should it stop reservations at full capacity or should it, anticipating a certain number of later cancellations and "no-shows" (passengers with valid reservations who do not appear by flight time), oversell? In the latter case it risks disappointing some holders of valid reservations in the event that the number of cancellations remains below expectations. The problem is complicated by the fact that usually there is some alternative demand, by persons kept on waiting lists or by last minute "standbys" (passengers without reservations who wait at the airport).

This is a situation not peculiar to the airlines alone. As a typical situation involving risk it merits some consideration.

\textit{Notation}

\begin{itemize}
  \item $a$, the penalty per excess passenger;
  \item $b$, the loss per unutilized seat;
  \item $c$, the capacity of the flight;
  \item $x$, the number of reservations demanded;
  \item $x'$, the number of reservations accepted;
  \item $y$, the sales limit;
  \item $u$, the number of late cancellations and no-shows;
  \item $v$, the number of additional passengers available;
  \item $F(x)$, the probability of demand for at most $x$ reservations;
  \item $P(u \mid x')$, the probability of at most $u$ late cancellations and no-shows out of $x'$ passengers with reservations; and
  \item $Q(v \mid x')$, the probability of a demand by at most $v$ additional passengers given that there have been $x'$ reservations.
\end{itemize}

\textit{Statement of the problem}

1. If $x' - u > c$ then there are $x' - u - c$ excess passengers and the penalty is $a(x' - u - c)$.
2. If $x' - u + v < c$ there are $c + u - x' - v$ seats left and the revenue lost is $b(c + u - x' - v)$.
3. If $x' - u \leq c \leq x' - u + v$ no loss or penalty arises.

For a given number $x'$ of reservations accepted the expected value of loss and penalty is therefore

\begin{equation}
    a \int_0^{x' - c} (x' - u - c) \, dP(u \mid x')
    + b \int_{x' - c}^{x' - u} \int_0^{c + u - x'} (c + u - x' - v) \, dQ(v \mid x) \, dP(u \mid x').
\end{equation}

Now $x' = y$ with probability $1 - F(y)$, and $x' = x < y$ with probability $dF(x)$, $x < y$.

\footnote{When the probability distributions involved are known; "uncertainty" is involved when the distributions are unknown.}
The unconditional expected loss (and penalty) is therefore

\[
a \int_{-\infty}^{y} \int_{0}^{y-c} (x - u - c) \, dP(u \mid x) \, dF(x) \\
+ a[1 - F(y)] \int_{0}^{y-c} (y - u - c) \, dP(u \mid y) \\
+ b \int_{0}^{y} \int_{y-c}^{x+u-y} (c + u - x - v) \, dQ(v \mid x) \, dP(u \mid x) \, dF(x) \\
+ b[1 - F(y)] \int_{y-c}^{y} \int_{0}^{x+u-y} (c + u - y - v) \, dQ(v \mid y) \, dP(u \mid y).
\]

(1.1)

This expression is to be minimized with respect to \( y \) where \( y \geq c \). If we assume the distributions to be differentiable so that \( dP(u \mid y) = p(u \mid y) \, du \) and \( dQ(v \mid y) = q(v \mid y) \, dv \), the condition that the derivative of (1.1) with respect to \( y \) should be zero assumes the form:

\[
[1 - F(y)] \left[ a \int_{0}^{y-c} dP(u \mid y) + a \int_{0}^{y-c} (y - u - c) \frac{\partial p(u \mid y)}{\partial y} \, du \\
+ b p(y \mid y) \int_{0}^{y} (c - v) \, dQ(v \mid y) \\
+ b \int_{y-c}^{y} \int_{y-c}^{x+u-y} (c - v) \, dQ(v \mid y) \, \frac{\partial p(u \mid y)}{\partial y} \, du \\
+ b \int_{y-c}^{y} \int_{y-c}^{x+u-y} (c + u - y - v) \, \frac{\partial q(v \mid y)}{\partial y} \, dP(u \mid y) \\
- b \int_{y-c}^{y} \int_{0}^{x+u-y} dQ(v \mid y) \, dP(u \mid y) \right] = 0.
\]

(1.2)

Obviously the factor \( 1 - F(y) \) may be eliminated from (1.2) leaving the condition independent of \( F(y) \). We conclude that the optimal \( y \) does not depend on the demand distribution \( F \). The conditions for the optimal \( y \) are considerably simplified when \( y \) is suppressed in the conditional probabilities \( dP \) and \( dQ \).

Since the sales limit \( y \) will differ little from the capacity \( c \), it may be argued that the precise value of \( y \) does not matter in these conditional probabilities, as it will be very close to \( c \). The probability distributions with \( c \) substituted for \( y \) will now be written \( P(u) \) and \( Q(v) \). The condition of optimality for \( y \) assumes then the simple form

\[
a \int_{0}^{y-c} dP(u) + b p(y) \int_{0}^{y} (c - v) \, dQ(v) - b \int_{y-c}^{y} \int_{0}^{x+u-y} dQ(v) \, dP(u) = 0.
\]

(1.3)

Since \( p(y) \), the probability that all of \( y \) reservations will be cancelled, is quite small we have as a reasonable approximation

\[
a \int_{0}^{y-c} dP(u) = b \int_{y-c}^{y} \int_{0}^{x+u-y} dQ(v) \, dP(u),
\]
which upon integration becomes

\[ \frac{1}{P(y - c)} \int_{y-c}^{y+c} Q(u - y + c) \, dP(u) = a/b. \]

(1.4) may be solved in terms of the proposed oversales \( y - c \).

An analysis of sales records kindly made available to us by one airline\(^4\) has shown that the demand for reservations and of additional passengers as well as the cancellations are well approximated by \( \Gamma \) distributions. These will be used in the following illustrative example.

**Example**

Let

\[ dP(u) = \frac{(\beta u)^{j-1} e^{-\beta u}}{\Gamma(j)} \beta \, du; \]

\[ dQ(v) = \frac{(\gamma e)^{i-1} e^{-\gamma v}}{\Gamma(i)} \gamma \, dv; \]

space available (DC 6):

\[ c = 58; \]

average late cancellation and no-shows:

\[ j/\beta = 5; \]

exponent of this distribution:

\[ j = 4; \]

average additional demand:

\[ i/\gamma = 3; \]

exponent of this distribution:

\[ i = 4. \]

The solution is then given by the table:

<table>
<thead>
<tr>
<th>ratio of penalty to revenue per seat....</th>
<th>( a/b )</th>
<th>100</th>
<th>75</th>
<th>50</th>
<th>25</th>
<th>10</th>
<th>5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal sales limit ....................</td>
<td>( N )</td>
<td>59</td>
<td>59</td>
<td>59</td>
<td>59</td>
<td>60</td>
<td>60</td>
<td>61</td>
</tr>
</tbody>
</table>

In practice the problem of setting the sales limit is not faced in this explicit manner,\(^4\) but the margin of admissible oversales is chosen more or less arbitrarily from among the small integers "on the basis of experience." However, a good policy in this respect cannot even be formulated without openly facing and properly weighing all implications. The lost revenue of a seat raises no difficulties; it is simply the price of a ticket; what an appropriate penalty for refusing a passenger should be must remain a value judgment to be rendered by the airline in the light of its "public relations" sensitivity.

3. **SELL AND RECORD**

We must now take into account that the demand for reservations becomes known to the selling organization not through just one office or agent, but through many spatially separate ones. To what extent do the communications

\[^4\] To be published separately.

\[^4\] Particularly as the situation is further confused by the "cushion problem" of accidental oversales to be discussed in Section 4.
requirements affect the sales limit decision problem? We begin with a description of the reservations systems in effect with most airlines today.

In the following, the words “sales” and “reservations” are used interchangeably. Sales are made by so-called reservation agents, the individuals who answer inquiries by telephone. The responsibility for keeping sales within the limits of the flight’s capacity rests with offices to be called space control. A reservations system is a set of sales decisions, and of communication rules between the reservation agents and space control—and also between space control and the airport agents at the departure stations.

The system adopted at present by all major airlines (with minor modifications) is that of “sell and record.” Under this system every agent may sell space on any flight (though not more than a certain maximum number of seats at one time) unless he has previously received a message from space control stopping sales for that flight. In this case the agent may put the passenger on a waiting list. Ordinarily each sale and cancellation is reported immediately to space control. When the accumulated sales have reached a certain level, space control issues a “stop sales” message to all reservation agents. If during the time between the sending of the stop sales messages by space control and the receipt of it by all stations (the “lag” period), sales have taken place to an extent exceeding a certain critical level, “protect sales” messages are sent by space control. Reservation agents then try to clarify the status of passengers who, by failing to purchase a ticket or otherwise, have indicated that they are potential no-shows. Cancellations thus obtained are reported to space control until space control puts this flight back on an ordinary “stop sales” basis.

Passengers on waiting lists are not reported to space control, but a message is sent that a waiting list is being kept. Space freed by local cancellations is filled up from the local waiting list. If a local waiting list has been emptied, space control is advised about this only when unused cancellations reach a considerable level. If space control wants to allocate unused cancellations, it sends a “report cancellations” message. It then keeps a waiting list centrally. But this is not usual. Any unused capacity when the flight goes on stop sale is allocated to the origin station of the flight and used by this station like space freed by cancellations.

A few hours before flight departure the control over the flight is handed to “departure control” at the airport station. This is done by sending a passenger list to the airport agent. The passengers actually carried on the flight differ from those manifested by various last-minute changes:

- late cancellations
- no-shows
- mistakes (passengers from other flights that are late)
- errors (passengers showing up with valid reservations which, by mistake, were not recorded)
- standbys
- removals (passengers with valid reservations refused transportation because of oversales).
After departure of the flight a corrected list is sent to the airport agent at the first stop, who on the basis of this information can then allocate space to standbys.

If the flight is nonstop, space control rests with the reservations office at the station of departure. For multistop flights space control is either centralized in one office for the entire system or in one office for each "region."

Various refinements to this basic system are in use. Since sales do not reach a large volume (ordinarily) before approximately one week prior to departure date, reports of sales are often not required until either a certain date or until a certain allotment has been reached. Alternatively sales will be reported in blocks of, say, five only until a "post advisory message" is received from space control. Thereupon all sales will be reported individually. Sometimes stations originating very few passengers to a flight are not included in the stop sales circuit and may either sell and record all the time, or must "request and confirm" any reservation to the flight in question.

Usually space control sends a "flight check message" to the reservation offices of the various originating stations listing all the reservations received from that office for a given flight in order to spot any errors. This is necessary only when the flight was "posted," i.e., was in a "stop sales" stage.

4. THE POSTING LIMIT

Disregarding all the more subtle features, we arrive at the following model:

Notation

\[ r, \quad \text{sales during the lag period;} \]
\[ G(r), \quad \text{the probability of not more than } r \text{ sales during the lag period;} \]
\[ s, \quad \text{the posting limit;} \]
\[ g, \quad \text{the cost of stop sales messages; and} \]
\[ h, \quad \text{the excess communication cost per reservation made after posting.} \]

In accord with actual practice let \( s \leq c \). Then the following situations must be distinguished:

1. If \( x \leq s \), i.e., demand is below the posting limit, then no oversales can occur and the only possible loss is from unsold seats, in which case \( x - u + v < c \) and the loss is \( b \cdot (c + u - x - v) \).

2a. If \( x > s \), seats will remain unsold provided \( s + r - u + v < c \). The loss is then \( b \cdot (c + u - s - r - v) \).

2b. The plane is oversold when \( s + r - u > c \) calling for a penalty of \( a \cdot (s + r - u - c) \).

A stop sale message is sent whenever \( x > s \) (situations 2a and 2b) at a cost of \( g \).

Request for seats after the flight has been posted occurs whenever \( x > s + r \), giving rise to an excess communications cost of \( h \cdot (x - s - r) \).
The total expected cost is now
\[ b \int_0^s \int_0^x \int_0^{e+u-x} (c + u - x - v) \, dQ(v) \, dP(u) \, dF(x) \]
\[ + b[1 - F(s)] \int_0^\infty \int_0^{s+r} \int_0^{e+u-s-r} (c + u - s - r - v) \, dQ(v) \, dP(u) \, dG(r) \]
\[ + a[1 - F(s)] \int_0^\infty \int_0^{s+r-c} (s + r - u - c) \, dP(u) \, dG(r) \]
\[ + g[1 - F(s)] \int_0^\infty \int_0^{s-r} (x - s - r) \, dG(r) \, dF(x). \]

The optimal \( s \) is determined by the condition that the derivative of (3.1) with respect to \( s \) should be zero, i.e.,
\[ b \cdot f(s) \int_0^s \int_0^{e+u-s} (c + u - s - v) \, dQ(v) \, dP(u) \]
\[ - bf(s) \int_0^\infty \int_{s+r-c}^{e+u-s-r} (c + u - s - r - v) \, dQ(v) \, dP(u) \, dG(r) \]
\[ + b[1 - F(s)] \int_0^\infty \int_0^{s+r} \int_0^{e+u-s-r} dQ(v) \, dP(u) \, dG(r) \]
\[ - b[1 - F(s)] \int_0^\infty \int_{s+r-c}^{e+u-s-r} dQ(v) \, dP(u) \, dG(r) \]
\[ - a[1 - F(s)] \int_0^\infty \int_0^{s+r-c} (s + r - u - c) \, dP(u) \, dG(r) \]
\[ + a[1 - F(s)] \int_0^\infty \int_0^{s+r-c} dP(u) \, dG(r) \]
\[ - a[1 - F(s)] \int_0^\infty \int_0^{s-r} dG(r) \, dF(x) = 0. \]

Here the second term represents a weighted average of the first, and the two very nearly cancel, for the distribution of \( r \) is concentrated near zero. The third, fourth, and sixth terms are familiar from equation (1.3). Finally the fifth, seventh, and eighth terms represent communications cost caused by posting; and, specifically, the fifth term expresses the risk of accidental oversales in the lag period. Since \( f(s) \) is small, however, relative to \([1 - F(s)]\), and since the interval from \( 0 \) to \( s + r - c \) tends to be small, this term may be neglected in a first approximation. Moreover since \( g \) and \( h \) are of small order relative to \( a \) and \( b \) under typical conditions, we are left eventually with
\[ \int_0^\infty \left[ b \int_{s+r-c}^{e+u-s-r} Q(c + u - s - r) \, dP(u) - aP(s + r - c) \right] \, dG(r) = 0. \]
To a first approximation the expression in brackets should vanish when \( r \) takes on its expected value \( \hat{r} \). Writing \( s + \hat{r} = y \) we obtain

\[
b \int_{y-c}^{y} Q(c + u - y) \, dP(u) - aP(y - c) = 0,
\]

which is our previous equation (1.4) for the sales limit.

We have shown that in a first approximation the stop sales posting limit should be chosen equal to the optimal sales limit diminished by the expected sales during the time required for the stop sales message. This expected number is typically not more than one or two.

While the communications lag thus has a certain effect on the sales decision problem, the numerical difference is negligible under certain assumptions and in view of the fact that the solution must be an integer. The communications system itself, however, presents interesting aspects and raises significant problems for consideration in the general theory of organization. Some of these points will be discussed in the following section.

5. COMMUNICATIONS RULES EXAMINED

In order to see the rationale of the sell and record system, we shall attempt a formal description in terms of Marschak's model of a team [2].

Space control and the reservation and airport agents form a team, i.e., they have a common goal. Presumably this goal is the maximization of the airline’s profits. From the emphasis that is often put on the "load factor," one may get the impression that this goal is to maximize the load factor subject to the constraint that oversales stay within prescribed limits, but regardless of communications cost. A closer inspection of the communications system, which is clearly designed with keeping an eye on communications cost, will dispel this impression.

The "sell and record" system is eminently suited for ruling out the particularistic interests of reservations offices. Under the former system of space allotments to various stations, the latter often felt a proprietary interest in their allotments and were reluctant to relinquish part of their quota to other stations since this would jeopardize their own sales records.

Profit, the goal of the team, must be understood as long-run profit. In order to translate long-run profit into the short run, a certain penalty must be attached to actions that tend to decrease long-run revenue, such as "protect sales" and removals of passengers. The profit to be considered by the "traffic and sales department" (including reservations, space and departure control) is actually revenue net of sales (reservation) cost, as operating cost may be regarded fixed by this department. Thus the proper measure of short-run profit is passenger revenue—(the price of the ticket) minus communications cost—minus penalties.

Within the communications system, the structural and functional aspects must be distinguished or, in Marschak's terminology, the network and the communication rules.

\footnote{The ratio of the average sales to capacity.}
The network is defined by the channels of communication that are open between the various team members. In "sell and record" the network is specified by the location of space control for a given flight and by the set of agents "in the circuit," i.e., that receive stop sales messages. These may include agents of other airlines. We have the somewhat unusual case that a team set up to maximize the profit of one firm has in it employees of other firms.

Communications cost includes the cost of a message. Typically the telephone or teletype service system or wire is leased at a flat rate and so the proper cost is the opportunity cost of using a scarce facility, a "shadow rent." While this varies with the business of each hour, an approximation may be had by pro-rating the entire rent of the facilities over the calls that must be handled during the peak hours; and by not imputing any cost at all to their use in the off-peak hours. For local messages these charges should not exceed the 10-cent rate applicable to private telephone use. Since the staff for handling calls is varied in proportion to volume, the wage cost of a message is more nearly determined by pro-rating payroll cost, giving a wage cost for this type of message of about 10 cents. The average cost per long distance communication from a reservation agent to space control (or vice versa) has been given as 4 cents per message (1954) by one airline official.

Finally, we have the cost of storing the communications received. The cost is probably of the order of 25 cents per message.

The sale of a seat by an agent located at space control costs therefore about 45 cents. If the selling agent must report to space control at a different office, this adds at least 15 cents. A stop-sales message by teletype and its posting may cost an amount of the same order. Even with the cost of checking and manifesting, the total reservations cost per reservation should thus not exceed $1. From published airline statistics I conclude that the total cost of the "traffic and sales" prorated over reservations is about $2. The extra dollar would seem to represent the considerable fixed cost of ticket and reservations offices.

In choosing between alternative means of communication—typically, teletype or telephone—for important messages such as stop sales, the airline is less concerned about money cost than about delay. For internal purposes there is thus an additive cost element to be considered: the value of the lost time. This value is essentially the probability of oversales multiplied by the appropriate penalty. The presence of this factor is implied by the priority rules that exist within the broader system of all airline communications. Stop sales messages

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8The quarterly reports of the airlines to the CAB contain (Schedule B-3) the expense totals of ticketing and reservations employees (account no. 32), and of telephone and telegraph services (account no. 37). Schedule B-4 lists the "traffic and sales expenses" by stations, and Schedule B-5 gives the corresponding ticket sales. I have not studied these data. From a comparison of expenses and the number of passengers as given in the printed "Certificated Air Carrier Financial Data," Civil Aeronautics Board Office of Carrier Accounts and Statistics, Systems and Reports Division, and "Certificated Air Carrier Mileage and Traffic Data," ibid., respectively, one may see that "traffic and sales expense" averages about $3 per passenger. Allowing for cancellations this would correspond to roughly $2 per reservation.
rank in priority after emergency, weather, and operations messages, but before all other reservations messages. The delay is usually a matter of minutes. (Five minutes is a figure given by Grossman.) Delays of the sales report messages may be more serious. Since only a small fraction of all sales reports are the critical ones, these messages should rightly rank behind the stop sales notices.

Costs due to delay play a part again in "sell and record" versus "request and confirm." For, since money costs are of such small order compared to the high price of unused seats on a flight, it might be thought that perfect communication should be aimed at even though it costs a little. This has been the rationale of the "request and confirm" system. Its principal defect is that it disregards the delay necessary in confirming a reservation to a passenger. This delay gives rise to an additional cost (or penalty) which is considerable: the probability of the loss of a passenger because of his unwillingness to wait. Under competitive conditions this cost is regarded to be prohibitive. The present sell and record system avoids this cost element.

We have entered now into the discussion of the communication rules proper. The rationale of some is evident: thus a "wait list" message is more efficient than a "request and confirm" message (after a flight is posted) because "unless it has available the requested space, and, unless the control station sends a message of confirmation, the selling station need not telephone the passenger" [1, p. 134].

The main attraction of a new electronic device, the "reservisor," is that it avoids delay between sales, sales report, and stop sales. Here all reservation agents selling certain flights can ascertain the availability of seats on a flight immediately by operating a keyboard that is connected by direct wire with the space control memory drum. System-wide reservisors are too expensive as yet, but may come into use at high density traffic stations.

While consideration of the delay cost, in terms of which "sell and record" is uniformly superior, has been prominent in the choice among alternative systems of communication rules, there are still areas in the world where the money cost of communication alone is large enough to be a factor of importance. Under these conditions a quota system, which requires fewer messages, can be more efficient and is being used. In "quota" versus "sell and record" we run into a special form of the "centralization–decentralization" antithesis. We have here indeed a general problem in organization theory: to specify the conditions on the demand and the cost side under which the administration of a resource is more efficiently carried out in terms of decentralized quotas (with some provision for the clearing of any residuals) rather than by centralization of the sales decision. It may be recalled here that quota systems are widely used for instance in the selling of show tickets.

In this form the problem is too broad for analysis here. Instead we propose to compare two simple alternatives, which are idealized versions of the quota and the sell and record system. Let the flight be nonstop and let there be \( n \) agents. The demands faced by each agent are assumed to have independent and identical distributions, later specified as \( \Gamma \)-distributions. Under system (1) \( 1/n \) of the flight's capacity is allocated to each agent; excess demand is
wait-listed. At a certain time prior to departure each agent reports to space control any available space or wait-listed demands. Space control then sends a message to each agent distributing the reported space in view of the reported demand. There is a fixed probability $\pi$ that a passenger will give up his demand, if asked to be wait-listed. The cost per message is $e$.

System (2) is sell and record, but without communications lag. Immediately after each sale, the agent reports to space control, and the stop sales message is sent without fail at the exact point of capacity. (No cancellations are considered). Under system (2) the net revenue (i.e., revenue minus reservations cost) is as follows:

$$
(4.1) \quad b \int_0^c x \, dF(x) + bc[1 - F(c)] - e \int_0^c x \, dF(x) - 2ec[1 - F(c)].
$$

Let the probability of a demand not exceeding $z$ at a given agent be $\Phi(z)$.

To calculate the net revenue under system (1) observe that the average demand not satisfied out of quotas, when demand deficits due to wait-listing are disregarded, equals the total average demand

$$
\int_0^c x \, dF(x) + c[1 - F(c)]
$$

minus the average demand filled within quotas

$$
n \int_0^{c/n} z \, d\Phi(z) + n \cdot c/n[1 - \Phi(c/n)].
$$

Considering that the wait-listed demand is only $\pi$ times this amount we obtain the expected value of revenue under system (2)

$$
(4.2) \quad b \left\{ n \int_0^{c/n} z \, d\Phi(z) + c[1 - \Phi(c/n)] \right\}
$$

$$
+ \pi b \left\{ \int_0^c x \, dF(x) + c[1 - F(c)] - n \int_0^{c/n} z \, d\Phi(z) - c[1 - \Phi(c/n)] \right\}
$$

$$
- 2ne[1 - \Phi(c/n)] = \pi b \int_0^c x \, dF(x) + (1 - \pi)bn \int_0^{c/n} z \, d\Phi(z)
$$

$$
+ bc - (1 - \pi)bc\Phi(c/n) - \pi bcF(c) - 2ne.
$$

The sell and record system (2) is therefore more profitable when

$$
(1 - \pi)b \left\{ \int_0^c x \, dF(x) - n \int_0^{c/n} z \, d\Phi(z) + c[\Phi(c/n) - F(c)] \right\}
$$

$$
> e \left\{ \int_0^c x \, dF(x) + 2c[1 - F(c)] - 2n[1 - \Phi(c/n)] \right\}.
$$

(4.3)

To be specific let

$$
d\Phi(z) = \varphi(z) \, dz = \frac{(\lambda z)^{k-1} e^{-\lambda z}}{\Gamma(k)} \, \lambda \, dz.
$$
Then

\[ dF(x) = \frac{(\lambda x)^{\alpha - 1} e^{-\lambda x}}{\Gamma(nk)} \lambda \, dx. \]

(4.4) \[ \bar{x} \frac{\Gamma_e(nk + 1)}{\Gamma(nk + 1)} + 2c \left[ 1 - \frac{\Gamma_e(nk)}{\Gamma(nk)} \right] - 2n \left[ 1 - \left( \frac{\lambda c}{n} \right)^{k-1} e^{-\lambda c / n} \right] > \frac{e}{(1 - \pi)b}. \]

The critical number \( e / (1 - \pi)b \) represents the ratio of the relevant sales costs.

**Example**

Let

- capacity (DC 6): \( c = 58 \);
- average number of passengers: \( \bar{x} = 50 \);
- exponent of distribution: \( k = 4 \);
- average demand per agent: \( k/\lambda = 50/n \).

The solution is then given by the schedule:

<table>
<thead>
<tr>
<th>If the number of agents is</th>
<th>3</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>then sell and record is superior for ( \frac{e}{(1 - \pi)b} \leq )</td>
<td>.35</td>
<td>.018</td>
<td>.0014</td>
</tr>
</tbody>
</table>

It is interesting to note that the case for centralization, i.e., for sell and record, is stronger the larger the organization, i.e., the greater the number of agents.

**6. Conclusion**

To conclude, the reservations systems of major airlines contain much that is relevant and instructive for the study of organization theory because with the enormous mass of information that must be handled in reservations every day, efficiency is at a great premium. The rules that are in current use are the outcome of many years experience and of much experimentation. Minor revisions occur all the time. With the increase in flight density and the introduction of electronic recording devices more drastic changes may well be ahead. These developments should be followed with great interest by students of organization theory.

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**References**

