$, thereby replacing Equation 4 with

$$\frac{\dot{\lambda}}{\lambda} = Ah - \beta A, \tag{30}$$

where $g'(h) = -A$. The linearized function in Equation 30 is depicted by the broken line in Figure 1. All other equations remain unchanged. We call the new system, including Equation 30, the "modified system."

We can without loss of generality normalize by setting $\lambda^* = \mu^* = 1$. Define $B(t)$ by

$$B(t) = \int_0^t \beta(v)\, dv - ht. \tag{31}$$

We know from Equation 30 that

$$\begin{aligned} \mu &= e^{ht}e^{B}, \\ \lambda &= e^{-AB}. \end{aligned} \tag{32}$$

Now consider an optimal path in the modified system with control variables $s(t)$ and $B(t)$. The Hamiltonian of the system is

$$H = e^{-\rho t}\{(1 - s)e^{ht+B}f(x) + q[se^{ht+B}f(x) - (\delta + n)k]\}, \tag{33}$$

where for simplicity we make use of the equality $x = ke^{-B(A+1)-ht}$. The necessary conditions for a maximum of the modified system are that there exists a continuous function $q(t)$ such that

$$\dot{q} = (\rho + \delta + n)q - e^{-AB}f'(x)\gamma, \tag{34}$$

$$\dot{k} = se^{ht+B}f(x) - (\delta + n)k, \tag{35}$$

$$\lim_{t\to\infty} e^{-\rho t}q(t) = 0. \tag{36}$$

Further, the control variables s and B must maximize H at every point of time. This implies that $\gamma = \max(1, q)$ as in the original system. To determine the maximum with respect to B, take the partial derivative of H:

$$\frac{\partial H}{\partial B} = \gamma e^{(h-\rho)t+B}f(x)(A + 1)\left[\frac{1}{1 + A} - \alpha(k, B)\right], \tag{37}$$

where $\alpha(k, B) = \alpha(x)$. If $\sigma < 1$, then H is maximized when $\alpha = 1/(1 + A)$. Similarly if $\sigma > 1$, then H is minimized for $\alpha = 1/(1 + A)$.

We can now show that for initial conditions $z(0) = z^*$, when $\sigma < 1$, the optimal path in the modified system is $[z(t) = z^*$ for all $t]$. If $z(t) = z^*$,

$q(t) = 1$, $B(t) = 0$, and $s(t) = s^*$, then $\alpha = 1/(1 + A)$ and all the necessary conditions of the modified problem are satisfied. Since both terms in Equation 33 are concave in the state variable k and control variables s and B—when the maximizing condition in Equation 37 and $\gamma = \max(1, q)$ are taken into account—the necessary conditions are sufficient. Therefore, if $\sigma < 1$, then $[z(t) = z^*]$ is the optimal path in the modified system.[8]

Given that z^* is optimal in the modified system, we can deduce, for $z_0 = z^*$ and for $\sigma < 1$, that $[z(t) = z^*$ for all $t]$ is optimal in the original system. Let $g(\beta)$ be the innovation possibility curve (IPC) in Equation 4 and $\tilde{g}(\beta)$ the linearized system in Equation 30. Assume that the closed sets to the south-west of g and $\tilde{g}$ are feasible regions. Denote U as the control region $[s(t), \beta(t)]$ for g, and $\tilde{U}$ the control region for $\tilde{g}$. The optimal path is seen to lie on the frontier in each region. Now for $z_0 = z^*$, the path $[z(t) = z^*]$ is the optimal path in $\tilde{U}$. Therefore $[z(t) = z^*]$ is the optimal path in any subset of $\tilde{U}$ that includes z^*. But since $z^* \in U \subset \tilde{U}$, then z^* is optimal in U.

Therefore, in the original system, if $\sigma < 1$ and if the system starts out in Harrod equilibrium, then the optimal path is to remain in Harrod equilibrium for all time.

If $\sigma > 1$, the result does not hold. Examining Equation 37, we see that since $\partial H/\partial B$ has the sign of $(\alpha - \alpha^*)$, the Harrod equilibrium is a minimum with respect to B in the modified system. Application of l'Hôpital's rule shows that $J \to \infty$ as $B \to -\infty$ and $\alpha \to 1$. We cannot, however, use this result to prove the inferiority of the Harrod equilibrium in the original system, since (using the notation of the previous paragraphs) the optimal path in $\tilde{U}$ will not be in U.

We can use a constructive approach to show the inferiority of the Harrod equilibrium in the original system. If $\sigma > 1$, pick the path that has Hicks-neutral technical change and a constant, positive savings rate. Then, as is shown in [1], the growth rate of output and consumption is unbounded. For long horizons the Hicks path will dominate the Harrod path, according to the criterion function of Equation 8.

The economic interpretation of the result for $\sigma > 1$ is that, as substitution of capital for labor occurs, the productivity of capital is so high that growth is unbounded. Robots are making robots at an ever increasing rate.

There are certain questions open at this point. We have shown only that for $\sigma < 1$ and for $z_0 = z^*$ the optimal path is to remain in Harrod equilibrium. We have not shown that in the general case (that is, for other initial conditions) the optimal path is to go to the Harrod equilibrium.

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[8] This sufficiency condition is similar to the strengthened Legendre condition in the calculus of variations. This result can also be seen by showing that all other paths except $z(t) = z^*$ violate the transversality condition in Equation 36. It can be shown that for all initial conditions in the modified system the optimal path will go to z^* in finite time.

4. The Optimal Rate of Technical Change

We have considered an economy that controls only the direction of technical change. It is not clear what is generating this technical change, although some authors have suggested that a fixed quantum of research activity or exogenously supplied inventors determines the IPC. How an economy or a firm might change the direction of technical change is not specified. A more natural formulation of the problem from the point of view of actual policy is to allow the rate or intensity of technical change to be controlled explicitly by varying the amount of resources devoted to technical change.

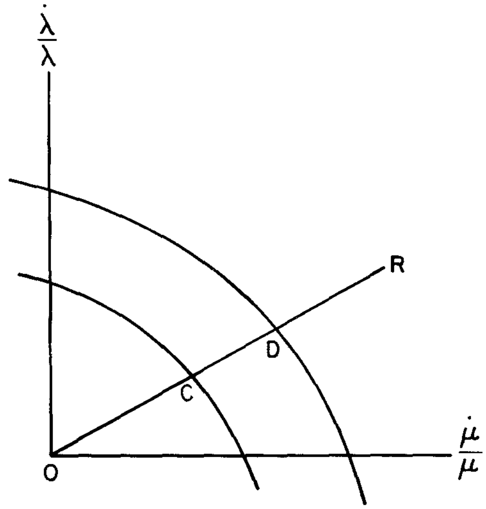

FIGURE 2. The IPC with variable rate of technical change.

In this section we follow Uzawa [9] in assuming that the central planners determine the intensity of technical change by allocating a percentage of the labor force to the research or educational sector.[9] A higher percentage of the labor force thus allocated pushes out the IPC in a homogeneous fashion, as in Figure 2. Thus all the IPC have the same shape but are blown up by a scale factor; in Figure 2, on an arbitrary ray from the origin, OC/OD is constant for any two IPC. Using Uzawa's notation, let researchers $= L_e = (1 - u)L$. We can rewrite Equation 4 as

[9] The assumption that technical change is a function of the relative size (either of the labor force or of production) devoted to research is not satisfactory. Empirical studies such as [5] indicate that the absolute amount of resources devoted to research is the appropriate variable. Unfortunately, the correct formulation does not allow a steady state equilibrium.

$$\dot{\mu} = \beta\phi(1 - u)\mu, \tag{38}$$

$$\dot{\lambda} = g(\beta)\phi(1 - u)\lambda, \tag{39}$$

where the function $\phi(1 - u)$ is the intensity of technical change that blows up the IPC in Figure 2. Normalize the system by setting $\beta = h$ when $g(\beta) = 0$. We assume that ϕ is sufficiently concave in u to ensure that the integral converges; thus $\phi(1) < \rho < \phi(0) + \phi'(0)$ with $\phi''(1 - u) < 0$.

To determine the optimal policy, we can use the same system as Equations 9 through 21 in section 1. We have control variables $s(t)$, $\beta(t)$, and $u(t)$. The Hamiltonian form is

$$H = e^{-\rho t}\left\{(1 - s)\mu u f\left(\frac{x}{u}\right) + p_1\left[s\mu u f\left(\frac{x}{u}\right) - (\delta + n)\frac{x\mu}{\lambda}\right] + p_2 e^{ht} g(\beta)\phi\lambda + p_3\beta\phi\mu\right\}. \tag{40}$$

When ϕ is constant and equal to 1, it is seen that the problem is identical to that in Equation 12. Necessary conditions for a maximum include Equations 9 through 11 and 16 through 21, making the modifications that the right-hand sides of Equations 10, 11, and 17 are multiplied by $\phi(1 - u)$ and that x/u replaces x. In addition, the price equations become

$$\dot{p}_1 = (\rho + \delta + n)p_1 + f'\left(\frac{x}{u}\right)\lambda\gamma, \tag{41}$$

$$\dot{p}_2 = [\rho - h - g(\beta)\phi]p_2 - \frac{f'(x/u)\, x\gamma\mu e^{-ht}}{\lambda}, \tag{42}$$

$$\dot{p}_3 = (\rho - \beta\phi)p_3 - \gamma\left[uf\left(\frac{x}{u}\right) - xf'\left(\frac{x}{u}\right)\right], \tag{43}$$

and we need the condition that u maximizes H, which is

$$\frac{\partial H}{\partial u} = \gamma(1 - s)\mu\left[f - \frac{x}{u}f'\right] - p_2 e^{ht} g(\beta)\phi'\lambda - p_3\beta\phi'\mu = 0. \tag{44}$$

To find the stationary equilibrium, note that for $u = u^*$ the Equations 9 through 11 and 16 through 21, with Equations 41 through 43, have the same equilibrium as Equation 29, with the modifications that x^*/u^* replace x^* and $\beta^*\phi(1 - u)$ replaces β^*. The equilibrium condition for the amount of resources allocated to the research sector is, from Equation 44,

$$\rho - \phi(1 - u^*) = \rho - h = u^*\phi'(1 - u^*), \tag{45}$$

which is exactly Equation 36 in Uzawa.

We have thus shown that the only long-run equilibrium in the economy where the rate and direction of technical change are controlled is the Harrod equilibrium with the rate of technical change given by Equation 45.

To prove that this new Harrod equilibrium is a maximum for $\sigma < 1$, we use essentially the same technique as in section 3. Call the new equilibrium $z^{**} = (z^*, u^*)$. By linearizing Equations 38 and 39 exactly as in Equation 30, we see that in this modified system the z^{**} equilibrium satisfies all the necessary conditions. Since the terms of the Hamiltonian are concave when $\sigma < 1$, the necessary conditions are sufficient. Thus, if $z_0 = z^{**}$, then $[z(t) = z^{**}$ for all $t]$ is the optimal path for the modified system. By exactly the same reasoning as in section 3, for $z_0 = z^{**}$ in the original system, the Harrod equilibrium z^{**} is the optimal path.

Similarly, for $\sigma > 1$, the new Harrod equilibrium z^{**} will be dominated by other paths and is therefore not the optimum.

5. Competitive Markets and Induced Technical Change

The results of this analysis are applicable to economies in which planning authorities direct capital accumulation and the rate and direction of technical change. They have a more limited application to competitive capitalistic economies than do most models of optimal growth. In most models a government pursuing the correct monetary and fiscal policy can make a competitive economy parrot the optimal plan of a centrally planned economy. Whether such mimicry can occur in the Kennedy model depends on the microeconomic framework.

Since the possibility of parallelism between a market economy and a planned economy depends chiefly on the existence of a competitive equilibrium, the logical question to ask is whether a Kennedy economy is compatible with competition. The answer depends on where the technical change comes from and whether it is costless. Suppose, to follow most writers,[10] that the IPC is generated by a fixed amount of research expenditures. There are a number of possible techniques that it is possible to invent through research, and the bias of the change in techniques is dictated by the IPC. It is implicitly assumed that the level of research is independent of the size of the firm, for otherwise the size of the firm would enter the maximum problem. But since constant returns to K and L are assumed by all authors, cost per unit output, or $(wL + rK + \text{R \& D})/Y$, will be a decreasing function of output of the firm. Competition will break down.[11] One could, it is true, dream up ways to preserve constant costs and competition, but this would only come at the

[10] Kennedy [4] gives no microeconomic interpretation. Samuelson ([7], p. 343) states, "Presumably a limited amount of resources available for research and development can be used to get a larger [increase in λ] only at the expense of a slower [increase in μ]." Drandakis and Phelps ([2], p. 11) explain that "firms can contrive to increase [λ] and [μ] or both by employing exogenously supplied inventors."

[11] The fact that competition breaks down should not come as a surprise at this point. The difficulty originates in the same place as our difficulties in showing sufficiency: the lack of convexity in the production set or increasing returns.

expense of realism, since technical change is endogenous and is produced by the firm.

In fact, about the only microeconomic framework that preserves competition is one in which a book of new blueprints falls from the sky every period—the new techniques given according to the IPC—and the entrepreneur chooses the best technique. In this case it would be quite misleading to say that technical change is induced. Rather, the IPC gives the technical possibilities at a point of time. The model is then just a disguised version of the neoclassical model with exogenous technical change.

The next step is to realize that if we cannot sustain competition, except in what is equivalent to the exogenous case, the behavior equations of the descriptive model are incorrect. A monopolist would minimize cost according to actual relative shares, not competitively determined relative shares that add up to greater than unity. He must then consider the elasticity of supply and demand. The theory becomes much more complicated, and the hope of getting some kind of steady state is dim.

There is one further reason to believe that in such a world a centrally planned economy would have certain advantages over a capitalistic economy. It is usually the case that a competitive economy economizes on the use of information. Sufficient information for a competitor to behave efficiently is the knowledge of current prices and of prices one period ahead. In the Kennedy model the firm needs to know all future prices over the infinite horizon to perform efficiently, since the firm cannot sell its investment in technology as it can its capital. Redundant information is worthless. The firm must make sure the infinite stream of quasi-rents on technical change covers costs. Each firm must assume a heavy computational burden.

None of these complications that burden a capitalistic economy with a Kennedy technology causes any additional burdens to the centrally planned economy.

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