THE NEW VIEW OF INVESTMENT:
A NEOCLASSICAL ANALYSIS*

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In 1956 appeared the first in a series of papers1 disputing the traditional thesis that capital deepening is the major source of productivity gains and conjecturing that we owe our economic growth to our progressive technology.

Thesis and antithesis were synthesized by 1960. Investment has been married to Technology.2 In the new view, the role of investment is to modernize as well as deepen the capital stock. Now investment is prized as the carrier of technological progress.

No criticism is made here of this “new view” of the role of investment. Nor is the need for accelerated investment, public and private, questioned. This paper is concerned only with the logic of certain conclusions which the new view has shown a tendency to inspire. In what sense does its new role make investment more important? What are the prospects of modernizing the capital

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stock through increased thrift? Does the new view of investment present any new reasons — should added ones be needed — for faster capital accumulation? The analysis is confined largely to investment-thrift policies described by a fixed saving ratio. The final section presents estimates of the rate of return to investment as implied by certain new-view assumptions. The results of the inquiry are summarized at the conclusion of the paper.

**THE BASIS OF INVESTMENT PESSIMISM**

The empirical work cited above spans a great variety of analytical methods and historical materials. One of the best known papers is that by Professor Solow. A number of other investigators followed much the same approach.

That method postulates aggregate output, \( Q_t \), to be a continuously differentiable function of capital, \( K_t \), employment, \( N_t \), and "time" (standing for the state of technology). If, further, technical progress is "neutral," then output is a separable function of time and the inputs, as follows:

\[
Q_t = A(t) F(K_t, N_t).
\]

Such a production function implies that technical progress is organizational in the sense that its effect on productivity does not require any change in the quantity of the inputs. Existing inputs are improved or used more effectively.

It follows that the growth rate of output is equal to the rate of technical progress plus a weighted average of the growth rates of the inputs. These weights are the elasticities of output with respect to capital and to labor. Assuming constant returns to scale, the weights add to one and we obtain

\[
\frac{Q_t}{Q_{t-1}} = \frac{\dot{A}_t}{A_t} + a_t \frac{\dot{K}_t}{K_t} + (1 - a_t) \frac{\dot{N}_t}{N_t}
\]

where \( a_t \) is the capital elasticity of output, that is \( \frac{F_K(K_t, N_t) K_t}{Q_t} \).

There are two unknowns in equation (2), the rate of technical progress and the capital elasticity. Solow, and later Massell, relied on an "outside" estimate of the capital elasticity and proceeded to focus on the rate of technical progress. Solow took capital's relative share of national income in year \( t \) as a measure of \( a_t \) and Massell, who assumed \( a_t \) was constant over time, used the average share going to capital. It is not known how close such approxima-


tions are. The practice presumes pure competition (which is not strictly implied by the model) as well as constant returns to scale.

The results of this approach produced a wave of investment pessimism. From a study of U.S. time series it was concluded that less than one-third of the average growth rate of output per worker in the last quarter century could be credited to the increase in capital per worker which occurred.5

Of course, it does not follow from this conclusion that capital deepening is ineffectual. It might mean only that over the time period investigated little capital deepening took place.6 For policy purposes, the effectiveness of additional investment is of greater interest. On this score too, however, the approach outlined above produces some gloomy results.

Consider the effect of doubling the (net) investment-income ratio from .09 to .18. If the capital-output ratio is about 3, then this increase in the saving ratio would in a year increase the capital stock by about 3 per cent (beyond what it would have increased otherwise). Now capital’s share in (net) national income is less than one-third. Therefore, according to equation (2), the 3 per cent increase in the capital stock would increase (net) output by less than 1 per cent (and it would increase output even less if the capital-output ratio rose).7 Solow has remarked of such a calculation: "This seems like a meager reward for what is after all a revolution in the speed of accumulation of capital."8

5. From equation (2) it is easy to derive the proportion of the growth rate of output per worker which is attributable to capital deepening. It is

\[ \frac{a_t(k_t - n_t)}{q_t - n_t} = \frac{a_t(k_t - n_t)}{r_t + a_t(k_t - n_t)} \]

where \( k_t, n_t, q_t, \) and \( r_t \) denote the (relative) growth rates of capital, labor, output, and technology respectively, at time \( t \). If there is no capital deepening, meaning \( k_t = n_t \), then the proportion is equal to zero. If there is no technical progress, the proportion is equal to one.

The Solow-Massell result is easy to explain. In the U.S. time series they employed, capital and output grew at approximately the same rate. But if \( k_t \) equals \( q_t \) then the proportion equals \( a_t \). Their factor share data put \( a_t \) at about one-third (or less).

6. The current alarm over the decline since the early twenties in the capital-output ratio rests on just such a counterinterpretation.


THE NEW VIEW

Just when the reputation of investment seemed at low ebb came the first signs of a new tide. Critics of the research described contended that new technologies generally require new kinds of capital goods. Therefore without positive (gross) investment productivity could hardly be expected to grow at all. Furthermore, it was argued, the higher the rate of gross investment, the newer and hence more modern and "efficient" will the capital stock become. Proponents of this new view of investment were apt to assign as much weight to capital modernizing as to capital deepening.9

In 1961 the new view of investment was embraced by the new administration. The President's Economic Message to Congress in January, 1961 stated:

Expansion and modernization of the Nation's productive plant is essential to accelerate economic growth and to improve the international competitive position of American industry. Embodying modern research and technology in new facilities will advance productivity, reduce costs, and market new products.1

Expansion and modernization are put on equal footing and the latter is stressed. A statement by the Council of Economic Advisers before the Joint Economic Committee in March 1961 amplifies this view:

One of the reasons for the recent slowdown in the rate of growth of productivity and output is a corresponding slowdown in the rate at which the stock of capital has been renewed and modernized... As has been confirmed by more recent research, the great importance of capital investment lies in its interaction with improved skills and technological progress. New ideas lie fellow without the modern equipment to give them life. From this point of view the function of capital formation is as much in modernizing the equipment of the industrial worker as in simply adding to it. The relation runs both ways: investment gives effect to technical progress and technical progress stimulates and justifies investment.2

To clarify the meaning of this new notion and to lay the basis

9. Two of the earliest documents taking the new view are Economic Survey of Europe in 1888, op. cit., and Growth in the British Economy, op. cit. They argue that rapid labor force growth—contrast Britain and Germany—will raise output per worker by stimulating gross investment. The stimulation required is not spelled out.
for the analysis to follow, we turn now to an important theoretical paper by Solow which adopts the new view. The purpose of that paper is to show that such neoclassical concepts as aggregate capital and the aggregate production function (containing aggregate capital) can be modified to accommodate the new view.

Solow postulates an index of technology, $B(t)$, which advances neutrally and exponentially at the rate $\lambda$. The nature of the technology so indexed is such that at every point of time it affects the efficiency only of new capital goods. Every capital good embodies the latest technology at the moment of its construction but it does not participate in subsequent technical progress. Thus "capital" becomes a continuum of heterogeneous vintages of capital goods.

The output rate at time $t$, $Q_s(t)$, of capital equipment of vintage $v$ is assumed to be given by a Cobb-Douglas function,

$$Q_s(t) = B_0 e^{\lambda t} K_\nu(t)^a N_\nu(t)^{1-a}$$

where $K_\nu(t)$ denotes the amount of equipment (in physical terms) of vintage $\nu$ surviving at time $t$ and $N_\nu(t)$ denotes the amount of labor employed on that equipment. Since technical progress is neutral, the elasticity parameter $a$ is the same for all vintages.

Solow then shows that if labor is allocated efficiently over the various vintages (by equalizing labor's marginal productivity on all equipment), aggregate output — the sum of the homogeneous outputs of the various vintages — is given by:

$$Q = B_0 J^a N^{1-a}$$

where

$$J = \int_{-\infty}^{t} e^{\lambda v} K_\nu(t) \, dv.$$  

The "J" variable might be called "effective capital." The integral adds up all the (surviving) capital goods like the conventional capital measure; but here the capital goods of older vintages (with their small $\nu$'s) receive a smaller weight than new capital goods.

For comparison with the old-fashioned model, let us specialize (1) in the same way. If all technological progress is organizational, neutral and proceeding at the constant relative rate, then

$$Q_i = A_i e^{\alpha t} K_i^{a} N_i^{1-a}.$$  

According to this classical view, old and new capital goods share alike in technical progress, so that "capital," $K_i$, is simply the sum of the homogeneous surviving capital goods. Hence (5) can be written:

\[ Q_t = A_0 \left[ \int_{-\infty}^{t} e^{y_i} K_y(t) \, dy \right]^a N^{1-a} \]

The encouragement drawn from the new view — as represented by (4) — as compared with the old view — represented by (6) is illustrated by the following example.

Suppose that existing machines are of just two vintages, \( v_1 \) (old) and \( v_2 \) (new), and that there are an equal number of machines of the two vintages.

According to (6) a 2 per cent increase in the number of machines of the current vintage, \( v_2 \), will bring about a 1 per cent increase in the value of \( K \) and of the bracketed expression in (6); we are weighting a 2 per cent and a zero increase equally.

Consider the case in equation (4). \( J \) is the weighted sum of the machines of the two vintages with the weight for the contemporary machines, namely \( e^{\theta_i} \), being greater. Consequently a 2 per cent increase in the number of machines of current vintage will produce an increase of \( J \) in excess of 1 per cent. Here current investment increases output per man partly through affecting the average modernity of the capital stock.

What if we lengthen our view and ask what happens as the program of capital accumulation continues? Pretty soon we will be confronted by a changed situation; large investments today will present us with a large amount of old equipment in the future. Investment must grow in order to maintain a constant average age of capital. And as we shall see, there is (under certain plausible assumptions) an average age of capital such that no smaller average age is tenable for long. The modernizing effects of expanded investment are limited.

Suffice it to say that the long-run consequences of a change in investment policy are not so clear as the immediate effect, and both are deserving of study. True, in the long run we are dead but our children will have to live in it. Can we control an important degree of the modernity of the capital stock they will inherit? Do we owe the modernity (such as it is) of our present stock to our ancestors' thrift? What significance has the new view of investment for the long run? This is examined now.

A Simple Model of Growth

We shall confine our analysis to the implications for output growth and productivity of investment policies which make gross investment a fixed proportion of gross output. The choice of an
investment policy is thus a matter of selecting the investment-output ratio $s$. Hence, where $I(t)$ denotes the rate of investment at $t$:

$$I(t) = s Q(t).$$  

Second, we assume that the labor force grows at the constant relative rate $n$:

$$N_t = N_0 e^{nt}.$$  

Finally we assume that all capital goods depreciate exponentially at the rate $\delta$ per annum. Hence

$$K_v(t) = I(v) e^{-\delta(t-v)}.$$  

If we think of $\delta$ as a mortality rate, then the average lifetime of capital goods is $\frac{1}{\delta}$ years.

Now our purpose is to compare the relation between investment and growth under the new and old view. We can do this by comparing a pure new-view model with a pure old-view model. But the simplest approach is to examine a single model which, by a variation of parameters, can be made to represent either pure or a mixture of both.

Thus we shall work with the following "general" production function which is simply a blend of (4) and (6):

$$Q_p = B_0 e^{\omega t} J_t^0 N_t^{1-a}$$  

where, as before

$$J_t = \int_{-\infty}^{t} e^{\frac{\lambda v}{\sigma}} K_v(t) \, dv$$  

or, by virtue of (9)

$$J_t = e^{-\beta t} \int_{-\infty}^{t} e^{\frac{\lambda v + \beta v}{\alpha}} I(v) \, dv.$$  

When we compare the new view to the old view we are comparing the behavior of the model with $\lambda > 0, \mu = 0$ against the behavior when $\mu > 0, \lambda = 0$. And if one believes in both kinds of technological progress then he can let $\lambda > 0, \mu > 0$ simultaneously. (In that

4. It may be (and has been) objected that it cannot be assumed that the other parameters, $\alpha, \delta$ and so forth, are invariant to the nature of the technology (i.e., whether it is the $\lambda$-type or $\mu$-type). But we find no implication in the new view that the nontechnological parameters differ from their supposed or implied values under the old view. That is, "$\lambda > 0, \mu = 0"$ implies nothing about $\delta$ and $\alpha$; to the contrary, the postulate that the embodied or $\lambda$-type technical progress is "neutral" implies that $\alpha$ is independent of $\lambda$. Whether empirical estimates of $\alpha$ and $\delta$ would be affected depends upon the method of estimation. Under neoclassical conditions it is common practice to take capital's relative share as an estimate of $\alpha$; this procedure is equally
case the efficiency of all capital goods may be said to rise at the
rate $\frac{\mu}{a}$ except the efficiency of new capital goods which rises at the
rate $\frac{\lambda + \mu}{a}$.

Differentiating $Q_t$ in (10) with respect to time yields (omitting the $t$
subscript):

\begin{equation}
\dot{Q} = \mu Q + \alpha B_0 \dot{a} \, J^{a-1} \, N^{1-a} \left[ \frac{\lambda}{e^\alpha} - \delta \dot{J} \right]
- (1-a) B_{0t} \, N^{-a} \, J^{1-a} \, \dot{N}
\end{equation}

where we have used the relation $\dot{J} = e^\frac{\lambda}{a} - \delta \dot{J}$ by virtue of (9).

Using (7), (8) and (9) (to express $J^{a-1}$ in terms of $Q$ and $N$) we
obtain the following differential equation governing the growth path
of output:

\begin{equation}
\dot{Q} = c_1 Q + c_2 \, Q \, a \, e^4
\end{equation}

where

\begin{align*}
c_1 &= \mu - a \delta + (1-a) n \\
c_2 &= a \, \delta \, B_a \, N_a \, e^a \\
c_3 &= \frac{2a-1}{a} \\
c_4 &= \frac{\lambda + \mu + (1-a) n}{a}
\end{align*}

This equation can be solved for the path of output. The
next section the long-run or asymptotic behavior of output will be
considered.

**Investment and Productivity in the Long Run**

These models have the convenient property that, starting from
the initial position, the path of growth will be asymptotic to a
balanced-growth, "golden-age" equilibrium growth path along which
path production, consumption, investment, and the capital stock
appropriate on the two views. One's assumptions about $\lambda$ and $\mu$
would affect $B_a$, the technology index at $t = 0$; we return to this in a footnote infra.

5. For the solution we are indebted to a regrettably unpublished paper
by Dernburg and Quirk, which analyzes an old-view Cobb-Douglas growth
model. T. Bernburg and J. Quirk, "Per Capita Output and Technological
Progress," Institute of Quantitative Research in Economics and Management,
Purdue University (1960).

Growth and Capital Accumulation," Economic Record, XXXII (Nov. 1956).
(of all ages) all grow exponentially at the same rate. This "equilibrium" output path is denoted \( \bar{Q}(t) \).

The limiting or asymptotic solution to equation (13) or (15) is

\[
(13) \quad \bar{Q}(t) = \bar{Q}_0 e^{\frac{c_s}{1-c_s} t}.
\]

Thus the growth rate, \( g \), tends in the long run to the constant \( \frac{c_s}{1 - c_s} \).

In terms of the original parameters:

\[
(14) \quad g = \frac{\lambda + \mu}{1 - \alpha} + n.
\]

It will be noticed that the limiting growth rate is independent of the investment ratio. This is a well-known property of old-style Cobb-Douglas models. It is not surprising to find this same property in the "new model," which allows \( \lambda > 0 \). Associated with this exponential growth pattern is a certain unchanging age distribution of capital. Capital which is \( (t - v) \) years old will grow at the rate \( g \) like most everything else; the proportion of capital which is \( (t - v) \) years old or less is constant over time. The fact that capitals of different vintages get different technical weights is immaterial in the determination of the exponential equilibrium growth rate.

Note also that the long-run growth rate depends only upon the total rate of technical change, say, \( \Delta = \lambda + \mu \), not upon the nature of the change. The reason is that the efficiency of capital \( (t - v) \) years old will, in exponential equilibrium, improve at the rate \( \Delta \) in either (pure or any mixed) case.

What then is the relation between investment and productivity in the long run? The higher the investment ratio that society chooses the larger will be its capital stock (at every point of time) in the long run. Thus the level of the "equilibrium" exponential growth path which the economy approaches is a function of the investment ratio. In short, \( \bar{Q}_0 \), the equilibrium value of \( Q \) at "time zero" (chosen arbitrarily), is a function of \( s \). This value is to be distinguished from the actual output at time zero, \( Q_0 \); the two will be equal only if the initial capital-output ratio happens to equal that ratio which the chosen \( s \) would have brought about.

   8. On the equilibrium path the "conventional" capital-output ratio is constant; both \( K \) and \( Q \) grow at the rate \( g \). But if \( \lambda > 0 \), "effective" capital \( \lambda \) grows at the rate \( g + \frac{\lambda}{\alpha} \) and so the effective capital-output ratio rises.
The solution for the long-run growth path is:

\[ Q_0 = \left[ \frac{(1 - c_0) c_2}{c_4 - (1 - c_3) c_1} \right]^{1/c_4} \]

or, in terms of the original parameters

\[ Q_0 = \left( \frac{\alpha}{1 - \alpha} \right)^{\frac{1}{1 - \alpha}} \left( \frac{1 - \alpha}{\alpha} \right) \left( 1 - \alpha \right) \left( \mu + \frac{\lambda}{\alpha} + (1 - \alpha) (n + \delta) \right) \]

What significance, we ask again, has the new view in relation to investment and productivity in the long run? From (16) one can see immediately that the elasticity of \( Q_0 \) with respect to \( s \) is \( \frac{\alpha}{1 - \alpha} \), independent of \( \lambda \) and \( \mu \). Whether one takes the new view or the old, it follows from this model that, in the long run, a 1 per cent increase in the investment ratio will yield asymptotically an output rate which is \( \frac{\alpha}{1 - \alpha} \) per cent in excess of what asymptotically it would otherwise have been (i.e., had the original investment ratio prevailed).

This result seems at first to contradict the little example of increased investment presented at the end of the "The New View" section. The explanation of the puzzle lies in the behavior of the average age — or more precisely, the age distribution — of capital. It has apparently been overlooked that, in exponential growth, the age distribution of capital depends upon the rate of growth and the rate of depreciation and upon nothing else. Since both rates are, in the long run, independent of the investment ratio, a once-for-all change in that ratio can have no permanent influence on the age distribution of capital. Consequently, in the long run, any increase in thrift must rely for its effectiveness upon the prosaic mechanism of capital deepening — of an equiproportionate deepening of capital of every age.

This is easily proved. Suppose the economy has been growing smoothly at the rate \( g \), along the growth path corresponding to the chosen fixed investment ratio, for quite some time. If, say at \( t = 0 \) (for convenience only), we were to look at the distribution of capital equipment by age we could summarize our findings by the exponential curves in Figure I. In order to obtain the amount of capital of vintage \( v \) still in use at \( t = 0 \), \( K_v (0) \), we have to multiply \( I(v) \) by \( e^{tv} \). This gives the lower curve.
To obtain the mean age and the other moments of the age distribution of capital, it is necessary to normalize the curve so that its area will equal 1. This requires dividing $K_0(0)$ by $I(0)/(g + \delta)$ for all $v$. Thus we obtain the formula for the proportion of equipment of age $v$:

$$f(v) = \frac{1}{g + \delta} e^{(\epsilon + \delta) v}.$$  

It is clear that all the moments of the equilibrium age distribution of capital are independent of the quantity of capital and the rate of investment. The equilibrium mean age of capital, for example, is simply

$$\bar{v} = \frac{1}{g + \delta} \int_{-\infty}^{\infty} g + \delta \, e^{(\epsilon + \delta) v} (-v) \, dv$$

Given the investment ratio, the mean age of capital depends in the long run only upon the rate of depreciation and the limiting rate of growth, and neither of these depend upon the investment ratio in this model.

9. $I(0)/(g + \delta)$ is the total area under the $K_0(0)$ curve, by the familiar "capitalization" formula.

1. Proof:

$$-\bar{v} = \int_{-\infty}^{\infty} \frac{g + \delta}{g + \delta} e^{(\epsilon + \delta) v} (-v) \, dv$$

$$= -\bar{v} \frac{g + \delta}{g + \delta} \int_{-\infty}^{\infty} e^{(\epsilon + \delta) v} \, dv$$

$$= 0 + \frac{e^{(\epsilon + \delta) v}}{g + \delta} \bigg|_{-\infty}^{\infty}$$

$$= \frac{1}{g + \delta}$$
Therefore a once-for-all rise in the investment ratio can significantly "modernize" the capital stock only temporarily. Ultimately the average age (or modernity) of capital must settle back toward its equilibrium level. A permanent modernization of the capital stock (starting from equilibrium) would require the investment ratio to increase without limit, a policy which is not feasible (without foreign assistance at any rate).

Of course, actual economies are never found in dynamic long-run equilibrium because of fluctuations in investment. An upswing in investment is usually associated with a downswing in the mean age of capital. But it should be understood that when the mean age of capital exceeds its equilibrium value, a decline in the mean age is bound to occur eventually no matter what investment ratio society elects to adopt. This leads us to digress briefly on the present mean age of capital in the United States and the direction in which it may be expected to move.

The Terbohrth-Knowles estimates of the average age of capital, which end at 1957, together with the experience of the past five

2. When the economy is out of equilibrium, the basic model is likely to forecast a different limiting growth path (corresponding to a given investment ratio) for every different value of $\lambda$ we should assign. If, for example, the mean age is below its equilibrium value then a new-view forecast, taking the eventual equilibrating increase in mean age into account, would predict a lower equilibrium path (whatever the investment ratio) than would an old-view forecast because the latter would attribute no significance to the eventual rise in the mean age of capital. This fact in no way invalidates the conclusions of this section concerning the long-run growth rate, the "investment elasticity" and the equilibrium mean age of capital, these relations being independent of the level of the equilibrium growth path.

A special case of some interest is that in which the economy has always traveled along the equilibrium path corresponding to the prevailing investment ratio. In this case the mean age of capital is in equilibrium and the value of $\lambda$ will not affect the predicted equilibrium growth path corresponding to any investment ratio, because $\lambda$ adjusts to satisfy (14) and B to satisfy (16).

Note that high $\lambda$ implies high $B$. Let $\eta$, $\delta$ and $s$ be recorded from direct observation and let $s$ be estimated from relative shares. Then $\Delta$ can be estimated simply from (14). If we believe some of this $\Delta$ is $\lambda$ then $\lambda \Delta$ rises so we have to make an upward adjustment of $B$ in (16) in order that the model be able to explain the actual level of output $Q_b = \bar{Q}_b$.

The adjustment of $B$ makes sense because the implied old-view estimate of $B$ is actually an estimate of the average level of technology embodied in all capital goods while the new view implies that the current level of technology is superior to the average. At time zero, the current (or best-practice) level of technology is measured by $B$. 
years suggest that the mean age today is about 17.5 years. Thus the postwar vintages comprise half the nation's capital. In 1975 the postwar investment boom will be working against a modern capital stock. Then all capital of vintages 1957 and earlier—particularly the heavy investments of 1946–57—will be older than 17 years. Between now and 1975 we apparently require an increase in investment comparable to the postwar increase in order to avert an increase in the mean age of capital.

Yet such an acceleration of investment is not unlikely, even without an increase of the investment ratio. Due to the expected rise in the rate of increase of the labor supply, many observers anticipate full-employment growth at 4½ per cent or more over the next decade—about 1 percentage point better than the postwar experience (in output and investment) to date. Therefore if investment should keep pace with output over the future, the mean age of capital may well hold steady or even fall.

Still, the major impression drawn from a study of the Terborgh-Knowles series is the remarkable stability of the mean age of capital. It took a depression and a war to raise the average age from 16.5 (in 1930) to 21.2 years (in 1945). This suggests that, given the technical and demographic factors which determine the limiting growth rate, it would be very difficult to reduce by means of investment the mean age of capital by more than 3 or 4 years. And, as we have seen, according to the model here this gain could not be indefinitely maintained. Eventually the mean age would slip back up to its natural long-run level.

Investment and Productivity in the Short Run

The foregoing analysis has some significance for "positive economics." For example, a sustained improvement in the modernity of the capital stock of a country should be ascribed (proximately) not to the level (rate) nor to the rate of growth of its investment but to a rise of the rate of growth of investment. The improvement can be expected to be permanent only if there have been (or will be) technical and demographic changes causing a rise in the limiting growth rate of output (thus averting a future deceleration of investment).


THE NEW VIEW OF INVESTMENT

The implications of the analysis for investment policy depend, of course, on the decision rules used by the policymaker. Many (all?) sensible rules will involve, among other things, the responsiveness of output in the short run to a policy of greater thrift and investment. The short run assumes considerable importance when we observe that our model economy approaches its limiting path only asymptotically. Even to get close to that path may take considerable time. It is worthwhile therefore to inquire into the speed with which the economy adjusts to a change in the equilibrium path brought about by a change in the investment ratio. It will be seen that the new view forecasts a faster transition from the old to the new equilibrium path.

This task requires the full solution of the differential equation in (12), for which we are indebted to the paper by Dernburg and Quirk. The complete solution is:

(18) \[ Q(t) = \left[ \left( Q_0^{1 - c_4} - \bar{Q}_0^{1 - c_5} \right) e^{\gamma t} \left( 1 - c_5 \right) t + \bar{Q}_0 \right] \frac{1}{1 - c_3} \]

where \( \bar{Q}_0 \) is given in equation (16).

Equation (18) implies that output will "approach" its equilibrium path, in the sense that \( \frac{Q'}{Q} \to 1 \) as \( t \to \infty \), if and only if \( c_1(1 - c_3) - c_4 < 0 \), in which case the model is said to exhibit absolute stability. This stability condition can be seen more clearly if we look at equation (18) in the form

(18a) \[ Q(t) = \bar{Q}(t) \left[ 1 + \left( \frac{Q_0}{\bar{Q}_0} \right)^{1 - c_5} - 1 \right] \frac{1}{\left[ \frac{c_1(1 - c_3) - c_4}{\delta} \right] t} \]

The condition \( c_1(1 - c_3) - c_4 < 0 \) means \( \mu + \frac{\lambda}{a} + (1 - a) (n + \delta) > 0 \) which is assumed here. Thus the latter expression determines the rate of approach to equilibrium. Given \( n, \delta \) and \( a \), the larger \( \mu + \frac{\lambda}{a} \) the faster is the approach. How does the new view affect it?

It is clear that if one were to start with a pure old-view model, with its rate of approach determined by \( \mu + (1 - a) (n + \delta) \), and

6. If only the limiting growth rate span (and not also the limiting path) is independent of initial conditions then the model possesses only "relative stability."
then proceeded to add $\frac{\lambda}{a}$ to this expression, as if $\lambda$ measured a neglected source of technical progress, the result would be a faster implied rate of approach to equilibrium.

But $\lambda$ and $\mu$ are not additive. An alteration of the model does not change the world but only the conception and estimation of its parameters. Suppose a pure old-view adherent, if one could be found, and a pure new-view supporter were dispatched out into the world to estimate the relevant parameters. Using conventional estimation procedures (based on neoclassical assumptions), they would return with identical estimates except for $A$ (the rate of technical progress) and $B$ (the level of technology), the latter fortunately being irrelevant to the question at hand.

A little reflection will indicate that the new view estimate of $\lambda$ will exceed the old view estimate of $\mu$ if the mean age of capital has been steadily increasing and will fall short of the old-view estimate if the mean age has been steadily falling. A fluctuating mean age complicates the picture. In any event, the estimates could not be presumed to be equal unless the economy had happened to be in long-run equilibrium.$^7$

Therefore, if the mean age of capital had been falling sharply, the estimate of $\lambda$ might be smaller than the estimate of $\mu$ by a factor of $a$ or more.$^8$ In this event, the old-view model would paint a more dynamic and adaptable economy than would the new-view model. It would predict a higher limiting growth rate ($\mu$ being larger than $\lambda$); and, with respect to the question posed in this section, it would imply a capacity to close a given disequilibrium more quickly and therefore to make the transition faster from a low equilibrium path to a higher one.

Circumstances are conceivable, therefore, in which a permanent increase in the investment ratio would appear — at least for a while — more attractive on the old view of the economy than on the new view. But it must be noted that when the economy has been out of equilibrium — and this is an essential part of those circumstances — the two models will imply different absolute levels of the equilibrium path corresponding to the prevailing investment ratio, and they will

---

$^7$ For details of this argument, see the extended footnote of the section above concerning the long run.

$^8$ It would be better to represent the new view by a mixed model allowing $\lambda > 0$. Then the $\mu$ estimate would be compared with the estimated sum of $\frac{\lambda}{a}$ in the new model.
imply different limiting growth rates. A rational investment policy may well take these factors into account together with the transition speed. Further, a wide discrepancy between the $\lambda$ and $\mu$ estimates could only be temporary. As the economy approached long-run equilibrium, they would have to come together.

This last observation reminds us once again that the mean age of capital does not move sharply and that estimates of $\lambda$ and $\mu$ (in alternative pure models utilizing the same data) do not differ much. Actually, estimates of $\lambda$ tend slightly to exceed estimates of $\mu$ in the United States because of the secular upward trend in the mean age of capital in this country. In point of fact, then, a permanent increase in thrift does appear to be more effective in the new view than it does in the old view.

**The Rate of Return on Current Investment**

This paper has studied the effect on the path of output of a once-for-all increase in the investment ratio under alternative models. Presumably the purpose of such an increase would be to raise the time path of consumption (public and private). A higher consumption rate could be sustained in all future years.

But what if it were desired to increase only the consumption of a single future period? In this case clearly and perhaps more generally, the rate of return on investment would be a desideratum of investment policy.

The marginal productivity of investment, in the sense of $\frac{\partial Q}{\partial I}$, is determined in the old model by

\begin{equation}
\frac{\partial Q}{\partial I} = \frac{a}{K} \quad \text{(from (5))}
\end{equation}

and the new model by

\begin{equation}
\frac{\partial Q}{\partial I} = \frac{a}{J} \frac{\partial J}{\partial I} = \frac{a}{J} e^{\frac{\lambda}{\nu} t} \quad \text{(from (4)).}
\end{equation}

Since the assignment of the zero point is arbitrary, we can consider $\frac{\partial Q}{\partial I}$ only at $t = 0$ without loss of generality. Since all $\nu < t$ are then negative, old (surviving) capital goods will be assigned weights $e^{\frac{\lambda}{\nu} t}$ smaller than unity in $J$ while all (surviving) capital goods receive unit weights in $K$. Hence $J < K$ (if there is any old
capital) and the marginal product of investment is higher in the new view.

Some rough new-view estimates of the marginal product of investment in the United States may be of interest. The President’s Council of Economic Advisers has compiled several time series of the fixed reproducible tangible capital in the business sector (excluding shelter and the output of government-owned enterprises) corresponding to different quality improvement rates of the $\lambda$ type. A little manipulation of these data and the addition of corresponding inventory estimates by Goldsmith yield the following table:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Excluding Inventories</th>
<th>Including Inventories</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>543</td>
<td>643</td>
</tr>
<tr>
<td>$J'$ 2% improvement rate</td>
<td>412</td>
<td>512</td>
</tr>
<tr>
<td>$J''$ 3% improvement rate</td>
<td>369</td>
<td>469</td>
</tr>
<tr>
<td>Q 4% unemployment</td>
<td>301</td>
<td>301</td>
</tr>
</tbody>
</table>


1 The improvement rate corresponds to $\lambda$ in equation (4). Hence, if $a = A$, 3% improvement implies $\lambda = 1.2\%$. This is small but not so implausible when it is recalled that the 3% is a correction for quality improvement not already reflected in the conventional $K$ series. The competition by producers of outmoded equipment tends to depress the capital goods price index below its appropriate level for $K$ calculations to the extent that old equipment will no longer be produced when inventories of them are depleted. Also, some quality improvements are usually taken into account in the deflation of investment expenditures. In addition, econometric models of the mixed variety will normally show some technical progress of the $\mu$ type.

Making use of (19), the following estimates of the marginal productivity of 1854 investment can be computed:

<table>
<thead>
<tr>
<th>Potential Marginal Productivity of 1854 Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = .15$</td>
</tr>
<tr>
<td>$K$: 70%</td>
</tr>
<tr>
<td>$J'$: 8.8%</td>
</tr>
<tr>
<td>$J''$: 9.6%</td>
</tr>
</tbody>
</table>

The $a$ values are for illustrative purposes only. They denote the elasticity of gross final potential business output with respect to effective business capital. Relative gross factor share data indicate that business before-tax quasi-rents as a ratio to business product at high levels of activity is somewhere between .25 and .40. This fact is a rough guide as to the value of $a$.

These marginal productivity estimates are equivalent to “gross earning” rates as defined by quasi-rents, $aQ$, divided by the market (equals replacement) value of the capital stock, $J$. This concept is gross of obsolescence and physical depreciation. To figure the net (social and private) rate of return to investment, we must deduct the annual proportionate decline in the real market value of the investment due to these causes.

By the net rate of return on investment we shall mean the marginal rate of transformation between next year's and this year's consumption minus one, subject to constancy of consumption possibilities in all subsequent periods. That is, let $-\frac{\partial C_1}{\partial C_0} - 1$, subject to $C_2, C_3, \ldots$ held constant, define the net rate of return on investment, where $C_t$ denotes consumption $t$ periods in the future.

Normalizing conveniently, and denoting the annual improvement factor by $\iota = \frac{\lambda}{a}$, we can write

$$
P_0 = C_0 + I_0 = F(J_0, L_0)$$
$$P_1 = C_1 + I_1 = (1 + \mu) \cdot F(J_1, L_0) = F\left( J_0 (1 - \delta) + I_0, L_0 \right)$$
$$P_2 = C_2 + I_2 = (1 + \mu)^2 \cdot F(J_2, L_2) = F\left( J_1 (1 - \delta) + (1 + \iota) I_1, L_0 \right)
$$

where $\delta, \mu$ and $\lambda$ measure simple annual rates. For simplicity we have assumed a constant labor supply. By $F(J, L)$ we mean the Cobb-Douglas function but we use this notation for convenience.

The consumption possibilities beginning two periods hence will be unchanged by this year's and next year's investment (consumptions) if and only if capacity output two periods hence, $P_2$, is constant. But this requires that $J_2$ be constant. Hence we have the constraint:

$$J_2 = [J_0 (1 - \delta) + I_0] (1 - \delta) + (1 + \iota) I_1 = \text{constant}.$$ 

Now if we consume less this year in order to invest more, we can consume more in the future for two reasons: We will get more $P_1$. 

And we will need less $I_1$ — to the extent the $I_0$ does not wear out — to meet our fixed $J_2$ goal. Algebraically,
\[
\frac{\partial C_1}{\partial C_0} = \frac{\partial P_1}{\partial I_0} = \frac{\partial I_1}{\partial I_0} = \frac{\partial P_1}{\partial I_0} + \frac{1 - \delta}{1 + \epsilon}.
\]
Finally we obtain the net rate of return:
\[
\frac{\partial P_1}{\partial I_0} - 1 = \frac{\partial P_1}{\partial I_0} - \frac{1 + \delta}{1 + \epsilon}.
\]
\[
\frac{\partial P_1}{\partial I_0}
\]
is the marginal productivity of base year investment (here 1954). Evidently it is necessary to deduct from this the rates of "obsolescence," $\frac{\epsilon}{1 + \epsilon}$, and "effective" depreciation, $\frac{\delta}{1 + \epsilon}$, to obtain the net rate of return on investment. By applying this result to the marginal productivity estimates of Table II (and neglecting the lag between $I_0$ and the increase of capacity it creates) we obtain the illustrative rates of return in Table III.

### TABLE III

<table>
<thead>
<tr>
<th>Potential Rate of Return on 1954 Investment</th>
<th>Net of Obsolescence and 3% Physical Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a = .15$</td>
</tr>
<tr>
<td>$K(i = 0)$</td>
<td>4.0%</td>
</tr>
<tr>
<td>$J'(i = 2%)$</td>
<td>3.9%</td>
</tr>
<tr>
<td>$J''(i = 3%)$</td>
<td>3.8%</td>
</tr>
</tbody>
</table>

The table shows that, with respect to the higher and more "reasonable" values of $a$, the new view yields higher estimates of the net rate of return on investment in the United States around 1954. But it is interesting and possibly important to note that this implication of the new view could not have been taken for granted on a priori grounds. Suppose that the 1954 capital stock had been so up-to-date that the $J$'s differed little from $K$. Or suppose we believed that $a$ was only .15 or less because we thought quasi-rents as a ratio to final output, while in the neighborhood of 25–40 per cent, contained a very large element of monopoly profit. In either case — Tables II and III verify the second case — the alternative marginal productivity estimates corresponding to different improvement factors would be much smaller and would differ so little among themselves that the net rates of return would be smaller the larger is the assumed rate of obsolescence!
Summary

A growth model has been constructed which accommodates two types of technical progress. The first type can be implemented by existing capital while the second type needs to be embodied in new kinds of capital goods. Comparison of the solutions of the model corresponding to these two types reveals that:

1. the limiting long-run growth rate depends on the rate of technical progress, not the type of progress;

2. the elasticity of the limiting exponential growth path with respect to the investment ratio depends only on the capital elasticity of output, which is independent of the type of technical progress;

3. no permanent, finite modernization of the capital stock can be achieved by increased thrift; in the model constructed here, the limiting equilibrium age distribution of capital depends only on the long-run rates of growth and depreciation and neither of these is affected by the fraction of income saved;

4. the anticipated rise in the labor force growth rate in the United States will lead to a more modern stock, given a fixed investment ratio;

5. normally, but not necessarily, the new-view model — which represents the second type of technical change — will paint a more adaptable economy, one faster to make the transition to the equilibrium growth path corresponding to a higher level of thrift;

6. however, empirical estimates of the rate of technical progress (and other parameters of the model) might differ depending on which type of technical progress was assumed; in this event the two variants of the model will predict different limiting growth paths (corresponding to any investment ratio) and different limiting growth rates; this complicates at least the answer to the question of which “view” of technical progress offers the larger investment incentive;

7. finally, the new view implies a higher estimate than does the old view of the “potential” net rate of return to 1954 United States business investment; however, this result need not hold in all countries nor in this country at all times; if the capital stock is sufficiently up-to-date or capital’s income share sufficiently small, then the new-view implication that additional investment today would satisfy future capital requirements which would be “cheap” to fill with the super-investments of tomorrow will operate to reduce our estimate of the rate of return to present investment.

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