KEYNES AND THE QUANTITY THEORY: A COMMENT ON THE FRIEDMAN-MEISELMAN CMC PAPER *

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Professor Friedman and Meiselman 1 recently have reported that a simple quantity theory model describes aggregate consumption more accurately than a simple autonomous expenditure model. They believe this result is evidence that the "quantity" theory is a better description of the American economy than the autonomous expenditure or "Keynesian" theory. 2

If their interpretation were correct, the Friedman-Meiselman paper would be one of the most significant economic studies in many years. But it is not correct. Friedman and Meiselman have represented the autonomous expenditure theory in a very unorthodox form. Their statistical comparisons are extremely sensitive to how the autonomous expenditure theory is represented.

Below, I employ a more conventional representation of the autonomous expenditure theory and demonstrate why Friedman and Meiselman's tests are misleading. Further, using this conventional model and some of their data, little empirical evidence is found which favors the quantity theory. Finally some other conceptual weaknesses of the Friedman-Meiselman tests are illustrated.

Briefly, Friedman and Meiselman compare simple, partial, and multiple correlation coefficients obtained from the following equations, estimated from annual (1897-1958) and quarterly (1945-1958) data for the United States:

\[ C = a_1 + \beta_1A \]  
\[ C = a_2 + \delta_2M \]  
\[ C = a_3 + \beta_3A + \gamma_3P \]  
\[ C = a_4 + \delta_4M + \gamma_4P \]  
\[ C = a_5 + \beta_5A + \delta_5M \]  
\[ C = a_6 + \beta_6A + \delta_6M + \gamma_6P \]

where:

- \( C \) = consumption in current dollars (durables, nondurables, and services);
- \( A \) = autonomous expenditure in current dollars (net private domestic investment, government deficit on income and product account, and the net foreign balance);
- \( M \) = money supply (currency in public circulation; adjusted demand deposits, and commercial bank time deposits); and
- \( P \) = the consumer price index.

The equations were estimated a number of times from data pertaining to various sub-periods. In nearly every subperiod the correlation coefficient \( r_{CM} > r_{CA} \); equation (2) had a better "fit" than equation (1). Similarly, the multiple correlation coefficient of equation (4) was almost always higher than the corresponding statistic of equation (3). Partial correlations between \( M \) and \( C \) were invariably positive and almost always exceeded the partial correlations between \( A \) and \( C \) in equations (5) and (6). The latter were occasionally negative. 3

Friedman and Meiselman also estimated equations (1)–(6) with first differences of each of the variables. Results of these calculations were not reported in detail, but for selected annual periods and in the postwar period of quarterly observations, Friedman and Meiselman indicated that correlations between first differences of consumption and the money supply exceed correlations between first differences of consumption and autonomous expenditure. 4

What happened? If one subscribes to textbook interpretations of the autonomous expenditure model, Friedman and Meiselman's regression of consumption on their concept of

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I am indebted to Milton Friedman, Arthur Okun, and James Tobin for remarks on an earlier draft of this comment. Albert Ando and Franco Modigliani are also preparing a comment on the Friedman-Meiselman paper which was independently conceived and suggests a somewhat different interpretation of the Friedman-Meiselman results. I, of course, am solely responsible for statements below.


2 Ibid., 187-188.

3 Ibid., 189-209.

4 Ibid., 203.
autonomous expenditure is quite objectionable.\textsuperscript{5} The government deficit and net foreign balance which Friedman and Meiselman include in autonomous expenditure, \( A \), are not likely to be exogenous variables. Further, the structure of the autonomous expenditure model has not been invariant over the past 60 years; for example, the composition and techniques of taxation have been strikingly altered.

The monetary theory is also tested in a very simple form. One can not infer, however, from the fact that a single equation approximation of a theory dominates a rival that a comparison of complete versions of the two theories will yield the same conclusion. Friedman and Meiselman concede this point, but adhere to an intuitive view:

The fact that we have done so [neglected refinements] makes it necessary to emphasize that our results cannot be decisive. On the simple level at which we propose to test the two theories, equation (1) might turn out to be better than or worse than equation (2) or conversely; whereas on a more sophisticated level, when additional variables are introduced, the relative advantage of the two might be reversed. This possibility cannot be ruled out, although the presumption would seem to be that the relationship which explains the most in its simplest version is the relationship that it will be most fruitful to explore and convert into a more sophisticated form.\textsuperscript{6}

In the present comment, attention is restricted to Friedman and Meiselman's comparison of simple correlations between consumption, \( C \), and [1] the money supply, \( M \), and [2] their definition of autonomous expenditure, \( A \) — i.e., attention is restricted to equations (1) and (2) above. Values of the money and autonomous expenditure multipliers are irrelevant to my criticism and are not reported. This criticism is not applied to Friedman and Meiselman's comparison of partial correlation coefficients because the extension is untidy and will not affect the basic issues.

The first objection to Friedman and Meiselman's regression of consumption on autonomous expenditure will now be formalized. They define a variable \( Y = C + A \).\textsuperscript{7} In national income terminology \( Y = \) net national product minus taxes (income taxes, corporate profit taxes, and indirect business taxes). At this point, advocates of the autonomous expenditure theory should have raised their eyebrows. When Friedman and Meiselman select a measure of autonomous expenditure, they implicitly assign a definition of income. Only an appropriate definition of income will permit the autonomous expenditure model to be represented fairly. To see this, define the following which are all in current dollars:

\begin{align*}
Y' &= \text{net national product}; \\
G &= \text{government expenditure on income and product account}; \\
T &= \text{income taxes, corporate profit taxes, and indirect business taxes}; \\
H &= \text{exports}; \\
I &= \text{net private domestic investment}; \\
Y_d &= \text{disposable income of consumers}; \\
M &= \text{imports}; \\
W &= \text{undistributed corporate profits minus transfers}; \\
D &= \text{capital consumption allowances}; \text{ and } \\
E &= \text{change in inventories}.
\end{align*}

By definition:

\begin{equation}
Y' = C + I + G + H - M. \tag{7}
\end{equation}

\begin{equation}
A = G - T + I + H - M. \tag{8}
\end{equation}

The usual textbook consumption function is:

\begin{equation}
C = a + bY_d \tag{9}
\end{equation}

In textbooks, taxes are often represented as a function of \( Y' \).\textsuperscript{5} For purposes of argument write:

\begin{equation}
T = e + dY' \tag{10}
\end{equation}

where \( d \) is the marginal rate of taxation.\textsuperscript{8}

International trade theory suggests that imports may also be a function of \( Y' \). This will be considered below; for the moment assume they are independent of \( Y' \). The system of equations, (7), (9), and (10) is not complete for:

\begin{equation}
Y' = T + Y_d + W. \tag{11}
\end{equation}

\textsuperscript{5} Dernburg and McDougal, \textit{loc. cit.}

\textsuperscript{6} I am indebted to Albert Ando for calling my attention to a misinterpretation of Friedman and Meiselman at this point in an earlier draft.

\textsuperscript{7} Dernburg and McDougal, \textit{Macro-Economics}, second ed. (New York: McGraw-Hill, 1963), chs. 5 and 6. Friedman and Meiselman list a number of other textbooks describing the autonomous expenditure model in a footnote on pages 187-188 of their paper. However, I have yet to see a modern textbook which defines autonomous expenditure to exclude tax financed government expenditure. An obvious implication of Friedman and Meiselman's \( A \) is that the balanced budget multiplier is zero.

\textsuperscript{8} Friedman and Meiselman, \textit{op. cit.}, 174.
The system becomes complete if one approximates $Y_a$ by:
\[ Y_a = g + (1 - d) Y' \]  
(12)
where $g$ is an estimate of $-(c + W)$.

In this interpretation, autonomous expenditure equals the sum of government expenditure, net private domestic investment, and the trade balance. This measure is an improvement on Friedman and Meiselman's $A$ for it recognizes the dependence of taxes on income. Denote the proposed measure by $L$.
\[ L = I + G + H - \bar{M}. \]  
(13)

Then,
\[ Y' = a + bg + b(1 - d) Y' + L, \]  
(14)
\[ Y' = a + bg + L, \]  
(15)
\[ C = \text{constant} + \frac{b - b_d}{1 - b + b_d} L. \]  
(16)

Equation (16) differs fundamentally from equation (1) which is the corresponding equation in the Friedman-Meiselman paper. The latter may be rewritten as:
\[ C = a_1 + b_1 (L - T) \]  
(17)
or, by successive substitution of (10), (12), and (9).
\[ C = a_1' + b_1 \left( L - \frac{d}{b(1 - d)} C \right). \]  
(18)

The correlation between $A$, or
\[ \left( L - \frac{d}{b(1 - d)} C \right), \]  
and $C$ is
\[ r_{CA} = \frac{\sigma_{LC} - \frac{d}{b(1 - d)} \sigma_{CC}}{\left\{ \left[ \sigma_{LL} - \frac{2d}{b(1 - d)} \sigma_{LC} + \left( \frac{d}{b(1 - d)} \right)^2 \sigma_{CC} \right] \cdot \sigma_{CC} \right\}^{\frac{1}{2}}} \]  
(19)

Setting $h = \frac{d}{b(1 - d)}$,
\[ j = \left\{ \left[ \sigma_{LL} - \frac{2d}{b(1 - d)} \sigma_{LC} + \left( \frac{d}{b(1 - d)} \right)^2 \sigma_{CC} \right] \cdot \sigma_{CC} \right\}^{\frac{1}{2}}, \]  
and differentiating (19) with respect to $h$,
\[ \frac{dr_{CA}}{dh} = -\frac{\sigma_{CC}}{j^2} \left( 2h \sigma_{CC}^2 \sigma_{LC} - h^2 \sigma_{CC}^3 - \sigma_{LC}^2 \sigma_{CC} \right) \]  
(20)

Because $\sigma_{CC}$ and $j$ are positive, the sign of the derivative depends on $(\sigma_{LC}^2 - \sigma_{LL} \sigma_{CC})$ which is negative unless the correlation between $L$ and $C$, $r_{CL}$, is unity. A high, but not unitary, value of $r_{CL}$ does not imply a high value of $r_{CA}$.

If $L$ is a sensible measure of autonomous expenditure, Friedman and Meiselman have stacked the cards against the Keynesian model in their comparisons by ignoring the fact that taxes are a function of income.\(^10\)

Of course, Friedman and Meiselman are fully justified in attacking the lack of a precise definition of autonomous expenditure in most discussions of this model. To the extent that $L$ can be divided into an exogenous and an endogenous component, say $x$ and $yC$, $r_{CL}$ will be too large (small) if $y$ is positive (negative). However, I believe most economists will agree that $L$ is a better measure of autonomous expenditure than $A$.

Other measures of autonomous expenditure are also appealing; three are considered. First, estimated capital consumption allowances are imperfect approximations for actual depreciation. Consequently, net private domestic investment has a substantial measurement error, for it equals gross private domestic investment minus capital consumption allowances. Random errors in variables bias correlation coefficients toward zero. Further, as noted above, imports are likely to be a function of $Y'$. By an argument completely analogous to that for taxes, failure to eliminate imports from $L$ will serve to misrepresent the autonomous expenditure model. A second definition of autonomous expenditure, $L'$, equals $L + \bar{M} + D$.

Second, Friedman has pointed out to me in correspondence that part of imports is included in consumption; spurious correlation exists between $\bar{M}$ and $C$. A third definition of autonomous expenditure, $L''$, equals $L' - \bar{M} = L + D$.

While there is good reason to think that imports are induced and exports exogenous over

\(^10\)Friedman and Meiselman do perform tests to see if certain variables are autonomous. For example, they argue that if exports are autonomous then $r_{CE} \cdot (I + H)$ should exceed both $r_{CA}$ and $r_{CN}$. This procedure is unreliable if a third autonomous variable, $G$, exists. Nonzero covariances between $G$ and either $I$ or $H$ make the procedure untrustworthy.
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periods as long as ten years, correlating $C$ with $L''$ does remove this spurious correlation.

Finally, Ando and Modigliani have observed that inventory investment is not likely to be exogenous over short periods of time.\footnote{Albert Ando and Franco Modigliani, "The Life Cycle Hypothesis of Saving," The American Economic Review, LII, No. 1 (March, 1963), 68.} In their view, variations in consumption may cause negative variations in inventories. This would impart a negative bias to estimated correlations between $C$ and $L'$, by the same reasoning as in the case of taxes above. A fourth definition of autonomous expenditure, $L'''$, equals $L' - E$.

Table 1 reports correlation coefficients between consumption and [1] Friedman and Meiselman’s annual series of the stock of money, $M$, [2] Friedman and Meiselman’s measure of autonomous expenditure, $A$, and [3] different measures of autonomous expenditure proposed in this comment. The coefficients are reported for periods suggested in Friedman and Meiselman’s paper during the years 1929–1938 when convenient Department of Commerce data were available. While a revision of national income data necessitated recomputing $r_{CM}$ and $r_{CA}$, their values were only trivially changed from values reported in the Friedman–Meiselman paper.

<table>
<thead>
<tr>
<th>Years</th>
<th>$r_{CM}$</th>
<th>$r_{CA}$</th>
<th>$r_{CL}$</th>
<th>$r_{CL'}$</th>
<th>$r_{CL''}$</th>
<th>$r_{CL'''}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1929–1939</td>
<td>.912</td>
<td>.917</td>
<td>.963</td>
<td>.957</td>
<td>.933</td>
<td>.976</td>
</tr>
<tr>
<td>1933–1938</td>
<td>.991</td>
<td>.935</td>
<td>.995</td>
<td>.992</td>
<td>.997</td>
<td>.997</td>
</tr>
<tr>
<td>1938–1953</td>
<td>.958</td>
<td>.397</td>
<td>.755</td>
<td>.837</td>
<td>.809</td>
<td>.817</td>
</tr>
<tr>
<td>1949–1948</td>
<td>.961</td>
<td>.713</td>
<td>.471</td>
<td>.566</td>
<td>.519</td>
<td>.527</td>
</tr>
<tr>
<td>1948–1957</td>
<td>.990</td>
<td>.755</td>
<td>.925</td>
<td>.964</td>
<td>.961</td>
<td>.969</td>
</tr>
<tr>
<td>1929–1958</td>
<td>.974</td>
<td>.706</td>
<td>.915</td>
<td>.953</td>
<td>.943</td>
<td>.949</td>
</tr>
</tbody>
</table>


The correlation coefficients are revealing: with the exception of the 1929–1939 period, the correlation between consumption and every proposed measure of autonomous expenditure exceeds $r_{CA}$ as expected. All the measures of autonomous expenditure proposed in this comment yield correlations which exceed .90 in non-World War II years. With rationing and other extraordinary circumstances coincident with the war, it is hardly surprising that the autonomous expenditure model failed during the years 1941–1945. Indeed it is remarkable that the monetary model failed to reflect these conditions more vividly. An alternative explanation for consistently high values of $r_{CM}$ is suggested below.

In more detail, the correlation coefficients support the previous discussion. With one exception, 1933–1938, $r_{CL'} > r_{CL}$. Using gross rather than net investment and using exports rather than the trade deficit improves the estimated consumption function. In non-World War II years $r_{CL'''} > r_{CL'}$, apparently supporting the Ando–Modigliani contention. This adjustment is likely to be much more important in Friedman and Meiselman’s quarterly data. With one exception, again 1933–1938, $r_{CL'} > r_{CL''}$. This may be due to spurious correlation between imports and consumption or due to the fact that exports are truly exogenous. There is no basis for identifying which of these explanations is correct. In any case, the differences between $r_{CL}$ and $r_{CL'}$ are small.

Finally, the correlations between any of the measures of autonomous expenditure proposed in this comment and consumption do not differ from $r_{CM}$ by more than .06 for the 30-year period. These differences cannot be tested for significance with conventional statistical tools.

No doubt other correlation coefficients can be computed for other definitions of autonomous expenditure, money supply, and consumption; it seems fruitless to report more. However, it is illuminating to compare the correlations of first differences of $C$ with first differences of $M$, $A$, and $L'$ reported in table 2. The other extraordinary circumstances coincident with the war, it is hardly surprising that the autonomous expenditure model failed during the years 1941–1945. Indeed it is remarkable that the monetary model failed to reflect these conditions more vividly. An alternative explanation for consistently high values of $r_{CM}$ is suggested below.

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<table>
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<tr>
<th>Years</th>
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<tr>
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<tr>
<td>1940–1948</td>
<td>.006</td>
<td>-.755</td>
<td>-.700</td>
</tr>
<tr>
<td>1949–1957</td>
<td>.494</td>
<td>.552</td>
<td>.780</td>
</tr>
<tr>
<td>1930–1958</td>
<td>.625</td>
<td>.009</td>
<td>.274</td>
</tr>
</tbody>
</table>

* Correlations of first differences have not been computed for all variables in table 1. One different conclusion emerged from the computations performed: $\Delta C'\Delta M > \Delta C'\Delta M$. In nonwar years, however, $\Delta C'\Delta L''$ was not consistently greater than $\Delta C'\Delta M$.\footnote{Albert Ando and Franco Modigliani, "The Life Cycle Hypothesis of Saving," The American Economic Review, LII, No. 1 (March, 1963), 68.}
previous correlations may be high merely because of a common linear trend; correlating first differences abstracts from such trends.

For the 29-year period, \( r_{\text{CM}} \) exceeds both \( r_{\text{MCA}} \) and \( r_{\text{CA\text{M}'}} \). However, if attention is restricted to non-World War II years, the inequality runs the other way. Even though \( \Delta \) is a dubious measure of autonomous expenditure, \( r_{\text{MCA}} > r_{\text{CM}} \) for the years 1934–1938 and 1949–1957. The correlations are virtually equivalent for 1930–1939. As expected, \( r_{\text{CA\text{M}'}} > r_{\text{MCA}} \) in all periods. Whether one uses \( \Delta \) or \( \Delta' \), the autonomous expenditure model outperforms the monetary model when predicting first differences of consumption in nonwar years.

Turning to a different issue, Friedman and Meiselman are aware that both the autonomous expenditure model and the quantity theory suggest that a strong correlation exists between \( M \) and \( Y \), and less directly between \( M \) and \( C \). High correlations between \( M \) and \( C \) are, in themselves, weak evidence for the quantity theory.

Also, the supply of money is, apart from treasury currency, determined by actions of the Federal Reserve Open Market Committee and individual banks. It seems likely that the supply of money provided by these organizations is an increasing function of the level of national income. If so, recognizing the close relationship between time series values of income and consumption, one might write the Friedman–Meiselman consumption function and approximate the aggregate supply function respectively, as:

\[
\begin{align*}
C &= l_0 + l_1M + u, \quad l_1 \geq 0 & (21) \\
M &= l_0 + l_2C + r, \quad l_2 \geq 0. & (22)
\end{align*}
\]

Then,

\[
\begin{align*}
\rho_{\text{CM}} &= \frac{l_1\sigma_{uu} + (1 + l_1l_2)\sigma_{uv} + l_2\sigma_{uu}}{\left[ (l_1^2 \sigma_{uv} + 2l_1\sigma_{uv} + \sigma_{uu})(\sigma_{uv} + 2l_2\sigma_{vu} + l_2^2\sigma_{uu}) \right]^{\frac{1}{2}}} \\
&= \frac{l_1\sigma_{uu} + (1 + l_1l_2)\sigma_{uv} + l_2\sigma_{uu}}{\left[ (l_1^2 \sigma_{uv} + 2l_1\sigma_{uv} + \sigma_{uu})(\sigma_{uv} + 2l_2\sigma_{vu} + l_2^2\sigma_{uu}) \right]^{\frac{1}{2}}} & (23)
\end{align*}
\]

In this case, Friedman and Meiselman have correlated two linear combinations of \( u \) and \( v \). Residuals of both equations enter symmetrical-ly. Only if residuals of one of the two equations had a small variance relative to the other, or if \( l_0 \) or \( l_1 \) were zero, would a high value of \( \rho_{\text{CM}} \) result. To argue that high values of \( \rho_{\text{CM}} \) support equation (21), one must plausibly defend the assumption that either \( l_0 = 0 \) or that \( \sigma_{uv} > \sigma_{uu} \).

In other words, a high correlation between \( C \) and \( M \) is consistent with a quantity theory model. It is also consistent with the transactions demand equation of the autonomous expenditure theory, and with a money supply function having net national product as an argument. The lower correlation between \( C \) and \( M \) during the thirties is consistent with the liquidity trap argument of Keynes, and with bank-failure-induced "noise" in the money supply function.

In summary, Friedman and Meiselman’s conclusion that a simple quantity theory performs better than a popular autonomous expenditure theory in correlation tests is false. In part, their result obtains from considering a peculiar definition of income which excludes tax-financed government expenditure. Second, by using net rather than gross private domestic investment, Friedman and Meiselman handicap the autonomous expenditure theory. While in principal net investment is the ideal concept, it is well known that depreciation measures are highly imperfect and that measurement errors bias correlation coefficients toward zero. The fact that we don’t know how to measure depreciation is not grounds for rejecting an autonomous expenditure theory.

Third, when correlations among first differences of the various aggregates are inspected, during the 29-year period 1930–1958, except in World War II years, the autonomous expenditure theory outperformed the quantity theory. Friedman and Meiselman do not report correlations among first differences for various subperiods.

Finally, as both autonomous expenditure and quantity models predict a high correlation between \( \Delta Y \) and \( \Delta M \) (and hence \( \Delta C \) and \( \Delta M \)), high correlations between the money supply and consumption are of little value in discriminating between the models.

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9 Friedman and Meiselman, op. cit., 167.