ECONOMIC IMPLICATIONS OF SOME COTTON FERTILIZER EXPERIMENTS

BY CLIFFORD HILDERETH

1. SOME GENERAL OBSERVATIONS

Too often experimental data relating to productivity of various agricultural practices are collected and subjected to extensive statistical analyses while the economic implications of the data are largely neglected. Recognizing this, a number of agricultural economists\(^2\) have recently undertaken economic analyses of technical data and in some cases have participated in the designing of technical experiments.

The typical procedure has been to use productivity data to estimate parameters in continuous production functions of specified algebraic form and to use the estimated functions to determine optimal combinations of inputs and outputs for specified combinations of prices. This is a natural and useful procedure in certain circumstances, but in other cases it may be better to regard the relevant production decisions as choices between discrete alternatives. The latter point of view is natural if the alternatives considered involve qualitative distinctions such as different breeds or varieties, and it may also have advantages in considering alternative levels of continuous inputs such as feed and fertilizer if there is considerable uncertainty about the appropriateness of various algebraic forms for representing the production function. Uncertainties about the appropriateness of possible algebraic forms for representing the fertilizer-yield relation of Section 2 led to the decision to analyze this problem as a choice among the several fertilizer combinations actually observed.

While economists typically have rather firm judgments about some properties (e.g., diminishing returns to certain inputs, technical complementarity or substitution relations between certain pairs of inputs, etc.) of production functions, they are seldom sure that a particular form is appropriate. That this problem has significance is indicated by the fact that, where alternative assumptions about form have been explored, they have often had markedly different implications.

The author has elsewhere\(^3\) considered the problem of estimating points on a

---

\(^1\) The author is indebted to W. L. Nelson, Agronomy Department, North Carolina State College, for the use of data from cotton fertility experiments, and to E. W. Constable, North Carolina State Chemist, and J. G. Sutherland and M. S. Williams, Department of Agricultural Economics, North Carolina State College, for suggestions as to typical fertilization practices. This paper was presented at a joint meeting of the Econometric Society and the American Farm Economic Association in December, 1953 and at a Cowles Commission staff meeting in January, 1954.


curve or surface of unknown form subject to qualitative restrictions like those mentioned above. Because the mean yields of Section 2 satisfy all the a priori restrictions (in this case, diminishing returns) that the author would feel safe in imposing, these new procedures are not needed for the present investigation. The principal point of interest in this paper is the development and presentation of economic implications of data on the results of applying a finite set of alternative production practices. Linear programming provides a suitable framework for many comparisons of discrete production alternatives and there is good reason to believe that suitable generalizations of linear programming can be developed for some of the cases in which the conditions for linear programming are not all fulfilled. When discrete alternatives are analyzed, necessary interpolations or extrapolations can be made on the basis of less formal judgments as the results of the analysis are applied. A liberal dose of judgment is, in any case, necessary at the stage of applying the results. Conditions faced by actual producers can never be exactly duplicated in experiments so the question of how and to what extent particular experimental results can be transferred to commercial situations always arises.

I do not intend to suggest that production functions should never be fitted to experimental data. It is the responsibility of the investigator to specify in each instance the assumptions that will underly his analysis. If he can be reasonably certain that an equation of a particular parametric form will furnish a good approximation to the underlying production function, then there are several possible advantages in using this form. There may be a gain in efficiency of the estimates of output for various combinations of input, and the procedure furnishes estimates for combinations that have not actually been observed. In principle the latter fact offers the possibility for a more precise determination of optimal combinations of inputs, but, if divergences between experimental and commercial situations are taken into account along with sampling errors in the parameters of the fitted relation, this may often be a minor advantage.

Against these possible advantages one must weigh the biases that will accrue if an inappropriate algebraic form is used. It would seem that production economists should be aware of possibilities under either approach and should choose according to their best judgments of the relative merits in particular analyses.

In the second section of this paper, the profitabilities of certain rates of fertilization for cotton in the Coastal Plain region of North Carolina are compared treating each observed rate as a discrete alternative and confining the comparisons to these alternatives. These may be regarded as a very simple application of linear programming. Each rate of fertilization is regarded as an activity and data are obtained on the inputs and outputs associated with each activity. The simplicity of this example arises from the fact that only one commodity is assumed to be limited. This reduces the problem of finding the optimal combination of activities to that of finding the single activity that yields highest net revenue per unit of the limited commodity. In this example the quantity of land is taken as given and we seek the highest net revenue per acre. It is then a convenience to let the unit level of each activity be that which uses an acre of land and to state other inputs and outputs in quantities per acre.
The comparisons of the next section are sufficiently simple that they could easily be explained without reference to linear programming. I take the trouble to relate the example to linear programming because the example will illustrate the use of a concept which, I believe, will prove useful in various linear programming analyses and which may be under-stressed in the present literature.

I should like to call this concept the price map. It is a partitioning of all possible combinations of prices for inputs and outputs into collections such that to each collection of price combinations there corresponds a combination of activities and therefore a combination of inputs and outputs that maximizes net revenue. Geometrically we may visualize a price space of say $K$ dimensions where $K$ is the number of commodities that enter as inputs or outputs in at least one of the activities in a linear programming model. The price of each commodity is measured along one coordinate of the space and a point in the space represents a price combination for the $K$ commodities.

The price map is then a partitioning of this space into sectors. For all price combinations in the interior of a particular sector, a corresponding combination of commodities (inputs and outputs) maximizes net revenue. For price combinations represented by points on the boundaries of sectors, two or more commodity combinations (and all convex combinations of them) yield the maximum net revenue.

The price map is the logical analogue of the set of individual firm demand and supply relations that one could obtain from a continuous production function just as the efficient point set derived from a set of discrete activities is the logical analogue of the production function itself. The price map tells, for each combination of prices, the activity levels to be established and therefore the quantities in which outputs will be produced and inputs used if net revenue is to be maximized in the case where technical possibilities are represented by a set of discrete activities. Where the technical possibilities are summarized in a continuous production function, the supply and demand relations for outputs and inputs perform this same function of connecting each combination of prices with the maximizing combination of quantities of commodities.

As was mentioned above, in the special case in which only one commodity is restricted, the optimal activity (or activities) may be found by comparing net revenues of the various activities at levels which involve equal quantities of the limited commodity. Consider such a case and let $a_{kn}$ be the quantity of commodity $k$ produced by a unit level of activity $n$, $k = 1, 2, \ldots, K$ and $n = \ldots$

---


4 In nonlinear extensions of linear programming the space will be partitioned into regions with hypersurfaces for boundaries.

4 The latter analogy may not be quite exact depending on the precise definition of a production function which is adopted. See Introduction to Koopmans, Activity Analysis of Production and Allocation, op. cit.
1, 2, · · ·, N. If commodity \( k \) is an input of activity \( n \), then \( a_{kn} \) is negative. Net revenue of a unit level of the \( n \)th activity is given by

\[
\pi_n = \sum_{k=1}^{K} p_k a_{kn}
\]

for a given set of prices \( p_1, p_2, \cdots, p_K \). If we are interested in only one set of prices, we may compute net revenue for each activity under this set of prices and pick the highest. If we are interested in comparisons under various sets of prices, then a price map is a useful construction.

The difference between net revenue from a unit of activity \( n \) and from a unit of activity \( m \) is

\[
\pi_n - \pi_m = \sum_{k=1}^{K} p_k (a_{kn} - a_{km}).
\]

If the \( a_{kn} \) and \( a_{km} \) are regarded as known quantities with \( a_{kn} \neq a_{km} \) for some \( k \), then the difference of net revenues will be positive for some sets of prices and negative for others. In the \( K \) dimensional price space there will be a hyperplane given by

\[
\sum_{k=1}^{K} p_k (a_{kn} - a_{km}) = 0
\]

which will divide the space into two halves, for sets of prices in one half activity \( n \) yields higher net revenue than activity \( m \) (\( \pi_n - \pi_m > 0 \)) and the reverse holds in the other half.

When the bounding hyperplanes for all pairs of activities are considered, they divide the space into sectors within each of which a particular activity yields the highest net revenue. This partition of the price space into sectors (and boundaries) is the price map. Ordinarily we are only interested in the positive orthant of the price space (i.e., we assume prices are nonnegative) and we may be willing to restrict our attention to part of this orthant. Hence, if there are a pair of activities \( n \) and \( m \) such that \( a_{kn} \geq a_{km} \) for all \( k \), then activity \( n \) will yield at least as much net revenue as activity \( m \) for all sets of nonnegative prices and will yield higher net revenue whenever there is a commodity \( k \) for which \( p_k > 0 \) and \( a_{kn} \neq a_{km} \). In this case we may say that activity \( n \) dominates activity \( m \) and exclude the latter from further consideration as soon as domination is noted.

A price map in more than three dimensions will generally be rather difficult to describe or use directly. It may, however, be possible to note some of its properties and to draw useful theoretical conclusions from these. It may also happen that in many practical applications there will be restrictions on the prices or on the commodity coefficients (the \( a_{kn} \)) or on both and that these can be used to transform the map to a space of lower dimension. The example of the next section is intended to illustrate the latter possibility.

Typically the restrictions would be expected to hold only approximately and
some precision may be lost in the transformation. There will be a matter of judgment in each case as to which restrictions are reasonable to impose. Examples of simple restrictions that may sometimes be used are:

(1) **Approximately constant prices.** If movements in the price of a particular commodity are expected to be small enough to have little effect on the net revenue comparisons, the expected level of this price might be substituted for the price variable, thus reducing by one the number of variables considered.

(2) **One price a function of other prices.** The function may be substituted for the price in question, thus reducing the number of prices by one.

(3) **Approximate equality of inputs or outputs.** If all of the activities considered use or produce the same or nearly the same amounts of a commodity, then its price will have very small coefficients in the difference of net revenue equations and these terms can be neglected with little loss of accuracy.

(4) **Approximately proportional variation in inputs or outputs.** If activities that use or produce more of one commodity tend to use or produce proportionately more of certain others, then prices of these related commodities may be combined. Suppose \( a_{jn} \) approximately equals \( \lambda a_{kn} \) for some \( j \) and \( k \) and for all \( n \) where \( \lambda \) is any constant. \( p_j^* = \lambda p_k^* + p_k \) could be defined and \( p_j^*(a_{jn} - a_{km}) \) can be substituted for \( p_j(a_{jn} - a_{jm}) + p_k(a_{kn} - a_{km}) \) in all of the boundary equations given by (1.3).

Other somewhat more general restrictions may also be used to transform the price map, but the above will be sufficient for the example to be presented and may suffice for many other practical applications. Simplified price maps can be seen to be useful expository devices when the technical data are fairly well determined and the principal uncertainties are attached to two or three prices.

To obtain a price map for a more complex case involving several restrictions on activity levels, one would proceed as above except that the \( a_{kn} \) would have to be interpreted as the inputs and outputs associated with extreme points of the admissible set of combinations of activity levels. A typical extreme point would involve positive levels of several activities instead of just one as in the special case described above. Charnes\(^7\) has outlined a procedure which can be used to find all of the extreme points. Since an investigator is, however, likely to be interested in mapping only part of the price space, he will ordinarily not be interested in all of the extreme points. Thus there would be some interest in the problem of trying to develop a practical procedure for identifying and finding the extreme points relevant to a particular part of the price space that would be shorter than investigating all extreme points.

### 2. A PRICE MAP FOR COTTON

In this example the activities consist of alternative fertilization treatments in cotton production. An activity is designated by a triplet of numbers which

\(^7\) A. Charnes, W. W. Cooper, and A. Henderson, *An Introduction to Linear Programming*, Wiley and Sons, 1963, pp. 68-70. The procedure is described as a way of obtaining all optimal extreme points corresponding to a given set of prices but it could readily be used to obtain all extreme points.
are, respectively, the pounds of nitrogen, phosphoric acid, and potash applied per acre of land. Estimated yields are obtained from experiments at the North Carolina Upper Coastal Plain Experiment Station in 1944 and 1945 and are listed in column (3) of Table I below. The observed yields in column (2) are averages of yields in the four replicates on which a given treatment was applied each year. The estimated average yield is obtained from a simple model of the form

$$y_{nt} = \alpha_n + \beta_t + u_{nt}$$

where $\alpha_n$ is the expected yield of a plot with treatment $n$ is an “average” year, $\beta_t$ is the year effect for year $t$, and $u_{nt}$ is a normally distributed random disturbance. If we normalize the $\beta$'s by setting $\beta_{1944} + \beta_{1945} = 0$, then an estimate of $\alpha_n$ for a treatment that was observed in both years is the simple average of the observed yields in the two years. For a treatment that was observed in only one year, the estimate is obtained by subtracting the estimated year effect from the observed yield. The estimated year effect for a given year is equal to one half the average of the differences obtained by subtracting the other year's yield from the given year's yield for all treatments that were observed in both years. In this case the average difference, $\frac{1}{2} \sum_{n=1}^{11} (y_{n,1944} - y_{n,1945})$, is 96 and we have $\beta_{1944} = 48, \beta_{1945} = -48$. Columns (4) and (5) of this table are discussed below.

The treatment designations specify inputs of nitrogen, phosphoric acid, and

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated Yields and Handling Charges for Fertilizer Treatments</td>
</tr>
<tr>
<td>![Table Image]</td>
</tr>
</tbody>
</table>

potash for each activity. The estimated average yields are used as measures of output. To estimate net revenue of a unit level of a given activity it would be necessary to estimate the quantities of all other inputs. To make comparisons of net revenues for several activities, however, one only need know the inputs which vary between activities. Other than the designated plant foods and the product, the only factors that vary significantly between these activities are the resources used in handling different amounts of fertilizer and cotton. In effect, the problem is to evaluate both fertilizer and cotton in the field. Some of the costs incurred between the dealer and the field depend on circumstances of an individual farm—distance from dealers, kinds of roads, etc. Fortunately these differences relate to minor items in the net revenue comparisons so that a price map developed with regard to fairly typical circumstances should not be far off on their account.

Fertilizer costs of the various activities may be divided into the cost of the basic plant foods and the cost of handling the necessary materials to obtain these feeds. Since fertilizer can be applied at various rates with conventional equipment, differences in costs of application are negligible and can be disre-

<table>
<thead>
<tr>
<th>Year</th>
<th>Prices of Plant Foods</th>
<th>Cotton Prices</th>
<th>Picking Cost</th>
<th>Ginning Cost</th>
<th>Daily Wage of Farm Labor ($)</th>
<th>Handling Cost</th>
<th>Net Price Seed Cotton</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nitrogen</td>
<td>Phosphoric acid</td>
<td>Potash</td>
<td>Lint</td>
<td>Seed</td>
<td>Seed Cotton</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(lbs)</td>
<td>(lbs)</td>
<td>(lbs)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1940</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>9.93</td>
<td>1.13</td>
<td>4.65</td>
<td>.67</td>
</tr>
<tr>
<td>1941</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>17.90</td>
<td>2.45</td>
<td>8.63</td>
<td>.95</td>
</tr>
<tr>
<td>1942</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>19.39</td>
<td>2.27</td>
<td>9.12</td>
<td>1.40</td>
</tr>
<tr>
<td>1943</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>20.19</td>
<td>2.50</td>
<td>9.88</td>
<td>1.75</td>
</tr>
<tr>
<td>1944</td>
<td>12</td>
<td>6</td>
<td>5</td>
<td>20.78</td>
<td>2.52</td>
<td>9.83</td>
<td>2.10</td>
</tr>
<tr>
<td>1945</td>
<td>12</td>
<td>6</td>
<td>5</td>
<td>23.22</td>
<td>2.38</td>
<td>10.72</td>
<td>2.30</td>
</tr>
<tr>
<td>1946</td>
<td>12</td>
<td>6</td>
<td>5</td>
<td>32.65</td>
<td>2.58</td>
<td>15.21</td>
<td>2.85</td>
</tr>
<tr>
<td>1947</td>
<td>15</td>
<td>7</td>
<td>6</td>
<td>32.59</td>
<td>4.02</td>
<td>15.45</td>
<td>2.95</td>
</tr>
<tr>
<td>1948</td>
<td>15</td>
<td>7</td>
<td>7</td>
<td>30.68</td>
<td>3.02</td>
<td>14.08</td>
<td>3.10</td>
</tr>
<tr>
<td>1949</td>
<td>15</td>
<td>7</td>
<td>7</td>
<td>29.16</td>
<td>2.11</td>
<td>12.93</td>
<td>2.70</td>
</tr>
<tr>
<td>1950</td>
<td>15</td>
<td>8</td>
<td>7</td>
<td>41.04</td>
<td>4.15</td>
<td>18.91</td>
<td>2.85</td>
</tr>
<tr>
<td>1951</td>
<td>16</td>
<td>8</td>
<td>7</td>
<td>38.75</td>
<td>3.46</td>
<td>17.58</td>
<td>3.35</td>
</tr>
<tr>
<td>1952</td>
<td>16</td>
<td>8</td>
<td>7</td>
<td>35.68</td>
<td>3.55</td>
<td>16.40</td>
<td>3.50</td>
</tr>
<tr>
<td>1953</td>
<td>16</td>
<td>8</td>
<td>7</td>
<td>33.00</td>
<td>3.57</td>
<td>14.74</td>
<td>3.35</td>
</tr>
</tbody>
</table>

Sources:
1. Fertilizer Analyses, North Carolina Department of Agriculture.
2. Statistics on Cotton and Related Data. USDA Statistical Bulletin No. 99. Units have been converted in some cases.
3. 40% of lint price + 60% of seed price.
5. Figured at 54 day per bale of 478 lbs. lint.
6. Price of Seed Cotton less Picking, Ginning, and Handling Costs.
garded. Prices of the basic plant foods vary somewhat with location and with the source material used. The North Carolina Department of Agriculture determines each year a typical price for nitrogen, for phosphoric acid, and for potash based on the costs of economical and commonly used materials. These prices are listed in Table II. To the costs of these elements must be added the dealer’s and farmer’s handling costs. The latter depend mainly on the total quantity of materials handled. In the present example an allowance of $7.00 per ton has been made for handling costs. Since this is a minor item and since many of its components are fairly constant over time, variations in this rate have not been considered.

The most common source of nitrogen for cotton fertilizer in North Carolina is ammonium nitrate which is 32.5% nitrogen. Other important sources are urea (46% nitrogen) and ammonium sulphate (20.5%). In view of this, it was assumed that for each pound of nitrogen applied three pounds of materials would have to be handled. Similarly it was assumed that 5 pounds of materials were needed for each pound of phosphoric acid (principal source, superphosphate, 18–20%), and 2 pounds of materials for each pound of potash (principal source, muriate of potash, 48–60%). Column (4) of Table I lists the pounds of material that would need to be handled per acre of land cultivated under each treatment and column (5) the corresponding handling charge.

If cotton is valued in the field net of picking and marketing costs, then net revenue at unit levels of two activities can be compared as follows:

\[
\pi_n - \pi_m = p_1 (a_{1m} - a_{1m}) + p_2 (a_{2m} - a_{2m}) + p_3 (a_{3m} - a_{3m}) + p_4 (a_{4m} - a_{4m}) + (h_n - h_m)
\]

(2.2)

where

\(p_1, p_2, p_3, p_4\) are the respective prices of cotton, nitrogen, phosphoric acid, and potash,

\(a_{1m}, a_{1n}\) are the respective quantities of cotton produced by activities \(m\) and \(n\),

\(a_{2m}, a_{2n}, a_{3m}, a_{3n}, a_{4m}, a_{4n}\) are the negative quantities of the various plant foods produced (i.e., \(-a_{1m}\) is a quantity used) by the two activities, and

\(h_n, h_m\) are the negatives of the handling charges for fertilizer materials used in the two activities.

The quantity coefficients and handling charges appear in Table I. The comparison of net revenues of activities 5 and 1, for example, is

\[
\pi_5 - \pi_1 = 710 p_1 - 25 p_2 - 30 p_4 - 47.
\]

(2.3)

If we set this quantity equal to zero we get a hyperplane in the four-dimen-

---

* This involves an allowance of $2.50 per ton for bags, $1.50 for local delivery, $1.90 for financing and bulk purchasing, and $1.10 for loading and transportation by the farmer. On a farm that was particularly unfavorably situated with respect to markets, the transportation would be higher changing slightly the constant term of (2.3) below; the net price of cotton, \(p_1\) in equation (2.3), would be a little lower.
sional space separating price combinations for which activity 5 yields the higher net revenue from those for which net revenue of activity 1 is higher. If comparisons are made for all pairs of activities, the space is divided into regions (except for the constant terms they would be sectors) in each of which a particular activity yields the highest net revenue. For nonnegative prices a particular activity can never yield higher net revenue than another if all of the quantity coefficients of the latter are at least as large as the corresponding coefficients of the former. If this holds we say the latter activity dominates the former. In this sense activity 6 dominates 7 and 13, and 12 dominates 14. 7, 13, and 14 are thereby excluded from further consideration.

The four-dimensional price map that could be constructed as outlined above would not be easy to visualize and would have little expository value. It can be simplified at the cost of making further approximations. In Table II it may be noted that prices of plant nutrients have tended to vary together. If we
were to set

\[ p_3 = p_3/2 \text{ and } p_4 = (p_4/2) - 1, \]

these relations would predict \( p_3 \) and \( p_4 \) to within half a cent in every year since 1943. Making the indicated substitutions would make each net revenue comparison depend only on \( p_1 \) and \( p_2 \). (2.3) for instance would become

\[ \pi_6 - \pi_1 = 716 \ p_1 - 40 \ p_3 - 17. \]

When these substitutions are made, the two-dimensional transformation of the price map can be plotted. Using the data summarized in Table I, the result is shown in Figure 1. As a step in the determination of the regions shown, partial net revenue functions were formed for those activities not dominated by any others in Table I. I use the term partial net revenue function of an activity to denote the sum of those terms in a complete net revenue function that vary from one activity to another. Call the partial net revenue of the \( n \)th activity \( v_n \); then \( (v_n - v_n) = (\pi_n - \pi_6) \) and the \( v \)'s will serve for the construction of difference of net revenue relations. The partial net revenue functions for the 12 activities considered at this stage follow.

\[ v_1 = 660 \ p_1 - 35 \ p_2 - 98 \]
\[ v_2 = 898 \ p_1 - 50 \ p_2 - 89 \]
\[ v_3 = 980 \ p_1 - 65 \ p_2 - 80 \]
\[ v_4 = 945 \ p_1 - 60 \ p_2 - 124 \]
\[ v_5 = 1376 \ p_1 - 75 \ p_2 - 115 \]
\[ v_6 = 1460 \ p_1 - 90 \ p_2 - 106 \]
\[ v_7 = 1187 \ p_1 - 85 \ p_2 - 150 \]
\[ v_8 = 1503 \ p_1 - 100 \ p_2 - 141 \]
\[ v_9 = 1679 \ p_1 - 115 \ p_2 - 132 \]
\[ v_{10} = 1092 \ p_1 - 130 \ p_2 - 123 \]
\[ v_{11} = 1348 \ p_1 - 65 \ p_2 - 19 \]
\[ v_{12} = 1733 \ p_1 - 155 \ p_2 - 211 \]

(2.6)

It is apparent from (2.6) that, under the assumptions of (2.4), activity 3 is dominated by 12 and activity 8 is dominated by 5. Among the remaining 10 activities, there are 45 possible comparisons of differences of net revenue. As one proceeds, however, additional activities are eliminated as it becomes apparent that they will not maximize net revenue for any set of nonnegative prices.
(they are dominated by mixtures of other activities). Fourteen comparisons were actually made in determining Figure 1. The six equations of equal net revenue that determine boundaries of regions are

\[
\begin{align*}
 v_{12} - v_{11} &= 41 p_1 - 25 p_2 - 88 = 0 \\
 v_{12} - v_{10} &= 54 p_1 - 40 p_2 - 79 = 0 \\
 v_{11} - v_{10} &= 13 p_1 - 15 p_2 + 9 = 0 \\
 v_{11} - v_{12} &= 344 p_1 - 65 p_2 - 104 = 0 \\
 v_{12} - v_{12} &= 331 p_1 - 50 p_2 - 113 = 0 \\
 v_{12} - v_{11} &= 688 p_1 - 30 p_2 + 79 = 0.
\end{align*}
\]

The two lower scales on the horizontal axis of Figure 1 show prices of phosphoric acid and potash that correspond to the indicated prices of nitrogen when the relations given by (2.4) hold. Using the data of Table II, prices received for cotton and paid for fertilizer by North Carolina farmers since 1940 are plotted on Figure 1 to show in which regions recent price combinations fall. Where the restrictions of (2.4) do not hold exactly, the abscissa of the plotted point is located approximately at the average of the locations that would be indicated by each of the three scales.

The market price of seed cotton (column 7 in Table II) is computed as 40% of the price of lint plus 60% of the price of seed. In the experiments summarized in Table I the lint percentages for various treatments varied from 37.1 to 42.6.

The net price of seed cotton is the market price less the indicated allowances for picking, ginning, and handling. As can be seen from Table II, market price and net price of seed cotton are highly correlated and market price runs very close to \( \frac{2}{3} \) times the net price. As long as this ratio is expected to hold approximately, a user of the price map can refer to the vertical scale for market price of seed cotton to the left of the scale for net price in the figure for a rough comparison between alternative activities.

In addition to summarizing in a convenient form information about activities for which data are available, the price map should often be useful in suggesting activities about which information is needed. In the present example it would be interesting to see how much of the area now covered by 10 and 15 would be taken over by an activity that used intermediate amounts of phosphoric acid and potash. Also, since much of the map is covered by activities using the highest application of nitrogen tried, activities involving still higher levels might be of some interest.

While it should be emphasized that the price map has been based on static comparisons, it may be noted that no activity which does not appear on the price map could yield best long-run returns under any pattern of price fluctuations or, equivalently, yield the highest expected net revenue for any probability distribution of prices.

*North Carolina State College*