THEORY OF AN EFFICIENT SEVERAL-PERSON FIRM*

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The firm of the classical theory is managed by a single person, the entrepreneur; he has complete knowledge of his present and future environment and is efficient in the special sense that he chooses plans maximizing his profit. More recent studies deal with a firm that is led by one or more persons. Each of them decides on the basis of partial information; and the decision may or may not be efficient.

Efficiency. In one respect, my exposition will not be quite so general. For I shall confine myself to the efficient firm, although in a broader sense. The firm will be assumed to have a consistent order of preferences. Its goal need not be maximum profit.

Our Chairman today, Richard Cyert, is one of those who have searched for significant theorems about inefficient behavior of firms. I submit that the analysis of efficient business firms, while less general and less closely portraying reality, is also useful, for two reasons: (1) It may well be that, on the whole, the efficiency assumption is not a bad approximation, especially for organizations that have survived over a long time; they have behaved as if they had pursued the goal of maximizing the chance of survival (not the profit); (2) we are often asked, not to describe how badly business people have solved their problems in the past, but to solve a business problem, as best we can; to act like engineers, not to engage in comparative zoology.

Subjective Probabilities and Utilities. Strictly speaking, the manager of A. Marshall’s theory, if he did not know the environment with certainty, knew the probability distribution characterizing it; and he computed and maximized the actuarial value (the mathematical expectation) of profit accordingly. When Frank Knight showed that relevant future events are, in the main, not repetitive, it was concluded that the theory of probability is of no interest to the theory of the firm. This made the analysis of decisions somewhat inarticulate. Yet in recent years the practical needs of business and military decision-makers made it necessary to take a second look. Modern statisticians, asked to advise on action without being able to collect large samples, have approached the problem in the economist’s spirit, as one of efficient

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behavior. Somewhat simplifying, we can say that the "personalistic" view of L. J. Savage and others has revived the concepts and behavior postulates made in the eighteenth century by Thomas Bayes, a founder of the theory of probability; thus: (1) if, for whatever reason, you bet 9 guineas against 1 on one of two alternative events, you behave as if you were assigning at least .9 probability to one of them, and were choosing that action (viz., to bet rather than not to bet) which maximizes the actuarial value of your gain; (2) if the trials are repeated, you will achieve maximum actuarial gain by raising, after each trial, the odds in favor of that event which has just occurred.

The definition of (subjective) probabilities given in the first clause reminds the economist of the definition of the consumer's subjective utilities: if, and only if, the decision-maker is consistent do those numbers, or ranks, exist; i.e., can be ascertained from his behavior. And because of the second clause, the subjective probabilities will approach the relative frequencies, and hence approach the objective probabilities, as repetitions become more numerous. Knight's risk—the case when probabilities are known to the decision-maker—is thus a limiting case.

The main step beyond Bayes made by modern thinkers (beginning with F. P. Ramsey) consists in replacing money by utility. This brings us, in fact, back to Marshall's view of gambling, but with probabilities reinterpreted subjectively. If business is gambling (as Mr. Baruch asserted against squeamish J. P. Morgan), it is less like roulette than like betting on horses.

The efficient man, then, behaves as if there existed two sets of numbers called, respectively, utilities (attached to the states of the decision-maker) and probabilities (attached to the states of the environment), whose sum of products (the expected utility) he maximizes. This takes care of the manager who is cautious in the sense that he assigns an almost infinite negative utility to bankruptcy; or a manager who merely aspires to survive, or to achieve some other "level of aspiration" or who is interested, not only in profits, but also in power or status. No tautology is involved, since consistency may be contradicted by facts.

By incorporating subjective probabilities, economic theory of choice becomes a theory, not only of consistent tastes, but also of consistent beliefs. On this basis, a large number of business problems have been submitted to analysis: inventory control, production planning, portfolio selection, quality control. Let me point out some important concepts and relations fundamental to these studies and to the theory of the several-person firm, and not so clearly perceived in older theory.

Pay-off Function. The action of the decision-maker, given the environment, determines his future state. Therefore, depending on the particu-
lar criterion applied—profit, sales, status, survival, or power, or any combination of these—action and environment jointly determine the utility, or pay-off. The table showing the pay-off for each action in each environment is called pay-off function. If, for example, a competitive firm is judged by the simple criterion of profits, the pay-off function coincides with the classical profit function, which essentially reflects production technology and market conditions. The actions are, in this case, the inputs of raw materials, labor, etc.; and the product and input prices constitute the environment.

Decision Rule. Under uncertainty, the firm has to determine not an optimal action but an optimal decision rule. The rule tells how to adjust action to information. To be sure, in the extreme case of uncertainty, when the man cannot learn anything beyond the probability distribution he believes in, he can do no better than fix some optimal routine—a constant action that maximizes expected pay-off. For example, the expected profit of a firm may be maximized by fixing output so that, on the average, marginal revenue and cost are equal, the actual prices being unknown. In general, uncertainty is not that extreme. The firm can adjust its action to varying information even though information is not a complete or precise statement about the environment (but may merely help to estimate it). An optimal decision rule calls for an action that maximizes the expected conditional pay-off, given the information; for then the absolute (nonconditional) expected pay-off will be, in the long run, higher than if information were ignored. In the previous example, the optimal decision rule would become: given the information, choose an input level at which the conditionally expected marginal cost and revenue are equated.

A particularly important case of partial information was pointed out by Albert Hart, the economist. In general, the firm's profit depends upon a time-sequence of actions; but the firm's best plan is not a time-sequence of actions but, more flexibly, a time-sequence of decision rules, each making the action at a given future time depend on the partial information that will be available at that time. In the theory of games a sequence of decision rules is called "strategy" (as distinct from a single action or move). In statistics it is called "sequential decision function." Its name in the current literature on operations research is "dynamic program." For brevity we shall use the term "decision rule" (in singular) to denote the whole sequence of such rules.

Information Cost. Information is not costless. A firm which, not contented with a rough idea of averages, bases its decision on a very close continual study of various markets, has to pay for it. The closer the information to the true state of the environment, the better will be the best action chosen, in the sense of a higher expected utility of the
outcome. That this gain may be offset by the cost of gathering information was first emphasized in statistical decision theory: large samples are expensive. Similarly all research activities of a firm, and also its internal communications, are costly, because they claim the manager's limited time or because a delayed decision may be ill-suited to a changed environment.

*Information Rule.* We shall call an "information rule" the schedule that tells, for each state of the environment, what the firm will know about it. Suppose the relevant aspect of the environment is the set of prices of all raw materials and products of the firm and its competitors. Under one information rule, the firm might learn all these prices daily, to the nearest half-cent. Under another information rule, the firm might learn some of these prices weekly, others monthly and with less precision, and still others not at all. The former information rule may contribute more to the expected profit but will also presumably cost more.

*Decision Cost.* Cost is also attached to each decision rule. The processing of information into decision may be a difficult mental task; it is the more costly the larger its claims on the available capacity of the manager, and the larger the loss resulting from delay.

*Organizational Form.* We shall define organizational form as a pair of two rules: the decision rule and the information rule. A schedule showing the cost of each organizational form may be called the "organizational cost function."

If information and decision were costless, the efficient firm's problem would be to find a decision rule that bases action on all available information and maximizes expected utility. Since information and decision are not costless, the problem consists in finding simultaneously the decision rule and the information rule—i.e., in finding the organizational form—that will maximize expected utility net of organizational cost.

The solution of the problem—the optimal organizational form—will depend on the given circumstances. And what are these givens of the problem? They are, of course: the pay-off function; the probability distribution of the states of the environment; and the organizational cost function. For remember that each information rule translates environment into (usually partial) information, and each decision rule translates information into action. Hence each organizational form translates environment into action. But the action and the environment determine jointly the achieved utility, in a manner described by the pay-off function. Now, if information and decision were costless, the
Circles are sets (variables)

Boxes are operators (functions)

Dotted circles are sets of controlled operators which can be chosen so as to maximize the net expected payoff, i.e., the difference between elements of radiant circles.

**Figure 1**

Determinations of Average Gross Pay-off and Average Organizational Costs
utility achieved in a given environment would depend on the pay-off function and the organizational form. Therefore the expectation of utility, i.e., its average taken over all possible states of the environment would depend on (1) their probabilities, (2) the pay-off function, and (3) the organizational form. The net expected utility depends, in addition, on (4) the organizational cost function. This cost function, the pay-off function, and the probability distribution are not under the firm's control. Given these three out of the four factors determining the net expected pay-off, the firm can choose the fourth—the organizational form—so as to maximize the net expected pay-off. (See Figure 1.1)

Illustrations. To illustrate the effect of the probability distribution on the optimal organizational form, suppose an external variable is subject only to small variations; then it may not pay to get information about them and to adjust one's actions to this information. Or suppose two variables are strongly correlated; then it may suffice to get information about one of them.

To illustrate the effect of the pay-off function, suppose that two purchased inputs are mutually strong complements or close substitutes; i.e., the marginal pay-off due to one of them strongly depends on the amount of the other. Then the firm benefits from knowing the variations in the prices of both; it will increase one input not only whenever the price of that input falls but also whenever the price of its complement falls (or that of its substitute rises).

To illustrate the effect of organizational cost: If it is large, the firm may prefer to pursue routine policy (mentioned by me earlier), and not to be kept informed about the variations of any of the external variables. It is because of high decision cost that a retailer uses a mechanical markup rule. It is because of high information cost that he does not spy on his competitors more extensively, and judges the market on the basis of his sales only.

Several-Person Firm. How is the problem modified if the firm consists of several—say, n—decision-makers (executives)? Each of them decides about different things and on the basis of different information. Our concept of organizational form has to be generalized. There are now n information rules, each translating true environment into some different, partial information, available to a different executive; and n decision rules, each translating the information of a given executive into his action. The set of n information rules—who learns what?—is generated by a communication network and the rules of operating it: Who talks to whom and when? Again, decision and information cost

will mainly consist of claims on the executives' time and of losses due to delayed decisions. It is mostly a fixed cost inasmuch as these persons are usually on long-term contracts.

To choose a good network of communication and good rules of operating it may require difficult analysis in any particular case. Economic theory never does more than establish some general results. Even these are, so far, fragmentary rather than systematic.

Suppose an outsider wants to organize or reorganize a firm according to his own criteria and beliefs. How will his choice of organizational form depend on his views of the pay-off function, the probability distribution, and the schedule of organizational costs? How do these factors affect the need for more or less communication between executives?

Properties of Pay-off Function: Complementarity. The allotted roles of two executives may be such that, with communication, they can increase each other's effectiveness; without communication, they may step on each other's toes. At first sight, there seems to be more complementarity, and hence more need for communication, when the different operations must be performed in succession—e.g., along a conveyor or at successive stops of an airline (studied by M. Beckmann)—than in the case of "parallel coupling" as among branch managers of a hotel chain. However, simultaneous operations may also imply high complementarity, if the branches have to compete for a limited capacity of some central facility, as with the salesmen of a bakery (studied by C. B. McGuire). Finally, a special case (emphasized in the theory of non-constant-sum games) occurs when the pay-off function has several maxima; e.g., two or more timetables are often equally good, but some "co-ordinator" has to choose one.

Person-by-Person Maximization. In the case of some pay-off functions, the maximizing decision rules can be found by step-wise approximations, person by person. This is true of the smooth pay-off functions beloved of classical economic theory: the summit of a smooth hill can be reached step-wise by moving due north and stopping at the highest point on that route; moving due east from there, and finding the highest point on that route; moving north from there, etc. Yet the hill representing the pay-off function may have a "ridge," as when, e.g., the sales of a nonstorables product are proportional to production or to demand, whichever is smaller; alternate adjustments by the manager of production and that of promotion will lead to one of the many points where production equals demand, but this need not be the highest one.

Properties of the Probability Distribution. We have already mentioned, for the single-person case, the effect of variances and of corre-
lations. The extension is obvious. As another example, suppose that the branch manager of a bank has power to decide on his own, except in emergencies; e.g., he can grant a loan only below a certain limit. Clearly, the optimal limit will depend both on the relative cost of the central and local officers' time and on the probabilities of applications for loans of various amounts.

Incentives and Leadership. So far, we discussed the goodness of alternative organizational forms chosen on the basis of the goals and beliefs of some outsider: an "organizer," a management consultant. The goals and the beliefs of the several executives themselves will, in general, differ. E.g., the goal of the owner-manager is not that of his officers. Goal divergence is diminished by appropriate incentives—positive such as a bonus or negative such as the threat of dismissal. A decision rule takes the form: Upon receiving information, proceed so as to maximize the actuarial value of utility to yourself, using as much additional information as you possess or can gather on your own. But even a complete identification of goals would not make beliefs identical. And an action optimal when the actuarial values are computed on the basis of one set of probabilities is not optimal under another such set. A leader is that member of the organization who imposes his goals and beliefs on the choice of the organizational form; he does so by setting incentives and thus controlling the actions of other members.

No theory is possible when concepts are vague. Yet an economist who takes seriously the multipersonal nature of the modern firm has at his disposal only ill-defined traditional concepts such as leadership, power, authority, co-operation, centralization. He is used to clearer ones; like "complementarity between factors of production." The present paper attempts to point a way to the necessary clarification.