Cowles Foundation Paper 75b
Reprinted from American Economic Review, 43(2), May 1953

WHAT HAS HAPPENED TO THE THEORY OF GAMES

By LEONID HURWICZ
University of Minnesota

Since the publication in 1944 of the first edition of the by now classic *Theory of Games and Economic Behavior* by von Neumann and Morgenstern (*1*)[1] there has been a minor flood of contributions to the various aspects of the theory of games and its applications; a recent book by McKinsey (*2*) provides a systematic treatment of some of the developments, while a collective volume edited by Kuhn and Tucker (*3*) indicates some of the directions of research in this field. Other valuable contributions, scattered in many journals, are too numerous to be listed or even referred to in this brief survey, but it may be helpful to mention that *2* and *3* contain very comprehensive bibliographies.

The theory of games thus has an undisputed fascination for, and a stimulating effect on, workers in many fields, ranging from (comparatively) pure mathematics to applied social sciences. There is a natural curiosity as to the directions in which the theory of games and its application have been developing; one also wishes to know which of the developments have found widespread agreement as against those which still are controversial. The limited scope and nontechnical nature of the present exposition can do little more than give a very sketchy indication of the nature of these developments.

The problems of the theory of games are characterized by the presence of two types of issues: the nature of individual behavior in situations involving (non-probabilistic) uncertainty; and the interactions in the behavior of individuals when everyone's action may affect the well-being of others. Since both of these present difficulties of their own, we shall start by discussing problems in which only issues of the first type are present.

Individual Behavior Under Uncertainty

There is no doubt that this type of problem is of great importance to the economist (e.g., in the study of investment decisions) and has been receiving in recent years an increasing amount of attention. One might question, however, the relationship of this problem to the theory of games. The answer is twofold. First, the problem of individual decision

[1] Numbers in italics refer to the list of sources cited, placed at the end of the paper. This paper will be included in *Cowles Commission Paper,* "New Series," No. 75.

making under uncertainty is an essential ingredient of most or all game-theoretic problems and hence, at the very least, has a very close conceptual link with the theory of games. Second, certain tools of the two-person theory of games have been found to be extremely useful in attacking even the one-person uncertainty type of problem. A brief discussion concerning the latter point seems in order.

It may be recalled that one class of two-person games has turned out to be of particular interest; viz., the constant-sum games where the (algebraic) sum of the gains of the two players does not depend on the way either of the players has played but always equals a certain fixed number. (This number is often chosen to be zero and the game is then called zero-sum.) For a given player, each strategy is characterized by the different (positive or negative) gains ("pay offs") he would obtain depending on the strategy chosen by the opponent. (To be precise we should substitute expected or long-run average gains for gains, since chance factors may be present.) The solution proposed by von Neumann and Morgenstern makes each player choose that strategy for which the minimal gain is at least as high as, and possibly higher than, the minimal gain guaranteed by any alternative strategy. Thus the player is maximizing the minimum pay off, or "playing the maximin." (Because of a customary formulation in terms of minimizing maximal loss or risk rather than maximizing minimal gains this principle is usually referred to as the "minimax" principle.) An interesting feature of maximizing the minimum expected gains is that it may lead to a "mixed" (i.e., randomized) strategy as superior to any "pure" (non-randomized) strategy.

Now the same principle (in its minimax form) had been suggested and applied to an important class of statistical problems by Abraham Wald (*4*).[2] Since there might be two or more minimax strategies not identical in their effects (but guaranteeing the same minimal risk), Wald combined the minimax principle with that of admissibility: to be optimal, a strategy (say s) must be admissible; i.e., there must not exist another strategy which promises sometimes a better, and never a worse, outcome than s.

Following the appearance of *1*, Wald reformulated (*7*) the statistician's problem as one of playing a zero-sum game "against nature," the nature representing the unknown properties of the universe from which the statistician draws his samples. Given this formulation, as well as the nature of the von Neumann-Morgenstern solution for two-

[2] The possibility of applying the minimax principle in statistical problems was mentioned previously in a contribution by Neyman and Pearson *(5)*. Von Neumann's initial publication in the field of game theory *(6)* antedates *4* and *5*, but the present writer is not aware of any influence it might have had on *5* or *4*, or, for that matter, of any influence that *5* might have had on *4*.

person constant-sum games, it was again natural for the statistician to minimax, so that the "game language" was not inconsistent with his earlier concept of optimality. However, the new language made it possible to pool the results obtained in two previously separate fields (game theory and statistics), with considerable gain for both.

The economic models involving decision making under non-probabilistic uncertainty are very closely related to those appearing in the problem of statistical decision making, hence here again the game language has attained considerable vogue.[3]

It should be noted, however, that recent discussions concerning behavior under uncertainty have led to a formulation of "optimal" behavior which is broader than the maximin principle. Among examples of alternatives to the maximin (minimax) principle one may mention the principle (formulated by L. J. Savage) of minimaxing the regret rather than the loss (or the disutility), the maximax principle of maximizing the maximal (rather than the minimal, as in maximin) expected gain (suggested by F. Modigliani), and the principle of maximizing some weighted average of the maximal and minimal expected gains (suggested by the present writer); all of these are to be interpreted as combined with the condition of admissibility defined above. None of these uncertainty behavior principles commands universal acceptance, since they all exhibit certain unsatisfactory properties. Just to what extent that is inevitable is not yet completely known. In the opinion of the present writer, one should not expect unanimous acceptance of some particular one among these principles, since it seems reasonable that some individuals might, say, find the regret principle a natural one, while others would feel inclined to maximax. In any case, however, if an individual is to have any consistent behavior pattern in non-probabilistic uncertainty situations, he must (implicitly or explicitly) follow some uncertainty behavior principle, whether it be minimax or something else.

Many-person Situations

Where two or more persons participate, we find, in general, that the problem of how an individual should act in a situation of uncertainty is still very much present, but that a rather peculiar type of uncertainty arises in an attempt to anticipate the others' behavior. The problem

[3] An example of a problem of this type treated by game-theory methods: the question of an optimal amount of insurance to be carried. (A Ph.D. dissertation by A. Morrison, Iowa State College, 1949.) Among ideas inspired by the game analogy is that of using randomized strategies as policy tools in situations where it is desired to achieve a certain measure of unpredictability without permitting arbitrary action (e.g., in open market operations by stabilization agencies). Cf. *8*, p. 418. The problem of how best to aggregate economic variables yields another example of the application of game-theoretic (or uncertainty decision-making) methods *(9)*.

of oligopoly is a classical illustration of this type of situation in economics. Von Neumann and Morgenstern further enriched the problem by considering the possibility of coalitions, threats, and compensations (the latter appearing very naturally in modern welfare economics), regarding these phenomena as unknowns rather than data. The concept of a solution (a set of mutually undominated imputations)[4] offered by von Neumann and Morgenstern for the general case involving an arbitrary number of participants and a wealth of possibilities of communication among players has not found universal acceptance.

Under the circumstances it was natural that a simpler class of game situations, viz., that free of intercommunication among players ("non-co-operative"), should be attacked separately; this was done by Nash (*11*).

Games without Communication

In order to appreciate the nature of Nash's proposed solution, it is desirable to return for a moment to the two-person constant-sum case. It was mentioned earlier that the von Neumann-Morgenstern solution in this case makes the two players use their respective minimax strategies. Hence the solution has the valuable property of postulating that each of the players is following a (fairly acceptable) criterion of behavior under uncertainty. A fundamental result, however, of the theory of games shows that (under appropriate assumptions) the two minimax strategies "meet" at a "saddle-point," the important implication being that when one of the players uses his minimax strategy, the other player cannot do any better than to use his minimax strategy. Thus if the two players, after having decided on their respective minimax strategies, revealed the choice, neither would have an incentive to change his strategy, provided that the other player was expected to stick to his (minimax) strategy. Consequently, the saddle-point enjoys a certain equilibrium property: there is a tendency to stay at the saddle-point once it has been reached. This combination of the two advantageous properties of the maximin strategies, viz., "rationality" of the players' behavior (in the sense of their following an uncertainty behavior principle) and the resulting equilibrium, is probably a good part of the reason for the favor which the saddle-point (minimax) solution of the constant-sum two-person game has generally enjoyed.

That (minimax) rationality and (saddle-point) equilibrium can be attained simultaneously is a special feature of the constant-sum two-person games. As soon as the game is two-person variable-sum or it involves more than two persons one must, in general, sacrifice at least

[4] Since the exposition of the general concept of solution would exceed the scope of this paper, the reader is referred to Sec. 30.1.1 of *1* for the relevant definition and to *10* for an exposition against the background of a three-person barter problem.

one of these two features. Nash's solution favors the equilibrium as against the rationality feature of the solution. In fact, his solution is defined in terms of the equilibrium property. For the sake of simplifying the exposition, we shall give the Nash definition for the case of a two-person game only. Let A and B be the two players and denote by *a* a stratetgy of the player A while *b* represents a strategy of the player B. The pair *a, b* of the two players' strategies is a Nash equilibrium point if the following is true: given that A plays *a*, B cannot do better than play *b*, while at the same time, given that B plays *b*, A cannot do better than play *a*.

It may be noted that if the game happens to be of the constant-sum type, the Nash definition yields the same saddle-point solution that is implied by the von Neumann-Morgenstern solution concept (i.e., it corresponds to both players maximining). A constant-sum game is, of course, "naturally" nonco-operative. It has been pointed out by Arrow and others that the Nash solution, when applied to the classical oligopoly problem (the mineral water example, for instance), essentially corresponds to the so-called "Cournot solution." It should be noted, however, that since the Nash definition avoids the reaction curve approach, it is immune to the Stackelberg objection: when all oligopolists but one follow the Cournot reaction curve behavior pattern, the one who does not can profit thereby; but if each of the oligopolists sticks to his Nash strategy, then (by definition) the remaining one could not possibly profit by abandoning his Nash strategy.

The present writer's inclination is to question the advisability of seeking solutions possessing the required equilibrium properties but sacrificing the rationality of behavior. To see the disadvantage of the equilibrium approach one only has to visualize an individual who is inclined to follow the maximin principle when faced by uncertainty situations. Such an individual (let us call him A) would be disinclined to count on the likelihood of his opponent B adopting (or retaining) his Nash strategy, since he (A) might suffer considerable losses were B (for whatever reasons) to follow some strategy other than that corresponding to the Nash equilibrium point.

One might, therefore, argue that in a two-person (or many-person) game without communication (Nash's nonco-operative game) each player should simply follow his usual uncertainty behavior principle (whether it be maximin, maximax, or anything else), subject to taking into account such information as he believes to have about his opponent's probable behavior. Thus it would seem safe to assume that the other player will not use a strategy which is inadmissible. This need not be true in a game with communication! The simplest solution then would be obtained by assuming that (after the elimination of all inadmissible strategies) each player selects his maximin strategy. The simplicity of the maximin (minimax) principle is mainly due to the fact that its nature has been explored much more thoroughly than that of most alternatives. Of course, this would not, in general, result in a Nash equilibrium point. However, the situation obtained would be one of equilibrium nature if the players are genuinely attached to the uncertainty behavior principle being followed. (One might note that on the latter assumption a Nash equilibrium point would lack the equilibrium property!)

It should be stressed that even if the Nash model is inadequate as a basis for a realistic description of what people actually do, and even if one would regard it unwise to advise an individual player to follow his Nash strategy, there is an important area in which the Nash model and solution concepts appear to be tools of great usefulness, both for descriptive and expository purposes. The area meant here is that of the allocative properties of the market mechanism. Thus a position of the market variables (outputs, inputs, consumption levels, prices) attained when every agent in the market finds his utility (profits) maximized, provided he assumes the variables (or reaction patterns) controlled by other agents to be fixed, is clearly a Nash equilibrium point. The basic results concerning the optimal properties of the market mechanism can roughly be summarized in the proposition that in a properly defined market economy a point is one of Nash equilibrium whenever it is Pareto-optimal and vice versa. (A situation is defined as Pareto-optimal if there is no way of raising anyone's utility without lowering that of someone else. For a game formulation of the allocative process, see *12.*)

It may be remarked that some of the controversies in the game-theory field result from the lack of distinction between the descriptive as against normative character of a theoretical model; we note that there is a third category of usefulness, viz., as a tool, which includes the utilization of a model for the purpose of setting up an organizational structure. Another source of difficulties, of relevance with regard to models claimed to be realistic, is vagueness as to observational implications of a given solution concept.

Games with Communication

In this field there is perhaps most still to be accomplished; in view of its complexity, this is hardly surprising. While there are some attempts at the application of the proposed solution concepts to economic and other situations, the lack of confidence in the solution concept itself has a tendency to undermine the interest in the applications. (Some contributions in this area are to appear shortly in the *Econo-*

metrica and in Vol. *2* of *3,* but are not available to the present writer at this time. Among papers of interest in this context one may mention *13* and *14.*)

Among attempts known to the present writer to seek an approach different from the solution of *1,* two deserve mention. One (Vickrey, *15*) appears to be relatively close to the formulation of *1,* differing from the latter in that it imposes an additional requirement that the solution be, in a certain sense, self-policing. Another, due to Nash (*11b,* page 295), is based on the idea of fitting the co-operative game into the general scheme of a nonco-operative game; this is to be achieved by introducing such elements as the bargaining process, as well as compensations, etc., explicity into the game, instead of leaving them on the outside as appears to be the case in *1.* (Once it is known how to reduce co-operative to nonco-operative games, there is still freedom with regard to which theory of nonco-operative games is to be applied to the reduced games.) With only very sketchy information available at this time, one must refrain from passing judgment on the chances of success of these proposals in terms of providing a workable and reasonably realistic tool for treating the multiperson games with communication.

Even if the case involving communication cannot yet be handled in a satisfactory manner, game theory (together with recent developments in the science of information and communication, including cybernetics) is to be credited for the increased attention being paid to information and communication aspects of social situations. (See *16* for an example related to certain problems of economic theory.) In the field of economics there is a class of problems very likely to profit greatly from increased emphasis on the information-processing aspects of the economy. Among problems of this class is that of the optimal size of the firm, as well as the more general one of optimal (internal) economic structure of an economic unit, especially with regard to the location of the limiting line between market structure (characteristic of interfirm relationships) and the administrative structure (characterizing—typically, but not without exceptions—the intrafirm relationships). An interesting special case is that of conditions under which a firm is likely to wish to introduce the pricing mechanism as a method of internal allocation of resources.

In a broader perspective, there is reason to hope that the development of an analytic approach to problems of economic structure (whether along the lines just indicated or in some other fashion) will tend to wipe out the traditional division between the theorists and the institutionalists. Recent developments in fields such as the theory of organization seem to add further justification to this hope.

While the present paper is in no way a systematic appraisal of the impact of game theory and related developments on economics, the writer does wish to express his opinion on one aspect of the manner in which economics has profited from the developments in the field of game theory. Even where the theory of games does not provide satisfactory answers, it has contributed to a more lucid, rigorous, and natural formulation of many problems. In many cases, it has led to the use of mathematical tools not involving calculus and often very close to intuitive thinking, thus creating the possibility of exploiting both the mathematical and common-sense (literary) type of talent available. A further development, which would seem at least partly due to the influence of game theory (as well as to that of statistical decision theory) is a trend toward axiomatic formulation of economic theory, with the consequent increase in rigor and greater transparency in the relationships between assumptions made and conclusions reached.

In the present writer's mind there is little doubt with regard to both the value of what the game theory (with its "relatives") has accomplished, and the length of the road that still remains to be traversed.

REFERENCES

1 J. von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior* (1st ed., 1944; 2nd ed., 1947; Princeton University Press).

2 J. C. C. McKinsey, *Introduction to the Theory of Games* (a RAND Corp. Research Study) (McGraw-Hill Book Co., 1952).

3 H. W. Kuhn and A. W. Tucker, eds., *Contribution to the Theory of Games, Annals of Mathematics Studies,* No. 24 (Princeton University Press, 1950).

4 A. Wald, "Contributions to the Theory of Statistical Estimation and Testing Hypotheses," *Annals of Mathematical Statistics,* 1939, pp. 299-326, especially p. 305.

5 J. Neyman and E. S. Pearson, "The Testing of Statistical Hypotheses in Relation to Probability A Priori," *Proceedings of Cambridge Philosophical Society,* 1933, p. 29.

6 J. von Neumann, "Zur Theorie der Gesellschaftsspiele," *Math. Annalen.,* 1928, pp. 295-320.

7a A. Wald, "Statistical Decision Functions Which Minimize the Maximum Risk," *Annals of Mathematics,* 1945, pp. 265-280.

7b A. Wald, *Statistical Decision Functions* (John Wiley & Sons, 1950); especially Sec. 1.6.3, pp. 26-28.

8 L. Hurwicz, "Comment on Business Cycle Analysis and Public Policy," in *Conference on Business Cycles* (National Bureau of Economic Research, 1951), pp. 416-420.

9 L. Hurwicz, "Aggregation in Macroeconomic Models" (abstract), *Econometrica,* July, 1952, pp. 489-490.

10 J. Marschak, "Neumann's and Morgenstern's New Approach to Static Economics," *Journal of Political Economy,* April, 1946; also circulated as *Cowles Commission Papers,* "New Series," No. 13b.

11a J. F. Nash, "Equilibrium Points in n-Person Games," *Proceedings of National Academy of Science,* 1950, pp. 48-49.

11b J. F. Nash, "Non-Cooperative Games," *Annals of Mathematics,* 1951, pp. 286-295.

12 T. C. Koopmans, "Analysis of Production as an Efficient Combination of Activities," in *Activity Analysis of Production and Allocation, Cowles Commission Monograph No. 13* (John Wiley & Sons, 1951).

13 J. F. Nash, "The Bargaining Problem," *Econometrica,* 1950, pp. 155-162.

14 M. Shubik, "A Business Cycle Model with Organized Labor Considered." *Econometrica,* April, 1952, pp. 284-294.

15 W. Vickrey, "Self-Policing Solutions in the Theory of Games." A paper presented at the Econometric Society meetings in Chicago, December, 1952. (Earlier versions circulated privately in 1949.)

16 M. Shubik, "Information, Theories of Competition, and the Theory of Games," *Journal of Political Economy,* April, 1952, pp. 145-150.