A STRUCTURAL APPROACH TO THE PROBLEM OF IMPORT DEMAND

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I

I propose, in this paper, to re-examine the problem of estimating import demand functions for the United States. To my mind, the most critical (and also most neglected) aspect of this problem is the choice of an appropriate estimating technique. I believe I can show that some elementary "structural" facts load the dice against the traditional least-squares method, and strongly suggest a new, alternative technique. Furthermore, the new technique, when applied, yields results far different from those we have been accustomed to getting. Whereas past results have led us to assume the price elasticity of United States import demands to be between 0 and −1, my work suggests that elasticities of −2 and even −3 are at least equally consistent with our experience.

My results are important, I feel, for the light which they shed on the postwar controversy over whether price elasticities in international trade are generally low or high. In this debate, the weight of empirical analysis has all been on the low-elasticity side; members of the high-elasticity school have heretofore been forced to argue solely from broad, qualitative presumptions. Moreover, with the empirical estimates in hand, some low-elasticity people have moved on to question even the possibility of a stable price equilibrium in international trade and have advocated discriminatory and restrictive policies which would be at best unnecessary and at worst downright detrimental if in fact the elasticities were high. My results tend to knock the empirical props out from under these recommendations of the low-elasticity school and lend support to trade policies that rely firmly on the free price mechanism.

Since my argument proceeds from the "first principles" of estimation, I shall start with a simple supply-and-demand example, in which

1 The general subject of this paper has been a matter of interest to me for some time, and my debts to others are correspondingly great. I am grateful to the Cowles Commission for Research in Economics, under whose auspices I first started my work on import demand, to the Social Science Research Council for its generous support, and to the members of the Cowles Commission Staff and of the economics seminar at Johns Hopkins for helpful comments and suggestions. I am particularly indebted to Carl Christ, Clifford Hildreth, and T. C. Koopmans for their long-standing interest in the econometric aspects of my work, and to Faye M. Goldware for her aid in the preparation of this paper. This paper will be reprinted as Cowles Commission Paper, New Series, No. 73.
the objective is to estimate a demand curve from four points observed at different times (Figure 1). Each point is presumed to represent the equilibrium of supply and demand for its particular time; hence it is clear that the demand curve must have shifted over the period of observation. But once the possibility (or necessity) of a shifting demand curve is recognized by the estimator, a Pandora's box of frightful possibilities is opened. For now it must be granted that the observed points could have come from a demand curve of practically zero elasticity shifting in one fashion (dashed lines), or from a demand curve of practically infinite elasticity shifting in another (solid lines), or, in fact, from a demand curve of any elasticity whatsoever. All we know is that each hypothetical elasticity (or slope) is associated with its own particular pattern of shifts in the demand curve. Clearly, the job of estimating elasticity is merely the other side of the coin from distinguishing between plausible patterns of demand shifts and implausible ones. And

![Figure 1](image)

by the same token, any defensible rationale for a particular procedure of estimation must be capable of translation into a reasonable criterion for distinguishing admissible from inadmissible shift-patterns.

There are two quite distinct ways by which the least-squares procedure, which has been the basis for past estimates of United States import demand elasticity, might be defended in terms of assertions about the nature of demand shifts. On the one hand, in cases where there is reason to believe that the function to be estimated is very stable, shifting hardly at all over time, no line which fits the observed points well is likely to yield a poor estimate. Hence, in such cases, the least-squares "line of best fit" can hardly be rejected. But instances in which we can assert that our function is likely in fact to be highly stable over time are rare indeed in economics, and, I would suggest, particularly rare in the field of international trade.

We may therefore turn to the second possible justification for the
least-squares method. It can be shown that least-squares yields correct estimates if the shifts in the function to be estimated are, over time, uncorrelated with changes in the variables treated as "independent" in regression. It may be surprising but it is nevertheless mathematically true that in our example the simple assumption of noncorrelation between demand shifts and changes in price reduces the possible values of the price elasticity from an infinite range to a single point. An assumption which, to the unpracticed eye, may seem to be a plausible approximation, acceptable on grounds of convenience in the absence of an obvious alternative, thus turns out to have tremendous empirical content. Clearly, we can be no more sure of the elasticity estimate that least-squares gives us than we are of the empirical validity of the assumption we use to get it. Yet it is hard to see, upon examination, how anybody could claim empirical validity (or even likelihood) for the assumption in question. For rightward shifts in demand tend directly to raise prices and leftward shifts to lower them. Hence we should expect a positive correlation between demand shifts and price changes and not the zero correlation which the least-squares method requires.

Thus we can justify the use of least-squares to estimate import demand functions neither on the grounds that shifts in demand are negligible in size nor on the grounds that the shifts are uncorrelated with changes in the "independent" variables. We must, it seems to me, seek an estimating scheme which is directly based on the assertions which we can plausibly make concerning the nature and effects of demand shifts. And, as a corollary to this approach, we must be prepared to accept a range of uncertainty in our estimates which corresponds to the range of unavoidable uncertainty in our assertions about demand shifts. The data themselves are fully consistent with an infinite range of price and income elasticities, and it is the estimator himself who narrows this range by the assumptions which his scheme imposes. This fact obviously warns against the use of arbitrary estimating procedures, derived from statistical cookbooks, whose implicit assumptions have not been explored and found applicable to the case at hand. It also throws considerable suspicion on techniques which yield precise estimates, or even precise estimates surrounded by normal curves of error. For we cannot afford to forget that precision of estimates quite typically merely reflects the rigidity of imposed assumptions.

II

Since shifts in demands, though unobservable, are so important in the process of estimating, we should certainly try to define them in the most meaningful way. The usual procedure of taking regressions of
absolute quantities of imports against absolute levels of real income and relative price, defines any year's shift in demand as the difference between the position of the demand curve in that particular year and its average position over the whole period of estimation. I think we can reason much more sensibly about year-to-year shifts in demand than about such deviations from a long-period average. Therefore, in my work, I take my variables as year-to-year percentage changes rather than as absolute quantities. This alteration in procedure has little effect on the least-squares estimates of United States import demand elasticity. The least-squares estimates which I obtained, using interwar data, are shown in row I of Table 1. They do not differ greatly from past least-squares results, except where the past estimates were based on a distinctly different definition of the relative price variable.⁸

As indicated above, the estimates of row I might be defended either by the assertion that the import demand functions in question were highly stable over the period of estimation (1923-39) or by the assertion that the shifts in import demand in this period were uncorrelated with changes in both relative price and real income. Since it is not possible to make either assertion with any degree of confidence, I feel that the estimates must be rejected. Stability is patently inconsistent with the data (even when the extent of the annual shifts, apart from trend, in the functions is minimized, they average about 10 per cent), and the assumption of noncorrelation between demand shifts and price changes runs counter to the expectations we derive from elementary supply and demand analysis.

As a first step in the search for better estimates, I shall assume that demand shifts are positively correlated with price changes, as our theory leads us to expect. But, tentatively, I shall retain the least-squares assumption that demand shifts are uncorrelated with income changes. This simple alteration of assumptions changes drastically the inferences that can be derived from the data. Far from being the central point in a range of plausible estimates, the least-squares estimate turns out to be the absolute lower limit to such a range. Any higher price elasticity is equally consistent with the data and the assumptions just made; the upper limit to the range is minus infinity.

To further limit the range of possible values of price elasticity, we must make more restrictive assumptions. If we were prepared to say, for example, that the income elasticity of import demand were sure to

⁸My price variable was import price deflated by the general U. S. wholesale price index. Quite naturally, the elasticity of demand with respect to such a price variable will be lower than the elasticity with respect to import price deflated by an index of the prices of close substitutes. Some previous investigators have used the latter deflation scheme, and have come up with elasticity estimates somewhat higher than those shown in row I, but these measure an elasticity different in concept from that which has become prominent in the theory of tariffs and of exchange stability and which I am attempting to measure.
### Table 1
PRICE-ELASTICITY ESTIMATES FOR UNITED STATES IMPORT DEMAND

<table>
<thead>
<tr>
<th></th>
<th>Total Imports</th>
<th>Crude and Semi-manufactured Materials</th>
<th>Crude Foods</th>
<th>Manufactured Foods</th>
<th>Finished Manufactures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>I. Least-square estimate</strong></td>
<td>-1.3</td>
<td>- .4</td>
<td>- .4</td>
<td>- .4</td>
<td>- .9</td>
</tr>
<tr>
<td><strong>II. Lower limit</strong></td>
<td>-1.3</td>
<td>- .4</td>
<td>- .4</td>
<td>- .4</td>
<td>- .9</td>
</tr>
<tr>
<td>Upper limit</td>
<td>-6.3</td>
<td>-5.0</td>
<td>-10.8</td>
<td>-1.4</td>
<td>-5.0</td>
</tr>
<tr>
<td><strong>III. Lower limit</strong></td>
<td>-1.3</td>
<td>- .4</td>
<td>- .4</td>
<td>- .4</td>
<td>- .9</td>
</tr>
<tr>
<td>Upper limit</td>
<td>-3.4</td>
<td>-7.6</td>
<td>-1.9</td>
<td>-1.1</td>
<td>-3.3</td>
</tr>
<tr>
<td>Assumed income elasticity</td>
<td>2.3</td>
<td>1.9</td>
<td>1.0</td>
<td>1.0</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>IV. Lower limit</strong></td>
<td>-1.9</td>
<td>- .9</td>
<td>- .5</td>
<td>- .5</td>
<td>-1.0</td>
</tr>
<tr>
<td>Upper limit</td>
<td>-3.5</td>
<td>-4.1</td>
<td>-1.9</td>
<td>-1.6</td>
<td>-3.3</td>
</tr>
<tr>
<td>Assumed income elasticity</td>
<td>3</td>
<td>2.5</td>
<td>1.5</td>
<td>.75</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>V. Lower limit</strong></td>
<td>-2.8</td>
<td>-2.1</td>
<td>- .6</td>
<td>- .2</td>
<td>- .6</td>
</tr>
<tr>
<td>Upper limit</td>
<td>-4.6</td>
<td>-4.5</td>
<td>-2.3</td>
<td>-2.8</td>
<td>-4.0</td>
</tr>
<tr>
<td>Assumed income elasticity</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>VI. Lower limit</strong></td>
<td>- .1</td>
<td>0</td>
<td>- .8</td>
<td>0</td>
<td>- .2</td>
</tr>
<tr>
<td>Upper limit</td>
<td>-45.5</td>
<td>-∞</td>
<td>-3.7</td>
<td>-∞</td>
<td>-28.6</td>
</tr>
<tr>
<td>Assumed income elasticity</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**Assumptions:**
- Least-square estimates (row I) are based on a) assumed zero correlation of demand shifts with price changes.
- Upper limit b) assumed zero correlation of demand shifts with income changes.
- Lower limit on income elasticity between .5 and 5.0.
- Limits in rows III-VI are based on a) assumed positive correlation of demand shifts with price changes.
- b) assumed positive correlation of demand shifts with income changes.
- c) income elasticities not explained by income.
- Income elasticities as shown.

(The income elasticities assumed in row III are those obtained by least-squares regression.)

**Data:** Year-to-year percentage changes in imports were fitted as a linear function of year-to-year percentage changes in the relative price of imports and real gross national product. The gross national product data were the Department of Commerce Series, expressed in 1939 dollars. The price deflator for import prices was the general wholesale price index. The series on quantities and prices of imports were obtained from Adler, Schlesinger, and van Westerborg, *The Pattern of United States Import Trade Since 1923* (Federal Reserve Bank of New York, 1952), pp. 81-82. The time period was from 1923 to 1939.

lie somewhere between .5 and 5.0, then we could limit the price elasticities as shown in row II of the table.

These ranges depend very explicitly on the rather implausible assumption that demand shifts were absolutely uncorrelated with changes in income over the period of observation. We can, however, get upper
limits without saying anything about the relation between demand shifts and income. We need only assume, in addition to positive correlation between demand shifts and price changes, that some particular income elasticity applied during our period, and that the shifts in demand were positively correlated with quantities of imports unexplained by changes in income. This last assumption comes right out of our theory; for just as rightward shifts in demand tend to raise prices, they also tend to raise quantities.

It seems to me that the only possible weakness in this approach is that, at the start, one has to choose a precise income elasticity. But this is not important for our problem of getting some notion of where the price elasticity of import demand lies. For regardless of what plausible income elasticity we assume, the ranges which emerge center very far

| Table 2 |
| Price Elasticity Estimates for United States Import Demand |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | Total Imports   | Crude and Semimanufactured Materials | Crude Foodstuffs | Manufactured Foodstuffs | Finished Manufactures |
| Lower limit     | -1.3            | -4.8            | -4.4            | -4.2            | -4.8            |
| Upper limit     | -6.3            | -4.8            | -2.3            | -2.5            | -4.8            |

Assumptions:
a) positive partial correlation of demand shifts with price changes, income held constant
b) standard deviation of annual demand shifts less than 20 per cent

above the traditional least-squares estimates. This can be seen in rows III-VI of the table. Each income elasticity has its own range of price-elasticity estimates, and these ranges do differ. But they are unanimous in lying overwhelmingly above the old least-squares estimates. I therefore conclude that the old estimates have given us a quite erroneous impression about the price elasticity of demand for imports.

Readers conversant with the statistical literature on estimation may notice the absence of probability statements in the above analysis, and, on reading the appendix, may wonder whether the results of my estimation procedures are thought to be independent of sample size. These apparent anomalies are simply explained. For my statements to the effect that "demand shifts are positively correlated with price changes," and like assertions, are intended to apply to the actual period of observation, not to a "population" from which the actual observations may be regarded as a sample. Naturally, the shorter the period, the more likely it is that fortuitous shifts in supply could make the actual correlation zero or negative, even though economic theory leads to a strong presumption to the contrary.
III

A possibility exists, which we have not as yet explored, of further narrowing the ranges given in Table 1. It was shown in the example given earlier how each hypothetical elasticity was associated with a particular pattern of shifts in the demand curve. We have so far used presumptions about the effects of demand shifts on prices and quantities to exclude as implausible some of these hypothetical elasticities. In this section we shall attempt to rule out additional hypothetical elasticities, on the grounds that they imply an implausibly great shiftability of the demand function.

The difficulty of successfully applying this criterion with respect to our present problem has already been alluded to. It arises from the fact that we have no good economic grounds for supposing that the United States import demand function is highly stable over time. Hence the estimates which we can rule out on the grounds that they imply excessive shiftability in our function are likely to have been already ruled out on other grounds.

My basis for concluding that we must admit of substantial instability in United States import demand is twofold. In the first place, as indicated earlier, even when estimates are so chosen as to minimize the variability of the function, the average annual shift, even after trend is removed, turns out to be around 10 per cent. Even without special presumptions in favor of instability we would have to set the maximum conceivable variability considerably above this level, which is, after all, the minimum possible. But there are very good independent reasons for believing that our import demand is quite unstable. They arise from the fact that we produce at home great quantities of goods which are practically homogeneous with our imports. Zassenhaus and Lovasy, in a recent unpublished study, have constructed an index of domestic production of items which compete directly with our imports of crude materials and semimanufactures. In value terms, this domestic production regularly amounts to between two and four times our imports in these two categories. The picture does not change when we look at our imports of foodstuffs and finished manufactures. Items like coffee, tea, cocoa, spices, and bananas, which we do not produce domestically, are the exception rather than the rule among our imports. A good 60 per cent of our total imports have close substitutes which we produce at home in great volume.

The fact of competing home production suggests that when we go about estimating an import demand function, we should not think purely in terms of a demand curve derived directly from the theory of consumers' choice. Rather, we should view import demand as a residual—as the difference between a total demand curve and a supply curve of
competing home production. Once this approach is taken, it becomes clear that the import demand function is likely to be highly unstable. For fairly small percentage shifts in either our total demand for import-like commodities or our domestic supply of import substitutes get magnified into fairly large percentage shifts in their difference, which is import demand. With these considerations in mind, it is with some reluctance that I set my "maximum conceivable" amount of shiftability in the import demand function at an average as low as 20 per cent per year. I would certainly find it difficult to argue for a significantly lower maximum.

The imposition of this restriction has very little effect on the ranges of price elasticities derived in the preceding section. In most instances it does not help to reduce at all the upper limits which we found on the basis of other assumptions. It does, however, bring the ranges of row VI into substantial correspondence with the ranges in the rest of Table 1. The upper limits in the five columns of row VI now become, respectively, -3.7, -2.7, -2.6, -1.7, and -4.0. In addition, the upper limit in row III for crude materials and semimanufactures is reduced to -3.4, and the upper limit in row V for manufactured foodstuffs is reduced to -2.0.

Restriction of the variability of import demand does, however, provide us with some additional comfort. For it is now possible to eliminate altogether the more arbitrary assumptions which were used in Table 1; that is, the assumption of zero correlation of demand shifts with income changes (used in row II) and the assumptions of specific values for income elasticity (used in rows III-VI). If we assume only that the partial correlation of demand shifts with price changes was positive (holding income constant) and that the standard deviation of demand shifts was less than 20 per cent, we can limit the admissible values of price elasticity as shown in Table 2. And it is worth while to note that even a reduction of the 20 per cent limit to 15 per cent would only slightly lower the upper limits shown. This analysis thus strongly reinforces my over-all conclusion that practically any combination of plausible assumptions about demand shifts yields ranges of price-elasticities centering far above the old least-squares estimates.

IV

The high price elasticities suggested by the data are also indicated by our theory, together with the fact of substantial import-competing domestic production. For if imports are to be viewed as the difference between total home demand and domestic supply of import substitutes, as shown in Figure 2, the price elasticity of import demand will be some large multiple of the component elasticities of total demand and of
domestic supply. In fact, if the home demand and supply curves are each of unit elasticity, the elasticity of import demand will be \(-19\) if imports are a tenth of total demand, \(-9\) if imports are a fifth, and \(-5\) if imports are a third of total demand. Thus the fact of competing home production by itself practically implies the high price elasticities that we find when we use a defensible technique of estimation.

The fact of competing domestic production may also have some implications about the income elasticity of import demand. On the surface it looks as if this income elasticity will also be high. For if income affects total home demand but not domestic supply, a 1 per cent rise in income will cause the home demand curve to shift to the right by a percentage equal to the income elasticity of home demand. And if the home supply curve does not shift, this will entail a magnified percentage shift in import demand.

But what about the home supply curve? It seems to me that in periods of high employment growth, like the middle and late twenties,

![Figure 2](image)

changes in real income merely reflect the aggregate shift to the right of all the supply curves in the economy, stemming from increases in labor force and productivity. I can find no reason to expect that import competing industries will not typically share in this general advance. So for periods of high employment growth, we have both total demand and domestic supply shifting to the right, and there is no reason to expect the income elasticity of import demand to be a large multiple of the income elasticity of total home demand. I come to just the opposite conclusion for periods of cyclical decline. For while income shifts the home demand curve sharply to the left in such periods, I find no reason to expect that the supply curve of import-competing goods should also shift to the left. Home suppliers should be willing to supply at least what they used to at the same relative price. They may even be prepared to supply more, owing to the availability of unemployed

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*The general formula for the elasticity of import demand, where imports \((M)\) are the difference between total domestic demand \((D)\) and home supply of import-like commodities \((S)\), is \(\eta_m = k\eta_d + (k - 1)\eta_i\). Here \(k\) is the reciprocal of the share of imports in total home demand and all the elasticities \(\eta\) are absolute values.*
resources and the increases in labor productivity which a depression induces. Import demand is thus likely to be squeezed from both ends, with foreign suppliers bearing the brunt of our depression. Hence the income elasticity of import demand should be quite high in periods of cyclical decline. In periods of cyclical rise, the home supply curve is likely to advance only slowly, and I would therefore expect the income elasticity of import demand to be somewhere between its relatively low prosperity value and the high figure it is likely to reach in depressions.

The foregoing argument presents a hypothesis which, though reasonable, is as yet untested, and a thorough test of it is beyond the scope of the present paper. I have, however, made a large number of empirical experiments, designed to ascertain whether the results obtained earlier in this paper would be endangered if, in fact, the income elasticity of import demand were not a constant but varied over the cycle as my hypothesis suggests. In these experiments, I followed the same procedure I used to obtain the ranges given in rows III-VI of Table 1, but instead of imposing constant income elasticities, I used three-valued income elasticities—highest in years of cyclical decline, lowest in prosperity, and in between for years of cyclical rise. The ranges which I obtained, though slightly narrower than those shown in Tables 1 and 2, differed from them in no essential way. Without exception, the new ranges for the price elasticity of import demand, like the earlier ones, lay overwhelmingly above the old least-squares estimates.4

V

Though the foregoing analysis establishes that the price elasticity of United States import demand is very likely to be substantially higher than was indicated by previous empirical work, it cannot claim to say the final word. The ranges obtained for price elasticity are fairly wide, and though it may not be possible to find additional plausible assumptions which narrow them significantly, some effort in this direction may be useful. More important is the fact that my ranges refer quite explicitly to the short-run (year-to-year) elasticity of import demand, while for many policy questions it is the long-run elasticity which is relevant. All we know about this important elasticity is that, because domestic supplies are more responsive to price in the long run, it exceeds the short-run elasticity which we have attempted to measure. Empirical work designed explicitly to estimate the long-run elasticity of import demand is practically nonexistent. Indeed, I doubt that it is amenable to estimation by time-series analysis of the type used in this paper. Even to approximate it will probably require a close survey of

4 Further information regarding these experiments will be made available upon request.
the American economy, with a view to ascertaining the extent of each industry's vulnerability to price competition from abroad or, conversely, its ability to raid the foreigner's share of our domestic market. The key to the problem of estimating long-run elasticity lies, I feel sure, in the supply side of the supply-demand picture. It stems directly from the fact that foreign and home suppliers do compete in the American market—a fact which, as we have seen, also plays a role in the estimation of short-run elasticities.

APPENDIX

A. Limiting Procedure Used in Table 1, Row II.

The equation fitted is of the form

\[ x_t = \alpha + \eta \rho_t + \varepsilon y_t + u_t, \]

where \( x_t \) is the percentage change in imports from year \( t-1 \) to year \( t \), \( \rho_t \) is the percentage change in relative price, \( y_t \) the percentage change in real national income, \( \alpha \) a trend term, and \( u_t \) the percentage shift in the import demand function, apart from trend. Taking moments with respect to \( \rho \) and \( y \), we obtain

\[ (2) \quad \Delta \equiv \eta \Delta_y + \varepsilon \Delta_y + \Delta_u, \]

\[ \Delta_y \equiv \eta \Delta_{yy} + \varepsilon \Delta_{yy} + \Delta_{uy}, \]

where \( \Delta_{ij} \) designates the sum of products of the deviations from their respective means of the variables \( i \) and \( j \), summed over the entire period of observation. The least-squares estimates are obtained by setting \( \Delta_u = \Delta_y = 0 \), and solving for \( \eta \) (the price elasticity) and \( \varepsilon \) (the income elasticity). The procedure used in row II follows least-squares in setting \( \Delta_y = 0 \), but with respect to \( \Delta_u \) merely asserts that it is positive. Under these assumptions, the "true" parameters \( \eta \) and \( \varepsilon \) can be expressed in terms of the least-squares estimates \( \tilde{\eta} \) and \( \tilde{\varepsilon} \) and an additional term. Thus

\[ (3) \quad \eta = \tilde{\eta} - \frac{\Delta_u}{\Delta}, \]

\[ \varepsilon = \tilde{\varepsilon} + \frac{\Delta_u}{\Delta}, \]

where \( \Delta \) is the determinant \((\Delta_{yy} \Delta_{yy} - \Delta_{uy}^2)\), which is necessarily positive. Since \( \Delta_{yy} \) is also necessarily positive and \( \Delta_{uy} \) is so by assumption, \( \eta \) (a negative number) must be greater in absolute value than \( \tilde{\eta} \). Furthermore, admissible combinations of \( \eta \) and \( \varepsilon \) must clearly lie on the straight line

\[ (4) \quad (\eta - \tilde{\eta}) = \frac{-\Delta_{yy}}{\Delta_{yy}} (\varepsilon - \tilde{\varepsilon}), \]
which permits the translation of assumed limits on \( \epsilon \) into derived limits on \( \eta \).

B. Limiting Procedure Used in Table 1, Rows III-VI.

The assumption of a specific income elasticity \( \epsilon^* \) permits the designation of a new variable \( z_t = x_t - \epsilon^* \mu_t \), which reflects the annual percentage change in imports unexplained by changes in income. The basic equation (1) can then be rewritten as

\[
(5) \quad z_t = \alpha + \eta \rho_t + \mu_t.
\]

Taking moments with \( \rho \), we obtain

\[
(6) \quad M_{\rho z} = \eta M_{\rho z} + M_{z\rho}.
\]

Since \( M_{z\rho} \) is assumed positive, the least-squares estimate \( (M_{z\rho}/M_{\rho z}) \) obtained from the regression of \( z \) against \( \rho \) is a lower limit to the set of admissible values of \( \eta \).

Taking moments with \( z \), we obtain

\[
(7) \quad M_{zz} = \eta M_{z\rho} + M_{\rho z}.
\]

Since \( M_{\rho z} \) is assumed positive, and \( M_{z\rho} \) is uniformly negative for the cases I examined, the least-squares estimate \( (M_{zz}/M_{z\rho}) \) obtained from the regression of \( \rho \) against \( z \) provides the upper limit to the admissible values of \( \eta \).

C. Limiting Procedure Used in Table 2.

From equations (2) we may derive

\[
(8) \quad \eta = \bar{\eta} - \frac{M_{\rho z} M_{\rho \rho} - M_{z\rho} M_{\rho \rho}}{(M_{\rho z} M_{\rho \rho} - M_{z\rho} M_{z\rho})},
\]

which in turn may be reduced to

\[
(9) \quad \eta = \bar{\eta} - \frac{r_{\rho z} - r_{\rho \rho} r_{z\rho}}{\sigma_p \left( 1 - r_{z\rho}^2 \right)}.
\]

The partial correlation of demand shifts with price changes can be written

\[
(10) \quad r_{z\rho} = \frac{r_{z\rho} - r_{z\rho} r_{\rho \rho}}{(1 - r_{z\rho}^2)^{1/2} (1 - r_{\rho \rho}^2)^{1/2} \sigma_u}.
\]

Since \( r_{z\rho} \) is assumed positive, it must lie between 0 and 1. Hence the bracketed expression in (9) must lie between 0 and \([1/(1 - r_{z\rho}^2)^{1/2}]\), and \( \eta \) must lie between \( \bar{\eta} \) and \( \bar{\eta} - \frac{\sigma_u}{\sigma_p (1 - r_{z\rho}^2)^{1/2}} \). The assumption of a specific value for \( \sigma_u \) completes the limiting procedure used in Table 2.