SYSTEMATIC AND RANDOM ELEMENTS IN SHORT-TERM PRICE MOVEMENTS*

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I. Are Price Changes Random?

Following Holbrook Working's precept in the preceding paper let me start by discussing the empirical observations that prompted the theoretical analysis in the second part of my article. These observations fall under two headings: the first concerning the results of certain trading policies in wheat and corn futures and the second some aspects of price changes in spot and futures prices of cotton.

The question whether prices in speculative markets move randomly has so far been answered mostly in the affirmative [3, 4]. It should be realized, however, that randomness can only be defined negatively; namely, as the absence of any systematic pattern. A particular test can detect only a particular pattern or class of patterns, and complete randomness can therefore only be disproved, not proved. The results just mentioned do show that any systematic pattern in price changes is not likely to be obvious or simple.

Indeed, we might be tempted to argue that if any pattern whatsoever persisted over a long time interval, it would be discovered by traders, who would proceed to use it to their advantage and thus destroy either the pattern or the market. This argument overlooks the fact that commodity price developments are watched by relatively few traders, most of them quite set in their ways; even in the most active futures markets the volume of serious research by participants seems to be quite small. It is therefore possible that systematic patterns will remain largely unknown for a very long time.

The particular pattern of which I will present some evidence results from the use of so-called "stop orders." An order to buy (or sell) may be of three varieties: it may be "at the market" (buy at any price), limited (buy if the price is no more than a given value), or "on stop" (buy if the price reaches a given value from below). The reasons for using stop orders and their effect on price behavior are discussed in the next section. Here I deal only with the profitability of stop orders as an indication of nonrandomness.

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Consider a trader who each year on the same date (say, February 1) buys a certain futures contract, say, September wheat, and who sells the contract on June 1. His profit or loss per bushel will be equal to the change in the September price between February 1 and June 1, less commission. Now suppose that rather than always maintain his position until June 1, he is prepared to sell at an earlier date, depending on the course of prices after February 1. More particularly, suppose that he tries to limit his losses by selling whenever the price falls a fixed percentage (say, 5 per cent) below the initial price; this means that he places a stop order to sell at 95 per cent of his purchase price.

If price changes were random, such a policy would not have any effect on his average profits over a number of years. A price fall of 5 per cent or any other figure would not affect the distribution of subsequent price changes and would therefore not provide any information according to which the trader should revise his plans. (It is true that this policy safeguards the trader against large losses, but we are only concerned with average profit at this point.) If, on the other hand, a price fall is on balance likely to lead to a further fall, timely liquidation will reduce losses without affecting profits and hence will increase average profit. In that case, everything will depend on the timeliness, that is, on the percentage, at which the stop order becomes effective. If the stop percentage is too small, insignificant fluctuations (“noise”) will lead to premature liquidation and the abandonment of profit opportunities. If it is too large, the losses suffered before liquidation may be too heavy to be balanced by profits.

In an attempt to clarify these matters the profitability of stop-order policies for varying stop percentages has been calculated for wheat and corn futures during the periods October 1, 1921, to October 1, 1939, and February 1, 1947, to October 1, 1956. Both long and short positions were considered. It was assumed that a trader bought (or sold) the May future on October 1, the September future on February 1, and the December future on June 1, each being liquidated after four months unless a stop order became effective at an earlier date. All positions were initiated at the closing price of the opening day, but intraday highs and lows were taken to release stop orders; liquidation was either at the stop price or at the closing price of the final day. Tables 1 and 2 show the financial results rounded off to cents per bushel over the period as a whole. A stop percentage of 100 means that no stop order was used, so that the position always remained open for the full four months. A stop percentage of zero means that the position was liquidated as soon as the price fell below (in case of a long position) or rose above (in case of a short position) the initial price. Commissions, which are the same for every policy, were not taken into account.
### TABLE 1
Trading Results in Wheat Futures (1921–39, 1947–56) for Various Stop Percentages, in Cents per Bushel

<table>
<thead>
<tr>
<th>Stop Percentage</th>
<th>May</th>
<th>September</th>
<th>December</th>
<th>Combined</th>
<th>May</th>
<th>September</th>
<th>December</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
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<td>+87</td>
<td>-127</td>
<td>+99</td>
<td>-59</td>
<td>-87</td>
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<tr>
<td>20...</td>
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<td>-126</td>
<td>+55</td>
<td>+33</td>
<td>-100</td>
<td>+112</td>
<td>-16</td>
<td>-4</td>
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<tr>
<td>15...</td>
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<td>-130</td>
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<td>+73</td>
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<td>-34</td>
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<tr>
<td>10...</td>
<td>+132</td>
<td>-123</td>
<td>+90</td>
<td>+99</td>
<td>-65</td>
<td>+90</td>
<td>+10</td>
<td>+32</td>
</tr>
<tr>
<td>7%...</td>
<td>+135</td>
<td>-74</td>
<td>+126</td>
<td>+117</td>
<td>-55</td>
<td>+57</td>
<td>+27</td>
<td>+29</td>
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<tr>
<td>5...</td>
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<td>-47</td>
<td>+140</td>
<td>+215</td>
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<td>+83</td>
<td>+79</td>
<td>+128</td>
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<tr>
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<td>+65</td>
<td>+183</td>
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<td>+53</td>
<td>+128</td>
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<tr>
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<td>+189</td>
<td>-19</td>
<td>+94</td>
<td>+70</td>
<td>+146</td>
</tr>
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<td>-4</td>
<td>+84</td>
<td>+122</td>
<td>-24</td>
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<td>+30</td>
<td>+77</td>
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<tr>
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<td>+45</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
</tbody>
</table>

These two tables call for several comments. In the first place, it will be noticed that without the use of stop orders (that is, for a stop percentage of 100), long positions were profitable in the May and December futures for both wheat and corn, and unprofitable in the September future; a consistently long position in all three futures was also profitable. This is remarkable because in the two periods of observation spot wheat and corn prices fell on the whole (about 28 cents a bushel for both wheat and corn). The rise in the May and December futures may be attributed partly to seasonality (which also accounts for most of the fall in the September future) and partly to "normal backwadation." Both these phenomena imply that price changes are not purely random but follow certain longer run trends (longer, that is, than the

### TABLE 2
Trading Results in Corn Futures (1922–39, 1947–56) for Various Stop Percentages in Cents per Bushel

<table>
<thead>
<tr>
<th>Stop Percentage</th>
<th>May</th>
<th>September</th>
<th>December</th>
<th>Combined</th>
<th>May</th>
<th>September</th>
<th>December</th>
<th>Combined</th>
</tr>
</thead>
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<tr>
<td>100...</td>
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<td>-79</td>
<td>+83</td>
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<td>+69</td>
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<td>+94</td>
<td>-36</td>
<td>-54</td>
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<td>+95</td>
<td>-94</td>
<td>+119</td>
<td>+120</td>
<td>-80</td>
<td>+91</td>
<td>-46</td>
<td>-37</td>
</tr>
<tr>
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<td>-73</td>
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<td>+151</td>
<td>-37</td>
<td>+107</td>
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<td>+23</td>
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<tr>
<td>7%...</td>
<td>+57</td>
<td>-97</td>
<td>+151</td>
<td>+111</td>
<td>-45</td>
<td>+103</td>
<td>-27</td>
<td>+31</td>
</tr>
<tr>
<td>5...</td>
<td>+21</td>
<td>-47</td>
<td>+127</td>
<td>+101</td>
<td>-23</td>
<td>+104</td>
<td>-19</td>
<td>+62</td>
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<tr>
<td>4...</td>
<td>+35</td>
<td>-34</td>
<td>+125</td>
<td>+126</td>
<td>-28</td>
<td>+117</td>
<td>-56</td>
<td>+33</td>
</tr>
<tr>
<td>3...</td>
<td>+36</td>
<td>-11</td>
<td>+139</td>
<td>+164</td>
<td>-12</td>
<td>+115</td>
<td>-68</td>
<td>+35</td>
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<tr>
<td>2...</td>
<td>+9</td>
<td>-17</td>
<td>+156</td>
<td>+148</td>
<td>-10</td>
<td>+105</td>
<td>-53</td>
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<tr>
<td>1...</td>
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<td>-24</td>
<td>+170</td>
<td>+175</td>
<td>-9</td>
<td>+66</td>
<td>-27</td>
<td>+32</td>
</tr>
<tr>
<td>0...</td>
<td>+29</td>
<td>2</td>
<td>+70</td>
<td>+100</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
daily fluctuations considered here). This does not greatly affect the problem of randomness in the short run.

The results for a stop percentage of zero are also interesting. In a small number of cases the initial price turned out to be lower than any subsequent price in the four-month holding period. In those cases there was consequently a profit, but there were no offsetting losses under the assumptions made here. These assumptions, however, may give a wrong impression of the profitability of a zero-stop policy: not only have commissions been ignored, but in reality it would not always be possible to liquidate a position at the stop price. Frequently a somewhat worse price (lower on stop-sell or higher on stop-buy orders) will prevail because of market imperfections or time lags.

The profitable instances just mentioned are fairly rare (three in May wheat, one in December wheat, two in May corn, one in September corn, and three in December corn out of twenty-seven or twenty-eight possibilities on the long side). Recent theoretical work [1, Chapter III] has shown that in randomly changing series such instances are not as rare as had previously been thought. Thus, if over the four-month period the price had changed a thousand times in either direction, a zero-stop policy would have paid off, whether on the short or on the long side, once in about thirty-five instances. The observed frequency is about twice as large and it is also strange that all observed instances are on the long side. Whether these peculiarities are by themselves sufficient to cast doubt on the hypothesis of randomness is not clear.

The results with stop percentages between zero and 100, however, provide somewhat more definite evidence against randomness. In every future, whether long or short, it is possible to do better by using some stop percentage than by using none (that is, by using 100 per cent). The improvement is not always very large and, on the whole, stop orders, as used in this analysis, seem to be more effective in reducing losses than in increasing profits. In no case can a stop policy with a fixed stop percentage turn an unprofitable operation (such as buying the September future) into a profitable one.

There seem to be regular patterns in the results for each position with different stop percentages, though they are not very marked. In the case of a long position in May corn, for example, fairly large stop percentages seem to give better results than small ones, but the opposite is true for December corn. The results for long positions in May and December wheat and for a short position in September corn are somewhat more cut in favor of moderate stop percentages. The irregularity of these patterns is no doubt due in large part to the relatively small number of observations.
The statistical significance of the results for stop percentages between zero and 100 is hard to assess in the absence of a more developed theory of randomly changing series. The influence of seasonality and normal backwardation noted previously is also a complicating factor. Nevertheless, I feel that Tables 1 and 2 indicate the existence of patterns of price behavior that would not be present if price changes were random.

In support of this conclusion it may further be pointed out that the assumptions made so far bring out no more than a part of the potential of stop-order policies. In the above calculations the starting date of each transaction was fixed, but if stop-order policies are at all effective, they could be used profitably to determine not only when to end but also when to start a transaction. Moreover, the policy could presumably be made more effective by letting the stop percentage refer not to the initial price but to the highest or lowest price in some recent interval. These refinements remain to be investigated.

II. Analysis of Daily Cotton Prices

Space limitations prevent a full discussion of the other set of data mentioned earlier, and only a few remarks must suffice. The data used are daily closing prices of spot cotton and the six nearest futures contracts from October, 1944, to July, 1958, excluding the period from January 27, 1951, to the expiration of the July, 1951, future which was affected by government price ceilings. The original purpose of the analysis was to estimate the parameters in a linear stochastic process by which the data might be represented, but the results obtained so far have made it advisable to reconsider this aim. Some of these results are:

The distribution of day-to-day changes in the logarithms of prices does not conform to the normal curve. It is not significantly skew, but highly leptokurtic (that is, there are more very large and more very small deviations than in a normal distribution with the same mean and variance). This phenomenon had also been noticed in similar data by Kendall [3]. It complicates the application of the available methods of time-series analysis, which are none too satisfactory even for the normal case.

The variance of price changes does not seem to be constant over time; thus it is about four times as large for the first half of the period (before the interruption in 1951) as for the second, and it also appears to be quite variable within shorter intervals. Very large deviations, in fact, seem to come in bunches. The leptokurticity mentioned above may be related to the changing variance.
The intercorrelations between spot and futures prices are higher in the first half of the period than in the second. Combined with the difference in the two variances this agrees with Working’s observation [5] that hedging is relatively more effective as a protection against large price changes than against small price changes.

The correlograms of the seven price series, which have been computed for lags up to 120, do not show any obvious pattern. More particularly they reveal little positive serial correlation, which might otherwise account for the profitability of stop percentages. It may therefore be suspected that a nonlinear stochastic process is at work, the detection of which requires different techniques of analysis. Work along these lines, especially the calculation of transition matrices, is now in progress.

III. Theoretical Discussion

The purpose of the following is to call attention to some theoretical peculiarities of short-term price movements in speculative markets and particularly to those peculiarities that may lead to the phenomena observed in the data. In the “market period” the price is determined by the physically available stocks (which are zero in the case of commodity futures markets) and the excess demand curve for these stocks. The latter curve is subject to shifts, which in the very short run contain a preponderant stochastic element. The price is given by those intersections of the vertical axis and the excess demand curve at which the latter has a negative slope; intersections at positive slope are irrelevant because any plausible tâtonnement process will overshoot them. Nevertheless, the possibility of upward sloping segments of the demand curve is of great importance for the explanation of short-term price movements.

To see this, consider Figure 1, where the excess demand curve has one “positive” and two “negative” intersections. If the previous price was somewhere near A or near C the new price (corresponding to the demand curve shown) will not be very different from the old one. But if the old price happened to be near B, a much larger price change is necessary to reach equilibrium. Upward sloping segments of the excess demand curve may therefore lead to disproportionately large price changes.

There are two reasons for the occurrence of upward sloping segments in actual speculative markets. The first is well known; it is so-called “movement trading,” especially the “explosive” variety in which a price rise leads traders to act as if they expect a further price rise, and similarly for a price fall. If price changes were random, how-
ever, there would be no profit in such behavior, nor would it become profitable because “everybody is doing it.” Explosive movement trading can reinforce any instability that is already present, but it cannot be the origin of instability. For the origin we have to look elsewhere.

![Diagram](image)

**Figure 1**

The second cause of upward sloping segments of the excess demand curve is familiar enough to the economic theorist but has rarely been applied to speculative markets. I refer to the Grifien paradox, which results if the income effect of a price change outweighs the substitution effect. This paradox has never been convincingly detected in data on consumption, but its chances of occurrence are much greater in speculation. Since in the present context we are concerned with stocks rather than flows, the income effect becomes a wealth effect.

The wealth effect is likely to be of importance to many speculators because futures are normally traded on very low margin. The equity which a speculator has to supply is rarely more than 10 per cent of the money value of the contracts he buys or sells; 5 per cent is a typical figure. This, however, is only the initial margin. If the price changes against the speculator, he has to provide a variation margin equal to the full change in the value of the contract unless he prefers to liquidate. Thus a speculative buyer of a cotton futures contract (50,000 pounds, currently worth some $16,500) is required to put up an initial margin of $750. If the price falls by a cent and a half, his initial margin will be wiped out; in fact he will be asked for variation margin well before this point. Because of this high gearing, the effect of price

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1 Another variety may be called “cyclical” movement trading; here a price rise would lead traders to expect a corrective fall later on. The explosive movement trader acts as if prices go up and up (or down and down); his cyclical counterpart as if they go up and down. The former will use stop orders so as to “cut his losses and let his profits run,” while the latter will “take profits.” Evidently explosive and cyclical movement trading will tend to offset each other.
changes on speculators' wealth, and especially on liquid assets, may be considerable; so they often protect themselves by stop orders. Whether such orders are used or not, if price changes are sharp and sudden, they will sometimes force even the strongest traders to liquidate and thus reinforce the original price change. As has been argued elsewhere [2], trading on low margin is an essential feature of viable futures markets.

The price change just considered was unfavorable. There is also a wealth effect associated with a favorable price change; namely, the increase in the equity of those traders who are already in the market. The paper profits may be used as initial margin for an extended position—a practice known as pyramiding. The formal symmetry between forced liquidation and pyramiding should not be carried too far, however; the element of constraint is absent in pyramiding, which may perhaps be more accurately regarded as a form of explosive movement trading.

The wealth effect does not concern those who are outside the market when the price changes. Their reaction to the price change, if it goes in the normal direction of a substitution effect, may offset the wealth effect. It cannot always be counted upon to do so, and if it does not, the excess demand curve will have an upward sloping segment.

It will be seen that quite a few conditions have to be fulfilled before the disproportionate price changes discussed in connection with Figure 1 can come about. The wealth effect on existing positions has to outweigh the substitution effect, or there has to be substantial explosive movement trading (or both); furthermore the upward sloping segment has to intersect the vertical axis and the previous price has to be near this intersection. This kind of instability is therefore not likely to arise frequently.

A disturbing conclusion may nevertheless be drawn from the above factual and theoretical analysis, sketchy and tentative though it is. If there is an original element of instability it will be profitable, at least to the quick witted, to reinforce it by stop-order policies of the kind discussed in Section I. Such destabilizing policies may in turn undermine the whole process of price determination. Should we rely on the well-founded expectation that few speculators take economists seri-

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9 Actually, stop orders as such are a symptom rather than a cause of destabilizing market behavior. Essential is the willingness of traders to sell when the price goes below a certain level, or to buy when it goes above it. Stop orders are merely a convenient device for implementing this type of behavior. Little or nothing would be gained, consequently, by prohibiting stop orders, as was proposed many years ago in the Federal Trade Commission's Report on the Grain Trade. The purpose now served by stop orders could be achieved with slightly more trouble by means of market orders.
ously? Or is there some unknown feature of competitive markets which would protect them even against this abuse?

REFERENCES