A KEYNESIAN ANALYSIS OF FORCED SAVING*

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1. INTRODUCTION

FORCED SAVING is an intriguing doctrine of long history. Hayek [4] traces the concept to John Stuart Mill, even Bentham. In the 14th century Bishop Nicole Oresme [10, (101)] expressed vehement opposition to the debasement of coinage by a country’s prince; he also asserted that in certain circumstances the community itself might appropriately alter its coinage.²

... if the community has great need of a large sum of money for a war or for the ransom of its prince from captivity... it might raise it by altering the money, and this would not be contrary to nature or usurious... in this arrangement about all the conditions desirable for such a levy seem to be combined; it brings in a large revenue in a short time, it is very easy to collect and assess without the services of many officials, and it involves little expense or opportunity for fraud by the collectors. Indeed, no other more equitable or proportional plan can be imagined; for he who has more pays more, and being relatively less felt, it is more tolerable without danger of rebellion and complaint by the people. It is also very general, for neither cleric nor noble can escape it.

In his Tract on Monetary Reform [6, (41)] John Maynard Keynes argued:

... by printing paper money... a government can... secure the command over resources—resources just as real as those obtained by taxation. The method is condemned, but its efficacy, up to a point, must be admitted.

Conventional wisdom warns that inflation gives rise to expectations of further price rises, to hyperinflation and inevitable economic collapse. In contrast, it was asserted by Keynes [6, (49)]:

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² It is interesting to note that the criteria enumerated by Oresme anticipate Adam Smith’s four canons of taxation.
Like other conveniences of life, the use of money is taxable, and although for various reasons this particular form of taxation is highly inexpedient, a Government can get resources by a continuous practice of inflation, even when this is foreseen by the public generally, unless the sums they seek to raise in this way are grossly excessive...

Although it is to be hoped that problems raised by forced saving will remain of only academic interest as far as the U. S. economy is concerned, the possibility of financing economic development by creating money constitutes a strong temptation for emerging nations. In his description of the Chilean experience, where "the government sector defended itself against inflation through the printing press," Joseph Grunwald [3, (100)] raised the following problem:

The curious aspect of Chile's inflation history of close to a century is that the country never experienced runaway inflation. One would think that, once a country reaches such high rates of price increases as Chile did, hyperinflation would follow almost automatically. There is no satisfactory answer to this. The fact is that not enough money was printed for hyperinflation to develop. But if the forces that made the authorities "print money" were so strong as to maintain a 20 or more per cent yearly inflation for many years, what stopped those forces from compelling a snowballing monetary expansion? Probably the social pressures were not strong enough, and perhaps public confidence was greater than is generally thought...

The evaluation of forced saving presented in this paper will concentrate upon the following questions: Given the proportion of real government spending to be financed by the printing press, will the rate at which prices rise converge to a particular value, or will prices spiral upwards at an ever increasing rate? How will the rate at which prices rise respond to increases in the proportion of real government spending financed by injections of new money? What determines the response of the speed of inflation to changes in the level of government spending?

Once it is decided that a given share of real government spending will be financed by a deficit, both the real and the nominal money supply must be regarded as endogenous variables. An essential step in the analysis of forced saving involves an examination of the influence of the rate of price change upon the quantity of money in circulation when the government resorts to inflationary finance. Section 2 of this paper examines the factors determining the money supply. In Section 3 various arguments concerning the impact of rising prices upon the level of effective demand are reviewed. In Section 4, where
the concept of a forced saving equilibrium is developed, it is shown that the question of whether a policy of forced saving will lead to a constant rate of price change rather than runaway inflation depends in part upon the response by holders of liquid assets of fixed nominal value to the tax levied by inflation. Properties of the function explaining the generation of effective demand which insure the existence of a stable forced saving equilibrium are spelled out. The concluding section examines the response of the rate of price change and the real money supply to adjustments in the level of government spending and the extent of deficit financing.

2. FORCED SAVING AND THE MONEY SUPPLY

In most studies of inflation the money supply is neglected, regarded as fixed in magnitude, or else assumed to grow at some fixed rate that is independent of price movements. Under conditions of forced saving the money supply cannot be regarded as exogenously determined. Rising prices will reduce the purchasing power of money already in circulation. In addition, as prices rise, more and more currency will have to be injected in order to finance a given volume of real government expenditure.

Suppose that a fully employed economy is disturbed by an increase in real government spending that is financed, at least in part, by an expansion of the money supply. Let us assume that the deficit $D$, measured in real terms, will be maintained at a constant proportion $\delta$ of real government spending $G$; we have $D = \delta G$ and real tax revenue $T = (1 - \delta)G$. The presence of the deficit leads to the creation of money. We may assume that the change in the nominal money supply, $dM^n/dt$, is determined by

$$
\frac{dM^n}{dt} = \rho p D = \rho p \delta G,
$$

(2.1)

where $p$ is the price level and $\rho$ is the coefficient of expansion of the monetary system. If the deficit is financed by the printing press, $\rho = 1$ for then the change in the money supply is equal to the deficit. When resort is made to the banking system in financing the deficit, $\rho$ may depart considerably from unity.$^3$

$^3$ In an interesting empirical study, Allan Meltzer [9] analyzed the relation between the money supply and central bank monetary liabilities in France during the period 1938-54 with quarterly data. When adjustments were not made for trend, he obtained estimates of $\rho$ of 2.03 when money was defined to include both currency and demand deposits; when savings deposits were also included within the definition of the money supply, the coefficient of expansion of the monetary system was estimated to be 2.52. While these estimates appeared to be quite precise, his empirical search did not disclose a stable value of $\delta$. 
In order to determine the impact of this forced saving policy upon the rate of change of the real money supply, it is first necessary to observe that by definition, \( M^* = pM \); differentiation yields the equality

\[
\frac{dM^*}{dt} = M \frac{dp}{dt} + p \frac{dM}{dt}.
\]  

(2.2)

If we denote by \( \dot{p} = (dp/dt)/p \) the speed of inflation, then (2.1) and (2.2) imply that the rate of change in the real money supply is

\[
\frac{dM}{dt} = \rho \bar{G} - \dot{p}M.
\]  

(2.3)

Clearly, the real money supply remains stable through time only if the injection of new money is precisely offset by the depreciation of the existing stock. In order to explain movements of the real money supply through time it is necessary to investigate the determinants of the speed of inflation \( \dot{p} \). This requires an examination of the various factors that govern the level of effective demand under inflationary conditions.

3. INFLATION AND EFFECTIVE DEMAND

For the purpose of studying the implications of a policy of forced saving attention must be focused upon the situation of demand-pull inflation, where capacity output \( Y_c \) falls short of the level of effective demand that would exist if prices were stable.

Effective demand, denoted \( Y_d(\dot{p}, M; G, \delta) \), is determined in part by two endogenous variables, the rate of price change \( \dot{p} \) and the real money supply \( M \); it is also influenced by the level of real government spending \( G \) and the proportion \( \delta \) of that spending which is financed by printing money. The analysis is to be strictly Keynesian in the sense that when \( \dot{p} = 0 \) the function determining effective demand, measured in real terms, reduces to the familiar equation

\[
Y_d(0, M; G, \delta) = \alpha Y - \alpha(1 - \delta)G + I(r(M, Y)) + G.
\]  

(3.1)

Here \( \alpha \) is the propensity to consume, \( \alpha(1 - \delta)G = \alpha T \) reflects the impact of taxation upon effective demand, and the last two terms represent investment and government spending respectively.\(^4\) When \( Y_d(0, M; G, \delta) - Y_c > 0 \), an inflationary gap exists, and the impact of

\(^4\) Although a possible direct influence of the real money supply upon consumption is neglected in this formulation, the argument that follows is in no way sensitive to the exclusion of the real balance effect. On the other hand, the real balance effect is to be distinguished from the role of capital losses discussed later in the paper.
rising prices upon effective demand must be carefully evaluated. In
developing the argument, the impact of capital accumulation upon
the investment schedule is ignored; this customary simplification means
that the implications of rising prices for the rate of economic growth
cannot be completely explored within the confines of this paper. The
complications of foreign trade must also be neglected.

When the Keynesian model is put in dynamic form for purposes of
business cycle analysis, any of a large number of alternative procedures
for introducing time may be applied. It is the same when the Key-
nesian framework is utilized in the study of inflation. Any of an
embarrassing variety of procedures may be utilized even when atten-
tion is restricted to models that are Keynesian in the sense that the
underlying equations have as static counterparts corresponding relation-
ships of the General Theory. Three dynamic complications arising
under conditions of rising prices will be considered in our equation
for effective demand,

\[ Y_d(\phi, M; G, \delta) = \alpha Y - \alpha(1 - \delta)G \]
\[ - \gamma \phi Y - \gamma \phi M + I[\tau^*(M, Y) - \psi \phi] + G . \]

These involve the impact of erroneous expectations, of adjustment for
capital losses, and the distinction between the real and the nominal
rate of interest. Since each of these complications has received at
least some attention in the literature, only a brief review is required
here.

Inflation illusion behavior is represented by the term \( \gamma \phi Y \) in equation
(3.2). Koopmans [8] and Smithies [14] have discussed in detail the
consequences of a misapprehension on the part of income receivers
who base their spending behavior upon the naive assumption that
current prices as well as income are stable. \(^6\) A second complication,
which may also be included in \( \gamma \phi Y \), concerns a possible effect of rising
prices upon the distribution of income, and hence consumption. If

\(^5\) This is the most frequently employed concept of the inflationary gap; it is the level
of excess demand that would exist if the price level were to remain stable; this virtual
gap is not to be confused with the actual level of excess demand. Walter Salant [13]
has discussed this and various other concepts of the gap.

\(^6\) One may assume not only that the current level of consumption spending is determined
by yesterday's income, but also that the number of dollars allocated to this function is
decided under the naive misapprehension that current prices as well as income will
remain at yesterday's level; in other words, the number of dollars of yesterday's income
allocated to consumption spending is \( p_t \alpha Y_{t-1} \); but then real consumption spending is
only

\[ C_t = \frac{p_t}{p_t} \alpha Y_{t-1} = \alpha Y_{t-1} - \alpha \left( \frac{p_t - p_{t-1}}{p_t} \right) Y_{t-1} . \]
wage rate adjustments fall behind price changes in periods of inflation, and if the consequent redistribution of income serves to reduce the level of consumption that takes place at any given level of aggregate real disposable income, then the process of inflation itself will help to reduce the level of effective demand. The sum of these two effects will be assumed to be proportional to the product of the speed of inflation times the level of income.

Capital loss adjustment response to inflation is represented by \( \gamma_\ell \dot{p} M \) in equation (3.2). This dynamic complication involves the consequence of the capital-loss which inflation inflicts upon the holders of money assets. When the price level doubles, an individual who holds $1,000 cash suffers a capital loss of $500. This is the difference between yesterday's and today's purchasing power of the money assets carried over from yesterday. It is the amount that would have to be set aside in order to restore the real value of liquid assets to their former level. The size of the capital loss measured in real terms, \( L_t \), is obtained by subtracting from the real value of yesterday's money supply its value in terms of today's price level,

\[
L_t = \frac{M^n_t}{p_t} - \frac{M^n_{t-1}}{p_{t-1}} = \left( \frac{p_t - p_{t-1}}{p_t p_{t-1}} \right) M^n_{t-1} = \frac{p_t - p_{t-1}}{p_t} M^n_{t-1}.
\]

When time is regarded as continuous the capital loss suffered at point of time \( t \) is \( \dot{p} M \). It seems reasonable to suppose that any capital loss inflicted by inflation is taken into account in planning current expenditure. Milton Friedman has argued [2, (254–55)]:

The price rise imposes, as it were, a tax on the holdings of... those net obligations of government that are expressed in nominal monetary units. The payment of this tax, as of any other, reduces the income available to consumers for spending or saving and so tends to reduce the fraction of their income, measured before payment of the tax, that they want to spend on consumer goods... and to bring it into equality with the fraction of resources available to produce consumer goods.

In equation (3.2) the symbol \( \gamma_\ell \) represents the proportion of capital losses that is replaced by curtailing current expenditure.\(^7\)

\(^7\) Franklyn Holzman [3] has analyzed in detail the effect of income redistribution during the inflationary process.

\(^8\) Empirical evidence on the magnitude of the impact of capital losses upon consumption behavior is sparse. J. J. Polak [12] reports estimates of the impact of capital gains and losses from the stock market upon consumption behavior. As a first approximation, \( \gamma_\ell \) might be taken to be equal to the marginal propensity to consume. If the inflation is regarded as a temporary rather than a permanent phenomenon, the capital
A final consequence of inflation upon effective demand concerns the distinction between the nominal and the real rate of interest. Since it is the opportunity costs of holding money that must be equated with the marginal gains from liquidity, the rate of interest determined by liquidity preference may be the nominal rate, \( r^* \). Investors, on the other hand, are concerned with the real rather than the nominal rate of interest, \( r = r^* - \hat{\rho} \), where \( \hat{\rho} \) is the anticipated speed of inflation. Writing the investment function as \( I[r^*(M, Y) - \hat{\rho}] \) emphasizes that an increase in the anticipated rate of price rise is equivalent to a reduction in the nominal rate of interest. Rather than complicate the analysis by attempting to incorporate the factor determining \( \hat{\rho} \), let us assume that the anticipated rate of price change is related by a coefficient of expectations \( \psi \) to the actual speed of inflation; more precisely, \( \hat{\rho} = \psi \rho \), and we have \( I[r^*(M, Y) - \psi \rho] \). If investors fail to recognize that prices are rising, \( \psi = 0 \), and investment is insensitive to \( \hat{\rho} \). With perfect expectations on the other hand, \( \psi = 1 \) and the impact of \( \hat{\rho} \) upon investment spending is of the same magnitude but of opposite sign from the nominal rate of interest.

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loss adjustment would constitute a reduction in transient income and \( \gamma_2 \) might be smaller than would be the case when a continued rise in prices is anticipated. It should also be noted that the effect of capital losses suffered on private holdings of government bonds is neglected in equation (3.2); this would be perfectly valid if only “index” bonds with both principal and interest guaranteed against price changes were issued by the government; on the other hand, when bonds with principal and interest expressed in nominal units are issued, \( \gamma_2 \) may well be larger than would otherwise be the case when this complication is omitted from the equation explaining the generation of effective demand; the magnitude of this coefficient also hinges upon how sweeping a definition of money is employed.

9 The importance of considering the distinction between the nominal and the real rate of interest in the analysis of inflation has been emphasized by Martin J. Bailey [1].

10 If one pays $130 back on a loan of $100, the nominal rate of interest is 30 per cent. But if the market value of physical assets rises by 25 per cent during the period of the loan, a five per cent real rate of return on investment would be sufficient to justify borrowing at this high nominal rate.

11 As is customary in Keynesian analysis, the impact of capital accumulation is neglected in this approach. It might be more accurate to assume that the equilibrium capital stock \( K^* \) depends upon the nominal rate of interest and anticipated price changes, and that investment is some function \( \lambda \) of the discrepancy between the equilibrium and actual stock; we would then have \( I = \lambda(K^*[(r^*(M, Y) - \hat{\rho}) - K]) \) but such an approach would increase the complexity of the problem by whole orders of magnitude.

12 While this approach has its obvious limitations, it does permit us to analyze in later sections the way in which the response of the price level to a forced saving policy depends upon the extent to which it is recognized that prices are rising. Observe too that any reader who believes that it is the real rather than the nominal rate of interest that is determined by liquidity preference is perfectly free to proceed without quarr el by simply setting \( \psi = 0 \).
In sum, our expression for effective demand involves three dynamic complications of the basic Keynesian equation. All the complications concern the impact of rising prices. Each of these effects has received at least some attention in the literature. The next section of this paper explores the implications of these complications for the process of inflation when the money supply itself is responsive to the rate of price change.

4. EXCESS DEMAND, FORCED SAVING EQUILIBRIUM, AND STABILITY

Let us consider the implications of the fundamental assumption that when an inflationary gap exists, prices rise rapidly enough to eliminate excess demand. Since excess demand is the surplus of effective demand over capacity, we require

\[ E(\dot{\rho}, M; G, \delta) = Y(\dot{\rho}, M; G, \delta) - Y_c \]
\[ = \alpha Y - \gamma_\rho Y - \gamma_\rho M + I[\psi'(Y, M) - \psi \dot{\rho}] \]
\[ + [1 - \alpha_t(1 - \delta)]G - Y_c = 0. \]

This assumption, which is untenable unless we also have

\[ \frac{\partial E}{\partial \dot{\rho}} < 0, \]

is recommended by expository convenience as well as numerous precedents in the literature.\textsuperscript{13} We shall see that assumptions (4.1) and (4.2) do not suffice to insure that a policy of forced saving will lead eventually to a stable rate of price rise.

The behavior of prices when the money supply is endogenously determined by a policy of forced saving is most easily derived by considering the relationship between the rate of price change and the real money supply implied by equation (4.1) for given \( G \) and \( \delta \). Let us suppose that curve \( \dot{\rho}(M; G, \delta) \) in Figure 1 represents the loci of all points at which excess demand is zero. It is drawn with a positive slope; later we consider the case \( \partial \dot{\rho}/\partial M < 0 \). The rectangular hyperbola in Figure 1 is obtained by considering the behavior of the money supply under forced saving as given by equation (2.3). If the real money supply is constant, if \( dM/dt = 0 \), we must have

\[ \text{\textsuperscript{13} Examples of the employment of this assumption are provided by Friedman [3] and Koopmans [8]. The assumption that prices always rise rapidly enough to eliminate excess demand is in no way essential to the argument to be developed in the text. As indicated in the next footnote, the stability conditions that will be derived also follow if (4.1) is replaced with the more general assumption that } \]
\[ \dot{\rho}/dt = \lambda E(\dot{\rho}, M; G, \delta), \quad \lambda > 0. \]
(4.3) \[ \dot{p}M = \rho \delta G. \]

The rectangular hyperbola is the loci of these points; combinations of \( M \) and \( \dot{p} \) to the southwest of this hyperbola yield an expanding money...
supply. On the other hand, points to the northeast of the hyperbola correspond to a contracting real money supply.

If at some point of time we have a real money supply $M$, the speed of inflation must be $\dot{\rho}$, as excess demand is zero. Since point $e$ is on the rectangular hyperbola, there is no tendency for the real money supply to change in magnitude. The injection of new money is precisely offset by the depreciation of the existing stock. This is a forced saving equilibrium, but it is a moving equilibrium characterized by a constant speed of inflation rather than an unchanging price level.

An equilibrium is of little interest if it is unstable. Let us see whether point $e$ in Figure 1 is stable. If the real money supply is below $M$, the resulting rate of price rise, read off the curve $\dot{\rho}(M; G, \delta)$ will lie below the rectangular hyperbola. Prices will not rise fast enough to prevent an increase in the real money supply. On the other hand, if the initial money supply is above the equilibrium level, prices must be rising rapidly enough to more than offset the injections of new money. Consequently, the real money supply always moves towards the unique equilibrium value $M_e$. This suggests that a policy of financing a given proportion of real government spending by creating money will necessarily lead to a unique equilibrium characterized by a constant rate of price change and stable real money supply if, as on the graph, the function $\dot{\rho}(M; G, \delta)$ is monotonically increasing in $M$.

A number of negatively inclined $\dot{\rho}(M; G, \delta)$ are plotted in Figure 2. Forced saving equilibria exist at points $s$ and $u$. Note the possibility of multiple equilibria, as with function $\dot{\rho}_s$. Although the forced savings equilibrium at $s$ is stable, those at $u$ are unstable, for a careful inspection of the graph suggests that stability requires that the speed of inflation function pass through the rectangular hyperbola from below as the money supply increases; in terms of the elasticity of $\dot{\rho}(M; G, \delta)$ we require\(^{14}\)

\(^{14}\) A forced saving equilibrium is stable in the small if there exists an $\varepsilon > 0$ such that $M > M_e$ implies $dM/dt < 0$ and $M < M_e$ implies $dM/dt > 0$ for all $0 < |M - M_e| < \varepsilon$; in other words $d(M/dt)/(M_e - M) > 0$ for $0 < |M_e - M| < \varepsilon$. Now $|M_e - M|$ sufficiently small implies by (4.1) that $\dot{\rho}(M, G, \delta) = \dot{\rho}_e + (\partial \dot{\rho}/\partial M)(M_e - M)$; in addition $\dot{\rho}_e M_e = \rho_0 G$. Consequently,

$$\frac{dM}{dt} = \rho_0 G - \dot{\rho} = \dot{\rho}_e M_e - \left[\dot{\rho}_e + \frac{\partial \dot{\rho}}{\partial M}(M_e - M)\right]M = \left(\dot{\rho}_e + \frac{\partial \dot{\rho}}{\partial M} M_e\right)(M_e - M).$$

Therefore, a forced saving equilibrium is stable in the small if and only if

$$\dot{\rho}_e + \frac{\partial \dot{\rho}}{\partial M} M > 0,$$

which is equivalent to (4.4).
\[ \frac{\partial \hat{p}(M; G, \delta)}{\partial M} \frac{M}{\hat{p}} > -1. \]

The necessary and sufficient condition for stability of a forced saving equilibrium can be formulated in terms of certain properties of the function explaining the generation of effective demand, equation (3.2), if we are allowed to assume, at first approximation, that the impact of rising prices and capital accumulation upon \( Y_e \) may be neglected.\(^{15}\) Differentiating (4.1) totally with respect to \( M \) yields

\[ \frac{\partial \hat{p}(M; G, \delta)}{\partial M} = -\frac{\partial E}{\partial \hat{p}} = -\frac{\partial Y_d}{\partial \hat{p}} - \frac{\partial Y_c}{\partial \hat{p}}. \]

Provided that \( \partial Y_d/\partial \hat{p} < 0 \) and \( \partial Y_c/\partial M = \partial Y_d/\partial \hat{p} = 0, \)

\[ \frac{\partial \hat{p}(M; G, \delta)}{\partial M} \frac{M}{\hat{p}} > -1 \]

if and only if

\[ \frac{\partial Y_d}{\partial M} \frac{M}{Y} > -\frac{\partial Y_d}{\partial \hat{p}} \frac{\hat{p}}{Y}. \]

Subject to the specified restrictions, a forced saving equilibrium is stable if and only if the elasticity of effective demand with respect to the money supply exceeds its elasticity with respect to the speed

\(^{15}\) If prices are stable in a Keynesian system experiencing full employment, the marginal productivity of labor must be equal to the marginal disutility of work; this condition can be satisfied only by a particular volume of employment, a volume that is independent of the level of prices. Capacity output \( Y_c \) is the output that can be produced with the full employment labor supply and the current supply of capital. Although capacity output is independent of the price level, it is conceivable for it to be an increasing function of the speed of inflation. If workers are misled by rising prices into thinking that real wages are higher than is actually the case, they may be induced to provide more labor services than they otherwise would at any given real wage. Again, if employers are misled by errors of accounting connected with inflation into confusing paper windfall profits with the actual gains of business, workers may in general be paid more than their marginal productivity. Furthermore, if there are lags in the adjustment of wages as prices rise so that real wages fall, the capacity level of output may be affected. All this was recognized by Keynes [7, (290)]. For the most part we shall neglect these complications and assume that \( Y_c \) is fixed. The interested reader will find that while part of our argument can be readily modified to take into account any assumptions one chooses to make about the impact of rising prices on capacity, certain policy issues may ultimately hinge upon a comparison of the effects upon efficiency of rising prices versus a higher level of tax rates.
of inflation.\footnote{18}

The conditions required for the stability of a forced saving equilibrium are not as restricted as those sometimes imposed in studies of inflation in which the nominal money supply is regarded as exogenously determined. Don Patinkin [11] has pointed out that when a full employment equilibrium has been disturbed by an injection of currency, rising prices may, under favorable conditions, lead to the elimination of inflationary pressure if the nominal money supply is stabilized at the new level. With a stable nominal money supply, \( M^e = M_0 \), the process of inflation may be self-correcting for with higher prices the constant nominal money supply will yield a higher rate of interest, less investment spending, and a reduction in the magnitude of the inflationary gap; prices continue to rise until \( E(0, M; G, \delta) = 0 \) and then, by (4.1), \( \dot{\delta} = 0 \). The reduction in the real money supply will indeed serve to curtail inflationary pressure if, in addition to conditions (4.1) and (4.2), we have

\[
\frac{\partial E}{\partial M} = \frac{\partial Y_t}{\partial M} = -\gamma_2 \dot{\delta} + \frac{\partial I}{\partial r} \frac{\partial r}{\partial M} > 0 .
\]

This last condition will definitely be satisfied if the only effect of rising prices upon effective demand is via the inflation illusion effects of inertia in adjusting wages and spending to rising prices described in Section 3, for then \( \gamma_2 = 0 \). Equation (4.5) reveals that if a constant nominal money supply would lead to the elimination of inflationary pressure, a stable forced saving equilibrium exists, for (4.7) and (4.2) imply \( \partial \delta(M; G, \delta)/\partial M > 0 \). On the other hand, the existence of a stable forced saving equilibrium does not imply that the adoption of the alternative policy of refraining from injecting new currency would necessarily serve to eliminate inflationary pressure; condition (4.7) is much stronger than (4.4).

\footnote{18} If we assume \( d\dot{\delta}/dt = \lambda E(\dot{\delta}, M, G, \delta), \lambda > 0 \) rather than (4.1), the conditions for stability in the small are unaffected. For \( \epsilon > 0 \) sufficiently small, \( |\dot{\delta} - \dot{\delta}_e| < \epsilon \) and \( |M - M_e| < \epsilon \) imply \( E(\dot{\delta}, M, G, \delta) = (\partial E/\partial \dot{\delta})(\dot{\delta} - \dot{\delta}_e) + (\partial E/\partial M)(M - M_e) \) and \( dM/dt = -M(\dot{\delta} - \dot{\delta}_e) - \dot{\delta}(M - M_e) \), or

\[
\begin{bmatrix}
\frac{d\dot{\delta}}{dt} \\
\frac{dM}{dt}
\end{bmatrix} = \text{diag.}(\lambda, 1)J \begin{bmatrix}
\dot{\delta} - \dot{\delta}_e \\
M - M_e
\end{bmatrix}, \text{ where } J = \begin{bmatrix}
\frac{\partial E}{\partial \dot{\delta}} & \frac{\partial E}{\partial \delta} \\
-M & -\dot{\delta}
\end{bmatrix}.
\]

Stability requires that the characteristic roots of the matrix diag \( (\lambda, 1)J \) have negative real parts for \( \lambda > 0 \); as is well known, this is independent of the speed of adjustment and requires that the principal minors of \( J \) be alternatively negative and positive in sign, conditions (4.2) and (4.4).
5. FORCED SAVING POLICY AND THE SPEED OF INFLATION

Although Keynes was, for the most part, preoccupied with the tragedy of unemployment when he wrote the General Theory of Employment, Interest and Money, the work does contain terse passages focused upon the problem of excess demand [7, (289)].

The conditions of strict equilibrium require, therefore, that wages and prices, and consequently profits also, should all rise in the same proportion as expenditure, the 'real' position, including the volume of output and employment, being left unchanged in all respects. We have reached, that is to say, a situation in which the crude quantity theory of money (interpreting 'velocity' to mean 'income-velocity') is fully satisfied; for output does not alter and prices rise in exact proportion to $MV$.

The price level does indeed rise in proportion to the nominal money supply when a Keynesian system involving the dynamic complications discussed in Section 3 of this paper is in forced savings equilibrium. Since a forced saving equilibrium is characterized by a stationary real money supply with output pushed against the full-employment ceiling, the income velocity of money must be constant. If one observed an economy in forced saving equilibrium, one would encounter no evidence suggesting that the most elementary version of the quantity theory should be rejected. Nevertheless, the quantity theory offers little help in the difficult task of evaluating the effects upon the speed of inflation and the money supply of changes in the level of real government spending $G$ and the extent $\delta$ to which it is financed by a deficit.

As a first step in the appraisal of the effects of adjustments of the two decision variables $G$ and $\delta$ upon the behavior of prices and the money supply through time, it is necessary to analyze their impact upon $M$ and $\dot{p}$, the equilibrium real money supply and speed of inflation. The effects of small changes in government spending may be evaluated by considering the total derivatives with respect to $G$ of the equation for excess demand (4.1) and the expression for the rate of change in the money supply (2.3)

\[
(5.1a) \quad \frac{dE}{dG} = \frac{\partial E}{\partial \dot{p}} \frac{d\dot{p}}{dG} + \frac{\partial E}{\partial M} \frac{dM}{dG} + \frac{\partial E}{\partial G},
\]

\[
(5.1b) \quad \frac{d\dot{M}}{dt} = -\frac{M}{G} \frac{d\dot{p}}{dG} - \frac{\dot{p}}{dG} \frac{dM}{dG} + \rho \delta .
\]

Since, by (4.1), these two derivatives must be set equal to zero in evaluating the effects upon a forced saving equilibrium of changes in $G$, we may solve these two simultaneous equations for
(5.2a) \[
\frac{d\dot{p}}{dG} = \frac{1}{k} \left( \dot{\hat{p}} \frac{\partial E}{\partial G} + \rho \hat{p} \frac{\partial E}{\partial M} \right)
\]
and
(5.2b) \[
\frac{dM}{dG} = -\frac{1}{k} \left( M \frac{\partial E}{\partial G} + \rho \hat{p} \frac{\partial E}{\partial \hat{p}} \right),
\]
where \( k = -\hat{p} (\partial E/\partial \hat{p}) + (\partial E/\partial M) M \).

Even with this simple two equation system, little can be said concerning the impact of changes in the level of government spending upon the forced saving equilibrium unless certain restrictions are imposed upon the parameters of the system. Since equilibrium solutions are of primary interest only when they are stable, it seems reasonable to confine our discussion to the case in which stability conditions are satisfied, in accordance with Professor Samuelson’s Correspondence Principle. Although it follows that \( k > 0 \), the effect of an increase in government spending upon a stable forced saving equilibrium speed of inflation still depends upon the relative magnitudes of two elasticities, as may be seen from the expression in parentheses in the following reinterpretation of the first equation of (5.2)\(^{17}\)

(5.3) \[
\frac{d\dot{p}_c}{dG} \frac{G}{\dot{\hat{p}}_c} = \frac{Y_d}{k} \left( \frac{\partial Y_d}{\partial G} \frac{G}{Y_d} + \frac{\partial Y_d}{\partial M} \frac{M}{Y} \right).
\]

An increase in government spending accelerates the equilibrium speed of inflation if the sum of the elasticity of effective demand with respect to increases in government spending and its elasticity with respect to the money supply is positive. Although the first of these elasticities is necessarily positive, the situation is ambiguous because it is conceivable that a larger money supply may reduce effective demand. It does follow from our assumptions, however, that \( d\dot{p}_c/dG > 0 \) whenever the speed of inflation is a positive function of the money supply; this may be seen from (4.5). In particular, this means that in the absence of the capital loss adjustment type of reaction to inflation discussed in Part 3, an increase in government spending would definitely lead to an accelerated speed of inflation.

The effect of changes in government spending upon a stable equilibrium’s real money supply also depends upon the relative magnitude

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\(^{17}\) Since \( k \) is the determinant of \( J \) (the \( 2 \times 2 \) stable matrix of the last footnote of Section 4), it must be positive.

\(^{18}\) That this elasticity interpretation is equivalent to (5.2a) may be easily seen if one remembers from (2.3) that \( dM/dt = 0 \) implies \( \rho G = \hat{p} M \) and that \( \partial E/\partial M = \partial Y_d/\partial M \) by the definition of excess demand when \( \partial Y_d/\partial M = 0 \).
of two elasticities, as may be seen from the following reinterpretation of the second equation of (5.2):

\[
\frac{dM_s}{dG} \frac{G}{M_s} = -\frac{Y}{k} \left( \frac{\partial Y_d}{\partial G} \frac{G}{Y} + \frac{\partial Y_d}{\partial \hat{p}} \frac{\hat{p}}{Y} \right).
\]

An increase in government spending increases the equilibrium real money supply only if the elasticity of effective demand with respect to the speed of inflation is larger in absolute value than its elasticity with respect to government spending.

The effect upon a stable forced saving equilibrium of small changes in the proportion of government spending financed by a deficit may also be evaluated in terms of the relative magnitudes of certain elasticities. Proceeding as before, one eventually obtains

\[
\frac{d\hat{p}_s}{d\hat{\delta}} \frac{\hat{\delta}}{\hat{p}_s} = \frac{Y}{k} \left( \frac{\partial Y_d}{\partial \hat{\delta}} \frac{\hat{\delta}}{Y} + \frac{\partial Y_d}{\partial \hat{p}} \frac{\hat{p}}{Y} \right)
\]

and

\[
\frac{dM}{d\hat{\delta}} \frac{\hat{\delta}}{M} = -\frac{Y}{k} \left( \frac{\partial Y_d}{\partial \hat{\delta}} \frac{\hat{\delta}}{Y} + \frac{\partial Y_d}{\partial \hat{p}} \frac{\hat{p}}{Y} \right).
\]

Since it is necessarily the case that \( \partial Y_d/\partial \hat{\delta} > 0 \), an increase in the proportion of government spending financed by creating money will lead to an acceleration of the inflationary process unless effective demand is particularly sensitive to the capital losses that accompany rising prices. The second equation reveals that the question of whether an increase in \( \hat{\delta} \) will lead to an augmentation of the equilibrium real money supply hinges upon the relative magnitude of the elasticities of effective demand with respect to \( \hat{\delta} \) and \( \hat{p} \).

That these elasticity conditions concern only the effects of small changes in the policy variables cannot be too strongly emphasized. A hypothetical example reveals that the task of evaluating the effects of adjusting \( \hat{\delta} \) or \( G \) is complex, and that a complete appraisal would require a detailed empirical investigation of the specific situation. Consider Figure 3; the curve \( \hat{p}(M; G, \hat{\delta}_i) \) represents the initial speed of inflation function; at every point on this curve excess demand is zero. The second speed of inflation curve \( \hat{p}(M; G, \hat{\delta}_i) \) corresponding to an increase in the proportion of government spending financed by creating money from \( \hat{\delta}_i \) to \( \hat{\delta}_1 \) yields, for any \( M \), a higher rate of price change; the magnitude of the shift depends in part upon \( \alpha_i \) and \( G \), as may be seen from equation (4.1). Because the rectangular hyperbola revealing combinations of \( M \) and \( \hat{p} \) at which the money supply would remain stable and the speed of inflation function have
both shifted, the new equilibrium at point $e_i$ corresponds to a much larger real money supply but a reduction in the speed of inflation. Since the new equilibrium is stable, the policy change results in only a temporary increase in the speed of inflation as the system moves towards $e_i$.

The hypothetical example suggests a second question. Suppose that instead of crossing the rectangular hyperbola at $e_i$, the new speed of inflation schedule was always below the new rectangular hyperbola. Then for $\delta_i$ there would exist no forced saving equilibrium, and the money supply would expand indefinitely. Because the speed of inflation schedule is downward sloping, the rate of price change would gradually fall. Thus the paradoxical result of the policy change, which shifted the speed of inflation function upwards, is a continued retardation in the rate at which prices rise.

In evaluating the implications of a policy of forced saving, it must be remembered that the speed of inflation provides only an imprecise index of the social costs of inflationary finance. After all, the net output left over to the private sector is $Y_c - G$, and this may be regarded as insensitive to increases in $\delta$ except insofar as the effects of decreased taxation and changes in $\dot{p}$ upon productive efficiency,
incentives, and hence upon $Y_a$ do not offset each other. As both Keynes and Friedman have emphasized, part of the incidence of inflation involves the tax levied in the form of capital losses upon the holders of monetary assets. Equation (3.3) reveals that this capital loss is $L = \dot{\rho}M$, where the real money supply as well as the speed of inflation depend upon the extent to which government spending is financed with a deficit rather than by tax revenues. The magnitude of the tax levied by rising prices is proportional to the deficit, provided that the economy remains in a forced saving equilibrium.\textsuperscript{19} Although the direction of the impact upon the speed of inflation caused by an increase in the proportion of government spending financed by the printing press depends upon certain properties of the effective demand function, there is no ambiguity concerning the increase that necessarily results in the tax that will be levied at the new equilibrium by inflation in the form of capital losses.

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\textbf{REFERENCES}


\textsuperscript{19} In equilibrium, $L = \dot{\rho}M = \delta \rho C = \rho D$ where $\rho$, the coefficient of expansion of the monetary system, is institutionally determined.
