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PRODUCTION RELATIONS IN THE RAILWAY INDUSTRY[1]

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A model of production relations is specified for two processes in railway technology. The parameters of the production function for each process are estimated by least-squares techniques from cross-section data. From these parameters, estimates are made of the coefficients of elasticity of factor use and elasticity of cost. A test of significance is then applied to the estimated coefficients.

THE QUESTION of increasing returns to scale in the railway industry has occupied the attention of economists since the last century, and justifiably so.[2] Knowledge of the existence of increasing returns is necessary before one can deal with such questions as whether to permit discrimination in freight rates; whether to regulate, encourage, or discourage nonrail competition; and, indeed, whether to regulate transportation industries at all.

This study has been designed to determine whether increasing returns actually exist. A model of production has been specified for two processes in railway technology, and a production function has been estimated from cross-section data for each process. The statistical results obtained indicate coefficients of elasticity of variable cost with values of unity or less. If the estimates are accurate, they indicate that increasing or constant returns exist.

The report of the study has been divided into two sections. The first deals with the specification of a model of production relations. In the second, statistical results are presented which have been obtained under two specifications of the production model. Estimates of the coefficients of elasticity of factor use and elasticity of cost are shown, as well as the results of a test of their significance.

I. SPECIFICATION OF THE MODEL OF PRODUCTION

The first step in constructing the production relations is the introduction of the hypothesis that railway technology consists of the simultaneous operation of three distinct processes which are physically connected

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[2] An early discussion of increasing returns is contained in A. M. Wellington, *The Economic Theory of the Location of Railways*, New York: John Wiley and Sons, 1893, pp. 109–111.

by a flow of empty and loaded cargo cars. The three processes are loading of cargo cars; switching of cargo cars; and line-haul operation, that is, the transfer of assembled trains between terminals.[3] That this distinction is not arbitrary is seen from the division of labor which takes place between transport firms at terminals handling a large volume of freight. One may observe that some firms specialize in the assembly of cargo, the loading of cargo cars, and the forwarding of loaded cars, while others specialize in the transfer of cargo cars between shipper and carrier or between carriers themselves.

Because a large proportion of loading activity is carried on by other than transport firms, models of production relations have been constructed for the switching and line-haul processes only. The variables chosen as the inputs and outputs of each process are shown below.

Switching process. Yard switching consists of the employment of switch locomotives in picking up, assembling, and sorting cargo cars into trains. In the switching process the following input services are employed: labor, fuel, equipment, and fixed plant. The output is the number of transferred cargo cars.

In order that a unique relation may be found between inputs and outputs it is also necessary to take into account certain obstacles to production which vary between switching firms. They are: (*a*) The distance from the assembly yard to the loading docks and sidings which contain cargo cars awaiting pickup. As this distance increases, an increase must take place in the number of miles to be covered by a switch engine as it transfers a given number of cars. This is measured by "switch-engine miles." (*b*) The degree of circuity in the routing of switch locomotives. Circuity of routing is made necessary by the scatter of cargo cars. If there are one hundred cars awaiting pickup, they may be at one hundred different locations, at the same location, or distributed in some intermediate fashion. Circuity implies a greater expenditure of engine time to pick up a given number of cargo cars over a route of given length. This is measured by "switch-engine hours."[4]

In order to derive an input-output relation from observations on firms which face the last two obstacles in differing degree, it is necessary to include them in the model as conditions which impede production. Therefore, switch-engine miles and switch-engine hours are viewed as services performed by the switching firm. These services are substitutes for the useful output of the switching process.

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[3] For a discussion of the services performed by railway firms, see *Rail Freight Service Costs in the Various Rate Territories of the United States*, 78th Congress, First Session, Document No. 63, Washington, D.C., 1943, p. 56.

[4] Switch-engine hours was later eliminated from the statistical model because it was found to be almost perfectly correlated with switch-engine miles and thus did not serve as an independent measure of circuity.

Line-haul operation. The line-haul process consists of the transfer of loaded and empty cargo cars between terminals. The following input services are employed: labor, fuel, equipment, and fixed plant. The outputs produced are loaded cargo-car miles; loaded, transported cargo cars; and empty cargo-car miles. Output of the line-haul process may be varied by changing either the number of loaded cars to be hauled or the distance over which they are to be transported. This is taken into account by the use of car-miles and loaded cars. It is also necessary to account for the flow of empty cars which are returned from consignees to points where they are to receive new cargo. Because of the lack of balance in the directional flow of loaded cars, empty cargo cars accumulate at terminals which are situated in "surplus" areas, that is, areas which receive a greater number of loaded cars than are sent out. These cars are transported, without return loads, back to terminals in "deficit" areas. For these reasons, the transport of empty cars will be viewed as a service performed by the railway firm; the proportion of a firm's resources devoted to the transport of empties will depend upon the geographic location of the railway firm with respect to the directional flow of loaded traffic.

Under present arrangements in the industry, each railway firm is obligated to return empty to deficit areas those cars which it delivered and for which it could not find return loads. In this way each road pays a proper share of the costs of returning empty cars, even if the optimal method of rerouting empty cars may not be achieved. By "proper share" is meant that each firm's share of the burden of returning empties is in the same proportion to the cost of originally transporting the loaded cars to surplus areas.

Adjustable and unadjustable variables. For the purpose of specifying those variables which are to be determined by the relations contained in the model, the inputs and outputs have been divided into three main categories: outputs, unadjustable inputs, and adjustable inputs.

The quantity of *outputs* to be produced is treated as being beyond the control of the firm. This follows from the observation that railway rate schedules are registered with a public authority and remain unchanged for long periods, while demand conditions are, for the most part, determined by conditions in the rest of the economy. The railway firm is obliged, as a legally defined common carrier, to transport whatever cargo

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[5] The problem of finding an optimal routing for the return of empty cars is similar to that of finding an optimal route for empty cargo ships. For a treatment of the latter problem, see T. C. Koopmans, "Optimum Utilization of the Transportation System," ECONOMETRICA, Vol. 17, Supplement, July, 1949, pp. 136–145. (Reprinted as Cowles Commission Paper, New Series, No. 34.)

[6] The rules regulating the return of empty cargo cars are contained in *Code of Car Service Rules*, Operations and Maintenance Department, Association of American Railroads, Chicago, Illinois, 1946.

is offered. This assures us that the railway firm will attempt to supply a quantity of transport services equal to the quantity demanded.

The *unadjustable inputs* are the variables which measure the capacity of railway firms. This category includes the roadbed, rolling equipment, and other types of capital equipment. Each firm has been endowed with these as a result of past investment decisions and their quantity cannot be altered in the short run. In the statistical model, therefore, no attempt is made to explain the relative quantities of capital equipment used by each firm. From this point of view, the railway firm is restricted in the short run to choosing the quantities of *adjustable inputs* necessary to produce the given output with a predetermined stock of fixed and rolling equipment.

Form of the production function. The statistical models have been constructed using the hypothesis that there is only one group of adjustable inputs which will, for given capacity, produce the desired output without waste. This is not to be confused with the hypothesis of fixed proportions, under which the inputs bear the same ratio to each other for all values of output. Rather, this is an assumption that an independent relation exists between each input and the set of output and capacity variables.

The form of this relation is

$$\begin{aligned} Y_1 &= f_1(Z_1, Z_2, \cdots, Z_n), \\ Y_2 &= f_2(Z_1, Z_2, \cdots, Z_n), \\ \cdots & \cdots\cdots\cdots\cdots\cdots \\ Y_m &= f_m(Z_1, Z_2, \cdots, Z_n). \end{aligned}$$

The Y represent the minimum values of the adjustable inputs, and the Z represent the outputs and the unadjustable inputs. In both the line-haul and switching models, four adjustable inputs are specified, so that there are four production equations for each process. It is also assumed that the form of these relations is linear.

It is clear from the specification of the form of the production relations that a sharp distinction has been drawn between the available stock of unadjustable inputs and the flow of services which may be derived from them in a certain period. In this view, both the stock and the flow are included as relevant variables, the former among the Z's, the latter among the Y's. Therefore, it is possible for an increase in output to call forth a greater flow of services from a given stock of capital equipment.

Choice of variables. In the line-haul model, the following variables have been chosen to measure the flow of input and output services and the stock of unadjustable inputs.

For the *line-haul process* the adjustable inputs are represented by Y_1, labor services, measured in man-hours employed in freight service;

Y_2, fuel consumption, measured in equated net tons of fuel consumed in freight service; Y_3, flow of equipment services, measured by the expenditures on maintenance of freight equipment, exclusive of depreciation; and Y_4, flow of track and structure services, measured by expenditures on maintenance of track and structure for freight service purposes, exclusive of depreciation. The outputs are represented by Z_1, loaded freight-car miles; Z_2, carloads of freight; and Z_3, empty freight-car miles. The unadjustable inputs are represented by Z_4, total tractive capacity of freight locomotives, measured by thousands of pounds of tractive effort; and Z_5, miles of main line track. In an alternative specification of the model, a variable was employed to measure all of the physical capital in use by the railway firm. This was E_1, cost of reproduction, at 1948 prices, of physical assets, exclusive of land.

In the *switching process* the following variables were used to measure the flow of input and output services. The adjustable inputs are represented by Y_5, Y_6, Y_7, and Y_8. The adjustable input categories are the same as those chosen for the line-haul process. In this case, of course, the input services are employed in yard switching rather than freight service. The outputs are represented by Z_7, yard switching locomotive miles; Z_8, yard switching locomotive hours; and Z_9, carloads of freight. The unadjustable inputs are represented by Z_{10}, miles of yard switching track; Z_{11}, total tractive capacity of yard locomotives, measured by thousands of pounds of tractive effort of yard locomotives; and Z_{12}, average number of freight cars standing on the line.

II. STATISTICAL RESULTS

In general, the following statistical results should be expected: first, that an increase in any output variable will call forth an increase in the services of the adjustable inputs; further, that an increase in the stocks of equipment will normally call forth an increase in the use of some adjustable inputs and a decrease in others. In balance, however, it is to be expected that an increase in the stocks of equipment would lower the total cost of hiring input services to produce a given output. If this were not the case, then too much equipment would be in use and resources would be wasted.

Statistical models. Two specifications of statistical models were set up and their coefficients estimated. In the first model, each adjustable input is treated as a linear function of the unadjustable inputs and of the outputs. This embodies the hypothesis that the level of employment of adjustable inputs can be explained by the level of output and capacity of firms. The second model was derived from the first by normalizing the variables on a measure of capacity. For the line-haul process, the variables were divided by the cost of reproduction of all capital equipment employed by the firm. For the switching process, the variables were divided by the miles of track devoted to yard switching. The second model embodies the hypothesis that the level of employment of adjustable inputs per unit of capacity can be explained by the level of output per unit of capacity.

LINE-HAUL PROCESS—MODEL I

Y	Z_1	Z_2	Z_3	Z_4	Z_5	Constant
Y_1	29.3	7.5	−26.4	*−3.5*	*238.9*	−1,860,000
	(8.15)	(1.06)	(13.28)	(45.0)	(217.8)	(484,000)
Y_2	2.02	*0.092*	*0.019*	25.74	28.5	*−24,800*
	(0.56)	(0.073)	(0.913)	(3.10)	(14.97)	(33,299)
Y_3	50.85	2.32	−23.61	259.38	−1,361.26	*−685,387*
	(7.18)	(0.93)	(11.71)	(39.81)	(191.98)	(426,964)
Y_4	19.20	2.78	21.74	−224.20	522.38	*−181,444*
	(6.707)	(0.869)	(10.94)	(37.19)	(179.36)	(398,900)

LINE-HAUL PROCESS—MODEL II*

Y'	Z_1'	Z_2'	Z_3'	Z_4'	Constant
Y_1'	20.43	*0.508*	*−13.18*	−97.57	0.0234
	(5.017)	(0.365)	(10.35)	(44.66)	(0.003)
Y_2'	2.469	*0.0245*	*−0.335*	16.272	0.0007
	(0.485)	(0.035)	(1.00)	(4.316)	(0.0003)
Y_3'	24.41	1.126	*−0.4533*	*57.52*	0.0103
	(7.056)	(0.5139)	(14.558)	(62.803)	(0.0043)
Y_4'	20.32	0.882	*1.792*	*−49.32*	0.0222
	(6.038)	(0.4398)	(12.45)	(53.74)	(0.003)

* The primed variables are the same variables as in Model I but normalized on E_1.

The data consist of observations on seventy-six Class I railways in the United States for the year 1948.[7]

The coefficients of the equations for each model were estimated by

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[7] Observations on Y_1, Y_5, Z_4, and Z_{11} were compiled from the individual annual reports of seventy-six Class I railways to the Interstate Commerce Commission for the year 1948.

Observations on Y_2, Y_6, Z_8, and Z_{12} were obtained from the report, *Comparative Statement of Railway Operating Statistics for the years ended December 31, 1948 and 1947*, Statement No. 49200, Bureau of Transport Economics and Statistics, Interstate Commerce Commission, Washington, D. C., July, 1949.

Observations on Y_3, Y_4, Y_7, Y_8, Z_1, Z_3, Z_5, Z_7, and Z_{10} were obtained from

SWITCHING PROCESS—MODEL I*

Y	Z_7	Z_9	Z_{10}	Z_{11}	Z_{12}	Constant
Y_5	1.06	*−0.12*	−2,332.9	323.22	1,624.4	*−77,397*
	(0.113)	(0.438)	(607.9)	(43.8)	(24.9)	(145,188)
Y_6	0.024	*0.0088*	*97.5*	*2.69*	*3.71*	*−19,912*
	(0.0097)	(0.038)	(52.3)	(3.77)	(2.14)	(12,500)
Y_7	0.099	*0.217*	1,141.7	53.7	−39.11	*−43,571*
	(0.044)	(0.171)	(237.7)	(17.12)	(4.74)	(56,792)
Y_8	0.27	1.20	−2,010 3	*8.41*	36 5	−206,121
	(0.071)	(0.28)	(382.7)	(27.6)	(15.7)	(91,419)

* Z_8 was dropped from the set of independent variables because it was found to be almost perfectly correlated with Z_7.

SWITCHING PROCESS—MODEL II*

Y'	Z_7'	Z_9'	Z_{11}'	Z_{12}'	Constant
Y_5'	1.09	*−0.094*	*37.36*	29.43	−1,069.97
	(0.055)	(0.0958)	(24.32)	(11.00)	(388.55)
Y_6'	0.059	*0.0206*	−16.59	*0.458*	*−12.85*
	(0.007)	(0.012)	(3.144)	(1.423)	(50.23)
Y_7'	0.45	*0.046*	2.626	4.716	−990.99
	(0.0784)	(0.1367)	(1.098)	(1.571)	(554.70)
Y_8'	*0.0329*	*0.0169*	*17.56*	14.64	*289.20*
	(0.0315)	(0.0549)	(13.94)	(1.99)	(222.66)

* The primed variables are the same variables as in Model I but normalized on Z_{10}. Z_8' was dropped from the set of independent variables because it was found to be almost perfectly correlated with Z_7'.

least-squares methods. Under the hypothesis that both outputs and unadjustable inputs are determined independently of the railway firms' actions, the least-squares technique will provide the best estimates of the

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Section A-1 of *Statistics of Railways in the United States for the year ended December 31, 1948*, Bureau of Transport Economics and Statistics, Interstate Commerce Commission, Washington, D. C., 1950.

Observations on Z_2 and Z_9 were obtained from *Freight Commodity Statistics, Class I Steam Railways in the United States for the year ended December 31, 1948*, Statement No. 49100, Bureau of Transport Economics and Statistics, Interstate Commerce Commission, Washington, D. C., 1949.

Observations on E_1 were obtained from Exhibit 1, Ex Parte 168, "Elements of Value of Property Used in Common Carrier Service, January 1, 1948, Class I Line-Haul Railways," Bureau of Valuation, Interstate Commerce Commission, Washington, D. C.

true coefficients.[8] The sample regression coefficients are shown above. Those coefficients are in italics which are not significantly different from zero at the 96% significance level. The estimates of the standard errors of the regression coefficients are written below the coefficients in parentheses.

From the estimated regression coefficients, elasticities of factor use have been derived. The elasticity of factor use is defined to be the percentage change in factor use accompanying a one percent change in all of the output variables, holding constant the stocks of equipment. Decreasing returns to scale are implied by a value of the elasticity coefficient which is greater than unity, while increasing returns are implied by a value of the coefficient which is less than unity. The elasticity of variable cost with respect to output has also been estimated. This is

Estimated Elasticity Coefficients with their Confidence Regions	Adjustable Input	Elasticity of Factor Use	Elasticity of Variable Cost	68% Confidence Region about Estimates
Line-Haul Process—Model I	Y_1	1.1531		±0.1364
	Y_2	0.5745		0.0734
	Y_3	0.9808		0.0895
	Y_4	0.9247		0.0908
			0.9376	±0.0983
Line-Haul Process—Model II	Y_1'	0.4528		±0.1193
	Y_2'	0.5508		0.0957
	Y_3'	0.6533		0.1346
	Y_4'	0.5389		0.1102
			0.5515	±0.1169
Switching Process—Model I	Y_5	0.9204		±0.1403
	Y_6	0.6659		0.0753
	Y_7	0.7749		0.1082
	Y_8	1.6082		0.3086
			1.0294	±0.1674
Switching Process—Model II	Y_5'	0.9687		±0.0595
	Y_6'	1.0346		0.1328
	Y_7'	0.4342		0.1701
	Y_8'	1.5430		0.0598
			0.9911	±0.1041

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[8] For a development of the conditions under which least-squares techniques lead to optimal estimates, and for definitions of these optimal properties, see Leonid Hurwicz, "Prediction and Least-Squares," Chapter VI in *Statistical Inference in Dynamic Economic Models*, Cowles Commission Monograph 10, T. C. Koopmans, ed., New York: John Wiley and Sons, 1950, pp. 266–300.

defined to be the percentage change in variable cost accompanying a one percent change in the output variables, holding constant the stock of equipment. The elasticity of variable cost with respect to output is a weighted average of the elasticities of factor use. The weights consist of the proportion of total variable cost expended on the particular inputs and were derived from the totaled expense accounts of the entire railway industry. The proportion of total variable cost spent on each input was found to be 25.56% for operating labor, 17.57% for fuel, 28.49% for equipment repair, and 28.38% for way and structure repair.

The elasticities of factor use for each adjustable input and the elasticities of cost with respect to output are shown below. Confidence regions for the elasticity coefficients are also indicated. A technique developed by M. A. Girshick was used to estimate the confidence regions.[9]

Both Models I and II of the switching process indicate a value of the cost elasticity coefficient which is not significantly different from unity, implying constant returns to scale. However, there is a considerable difference between the cost elasticity coefficients indicated by the two models of the line-haul process; the first model indicates a coefficient close to unity, while the second model shows a coefficient of one half, implying increasing returns to scale.

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[9] M. A. Girshick, "The Application of the Theory of Linear Hypotheses to the Coefficient of Elasticity of Demand," *Journal of the American Statistical Association*, Vol. 37, June, 1942, pp. 233–237.